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VALIDATING THE AUTOREGRESSIVE MODEL OF THE ANGAT RESERVOIR MONTHLY INFLOWS

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ABSTRACT

Thirty six years of historical streamflow records (1946 to 1985), before and after construction of the Angat Reservoir, were used to identify the time series model that can forecast the Angat Reservoir monthly inflows. After more than twenty (20) years when the autoregressive model of the Angat Reservoir inflows was identified in a previous study by this author, its effectiveness and practicality to forecast the monthly inflows to the Angat Multipurpose Reservoir is validated by comparing the generated model outputs with recent observed measurements from 1986 to 2008. The paper presented the ARMA model selection process and showed the validity of the selected autoregressive model..

1. INTRODUCTION

A collection of observations of a hydrologic process made sequentially in time constitutes a time series. Modeling a hydrologic time series is generating a synthetic sequence that can be used in the design and operation of a water resource system such as the Angat Multipurpose Reservoir. If properly calibrated, the time series model of the Angat Reservoir inflows can be used in the analysis, planning, and help in real-time operation by forecasting future reservoir inflows. The main requirement is that the generated flows should agree with the historical data with respect to their population means, variances, correlations, and other simple statistical properties.

If a time series can be exactly computed using a mathematical model, such model is called a deterministic model. However, most time series are stochastic in that future values are only partly determined by past values. A time series model that is used to compute the probability of a future value lying between two specified limits is called a probability model or a stochastic model.

In time series modeling, stochastic or time series models are fitted to a hydrologic time series such as sequences of streamflows and precipitation. The study shows the results of the stochastic modeling derived from 36 years of historical data of the Angat Reservoir inflows, 18 years (1946 to 1963) before and 18 years (1968 to 1985) after construction of the reservoir.

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After more than twenty (20) years when the autoregressive model of the Angat Reservoir inflows was determined, its effectiveness and practicality to forecast the monthly inflows to the Angat Multipurpose Reservoir is validated by comparing the generated model outputs with recent observed measurements from 1986 to 2008. The detailed ARMA model selection process is presented and the validity of the selected model is determined.

2. ARMA(p,q) MODELS

Consider the stationary time series x_t, x_{t-1}, x_{t-2} . . ., normally distributed with mean μ and variance σ^2, observed at equally spaced times t, t-1, t-2, ... The stationary time series has the property of remaining in equilibrium about a constant mean and variance, that is, shifting the time origin by an amount k does not affect the properties of the time series. According to Box and Jenkins [1966], the series, in which observations are highly correlated, can be transformed to an independent series of random shocks a_t, a_{t-1}, …. These uncorrelated normal shocks, with mean zero and variance σ_a^2 are called the white noise of the stochastic process. The noise a_t is also called model residual or error. This transformation can be accomplished in two ways.

First, the standardized observation, $z_t = (x_t-\mu)/\sigma$, can be made linearly dependent on finite previous observations z_{t-1}, z_{t-2}, … and on the noise a_t. This is called an autoregressive model of order p, denoted by AR(p), and may be written as

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots + \phi_p z_{t-p} + a_t \qquad (1)$$

where ϕ_1, . . .ϕ_p are the autoregression coefficients of the AR(p) model. The order of the AR model tells how many lagged past values are included. Defining a backward shift operator B,

$$Bz_t = z_{t-1} \text{ and } B^2 z_t = BBz_t = Bz_{t-1} = z_{t-2}$$
$$\text{or in general } B^m z_t = z_{t-m}$$

and the AR(p) model using the backward shift operator B is,

$$z_t - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \ldots - \phi_p z_{t-p} = a_t$$
$$(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)\, z_t = a_t$$
$$\phi(B) z_t = a_t \qquad (2)$$

The other way of transforming the time series z_t is by letting the series be a linear combination of a finite number q of previous random shocks a_t's. This is called the moving-average model of order q, MA(q). It can be written as

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \qquad (3)$$

where $\theta_1, \ldots \theta_q$ are the moving-average coefficients of the MA(q) model. Using the operator B,

$$z_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)\, a_t$$

$$z_t = \theta(B) a_t \qquad (4)$$

To select the best model for the stationary time series, sometimes it is necessary to have both autoregressive and moving-average terms in the model. An autoregressive model AR(p) and a moving-average model MA(q) can be combined to obtain the autoregressive moving-average (ARMA) model of order (p,q). The ARMA(p,q) model is defined as

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots + \phi_p z_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \qquad (5)$$

$$\phi(B) z_t = \theta(B) a_t \qquad (6)$$

3. STEPS IN MODELING

The ARMA modeling process is generally an iterative, trial and error process. Thus, it is necessary to use the least possible number of parameters that will adequately produce forecasted values with similar statistics of the historical data. Applying this principle of parsimony will result in a model with the smallest possible error.

