

Stochastic Modelling of Interest Rate Dynamics: An Expository Note

Rogemar S. Mamon

This paper discusses the modelling of the term structure of interest rates, which refers to the relationship between the interest rate and the term to maturity of an underlying financial instrument such as a zero-coupon bond, a Treasury-note or a Treasury-bill. Interest rate modelling plays an important role in economic and financial theory and has long been a topic of concern to economists.

On the microeconomic level, interest rate is the most significant risk that any financial institution has to cope with. Therefore, an understanding of the yield curve movement is necessary in pricing interest rate sensitive securities and in the management of the associated interest rate exposures. Knowledge of interest rate dynamics is also invaluable to market practitioners. Market rates such as returns on risk-free and liquid securities (e.g. non-callable US Treasury bills and bonds) are used by investment banks as benchmarks for examining other key financial rates. As well, term structure of interest rates is of a paramount importance to corporate treasurers, who must decide whether to borrow by issuing long- or short-term debt, and to investors who must decide whether to buy long- or short-term bonds. On the macroeconomic level, previous research studies show that a forecast of economic growth can be made based on the short-rate and long-term interest rate behaviours.

Statistical investigations have documented modest, but reliable positive correlations between the slope of the term structure (differences between long- and short-term interest rates) and futures of economic growth. This is further substantiated in Bomhoff (1994), Estrella and Hardouvelis (1991), and Harvey and Campbell (1991). In addition, findings from previous studies reveal that the term structure of interest rates can reasonably determine market expectations of future interest rates and inflation. Information derived from market expectations is translated to major economic indicators for central banks and the Federal Reserve Bank, guiding them in their function to control and influence the levels of interest rates or when adopting monetary policies (see for example Estrella and Mishkin, 1995; Babbs and Weber, 1997; El-Jahel, Lindberg and Perraudin, 1997; and Hardouvelis, 1998). These macroeconomic and microeconomic reasons make modelling of interest rate dynamics one of the most fertile research areas in economics and finance.

Prior to the theory of dynamic asset pricing that was developed using the fundamental ideas of Black and Scholes (1973) and Merton (1973), the most prevalent explanation of the term structure is the expectations theory. A comprehensive examination of this theory can be found in Malkiel (1966). This theory asserts that the expected returns on bonds of different maturities are equalised, or that they differ only by constant term premia. It also propounds the argument that long rates are given by the current and expected short rates. Accordingly, the spread between long rates and short rates reflects the market's forecast of changes in short rates. A relatively extensive literature deals with this point of view and related works attempt to understand the determinants of interest rates from this perspective.

On the other hand, the topic has attracted increased interest in recent times due to its close link with the valuation of interest rate derivatives and bond pricing under the risk-neutral framework. The approach used in this setting is to start with a plausible stochastic process for the short-term rate and explore the process implied for bond and option prices. Heath, Jarrow and Morton (1992) proposed