ARMA modeling proceeds by a series of well-defined steps. The modeling process is divided into four main steps; (1) model identification, (2) parameter estimation, (3) model verification or diagnostic checking, and (4) forecasting. Identification consists of examining the data to see which model appears to be most appropriate, based on the comparison of the computed statistical properties of the original data and known theoretical behavior of often used ARMA(p,q) models. During the second phase, the coefficients of the candidate models identified in the initial phase are estimated. Under model verification, the candidate models are checked for possible inadequacies and lack of fit, selecting the most adequate and parsimonious model in the process. And in the final phase, after selecting the best model, the results of the forecast functions are compared with the observed model verification data.

4. THE ANGAT RESERVOIR INFLOWS

The Angat Hydroelectric plant is located in Sitio Bininit, San Lorenzo, Bulacan. The Angat watershed , with a drainage area of 568 km^2, is bounded by the Umiray River Basin in the northeast, the Sierra Madre Mountain Range in the east, the Kanan River Basin in the southeast, and the Marikina River Basin in the south. The Angat River project is basically a power project, but is multipurpose in concept. It was operational since 1967 and the plant is the first project intended to fully utilize the resources of the Angat river for power generation, irrigation, water

supply, and flood control. To meet the requirements of the various functional activities and at the same time maximizing the water releases from the Angat reservoir, the need to develop forecast models to predict the streamflows should be taken into full consideration.

Initially, a total of 36 years of monthly streamflows from the Angat river were gathered for the study. The pre-construction inflows, from January 1946 to December 1963, were collected from the NPC main office, while the post-construction inflows, from January 1968 to December 1985, were computed using the daily measurements of reservoir releases and elevations collected at the Angat Reservoir damsite. The Angat Reservoir ARMA inflow model was developed using these monthly inflows and the computer programs that were developed by Jose D. Salas and Ricardo A. Smith of Colorado State University. Some of the programs were modified and additional programs were developed by this author to suit the modeling procedure followed in the study.

Eighteen years (1946-1963) of historical records before construction of the Angat Reservoir were used for model calibration. During construction, there was no available data, hence, 18 years of monthly streamflow data (1968-1985) were used for model verification.

After more than twenty (20) years when the autoregressive model of the Angat Reservoir inflows was determined in 1987 by this author, the selected forecast function is validated using the observed data from 1986 to 2008. The detailed ARMA model selection process and the results of a previous study by this author are presented.

5. ARMA MODELING OF THE ANGAT RESERVOIR MONTHLY INFLOWS

5.1 Step 1 Model Identification

5.1.1 Preliminary Analysis

Series of monthly streamflows and other hydrologic variables usually have periodic components equal to 12 months in both means and standard deviations. When these periodic components are removed, the resulting component of the non-stationary series can be considered a stationary series. A time series can be modeled by the ARMA process only if the series is stationary and assumed to be normally distributed. Normally distributed data should possess no significant skewness.

One way of removing the non-stationarity of the series is by cyclic or seasonal standardization. This is accomplished by subtracting the seasonal mean on the series and dividing the result by the seasonal standard deviation. To normalize a time series, a suitable transformation of the series is sometimes needed. A simple logarithmic or power transformation is usually applied in ARMA modeling.

An examination of the Angat monthly streamflows in megacubic meters (MCM) shows the two seasons prevailing in the country; the dry and wet season. Streamflows for the months of February to May (dry season) are low compared to the months of July to December (wet season). To correct this, the common approach for modeling a seasonal time series is to first deseasonalize the series by standardization.

5.1.2 Autocorrelation Function (ACF) of Untransformed Series (1946-1963)

The autocorrelation function, sometimes called the correlogram, measures the amount of linear dependence between observations in a time series that are separated by lag k. The ACF, which is a plot between the autocorrelation coefficient ρ_k as ordinate against lag k as abscissa, shows the general character of a time series.

The autocorrelation function (ACF) of the original data, without transformation, was computed for a maximum lag of 24 and a confidence level of 95%. An examination of the plot shown in Fig. 1 revealed that the ACF exhibited a wave pattern with peaks at lags 12 and 24 and lows at lags 6 and 18. The pattern was repeated for a period of 12 lags. The behavior of the ACF was due to the effects of the low inflows during the dry season and high inflows during the wet season.

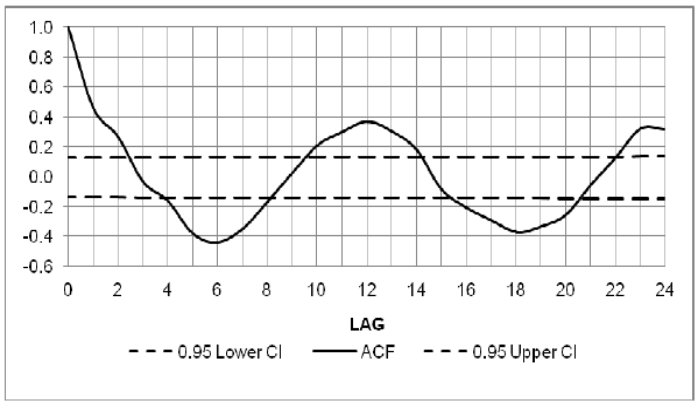


Fig. 1 Autocorrelation Function of Original Series

5.1.3 Transforming and Standardizing the Series

To make the monthly time series approximately normal and stationary, the model calibration data was transformed logarithmically and cyclic standardization was applied with a period equal to 12.

(1) Logarithmic Transformation

$$y_t = \log (x_t) \qquad (7)$$

where y_t is the transformed monthly inflow and x_t is the original inflow in MCM

(2) Standardization : seasonal standardization

$$z_{D,W} = \frac{y_{D,W} - \mu_W}{\sigma_W} \qquad (8)$$

where $z_{t,w}$ is the standardized inflow, μ_w is mean , and σ_w is standard deviation of month w

Table 1 lists the monthly means and standard deviations for the logarithmically transformed series. The log-transformed periodically standardized series had an overall mean of zero and standard deviation of 0.974. A visual inspection of the graphs showed that the new series is relatively stationary and a check on the normality for a confidence level of 95% showed that the series is normal.

Table 1 Mean μ_w and Standard Deviation σ_w of Log-Transformed Series

Month	Mean	Standard Deviation
January	2.19	0.18
February	1.89	0.17
March	1.83	0.20
April	1.64	0.25
May	1.64	0.26
June	1.92	0.26
July	2.21	0.26
August	2.45	0.26
September	2.41	0.20
October	2.41	0.33
November	2.42	0.31
December	2.39	0.31

Plots of ACF, PACF, IACF, and IPACF

Salas and Obeysekera [1982] described the properties of the autocorrelation (ACF), partial autocorrelation (PACF), inverse autocorrelation (IACF), and the inverse partial autocorrelation (IPACF) functions for different cases of ARMA models. Plots of these functions help determine the number of autoregressive and moving-average parameters that are needed for the model. Schematic forms of these functions are shown in Fig. 2 .

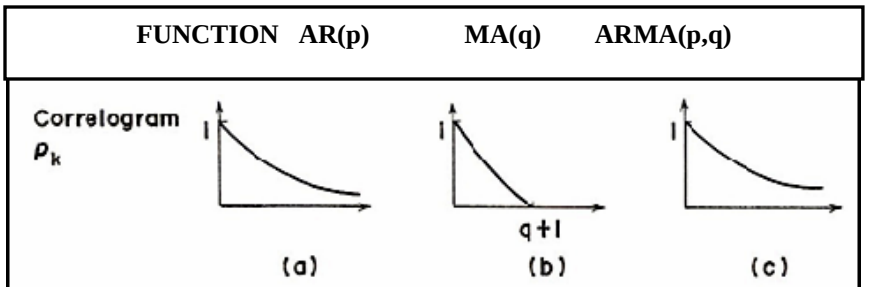
5.1.5 Visual Inspection of ACF, PACF, IACF, and IPACF of Standardized Series

The autocorrelation (ACF), partial autocorrelation (PACF), inverse autocorrelation (IACF), and the inverse partial autocorrelation (IPACF) functions were calculated for lags 0 to 24 using a 95% confidence level to identify the candidate models to be entertained. The ACF, shown in Fig. 3, did not truncate but rather damps out, suggesting the presence of autoregressive (AR) terms. It was positively significant at lag 1 and had values at lags 2 and 3 that touched the confidence limits.. These observations were also illustrated in the plot of IPACF. The PACF and IACF (Fig. 4) possessed significant values at lags 1 and 3. From these observations, it was appropriate to try the AR(1), AR(2), and AR(3) models. A closer look at the PACF showed that the plot did not truncate as in the case of a pure AR(p) model, therefore, it appeared reasonable to include the ARMA(1,1) and ARMA(1,2) models.

Step 2 Parameter Estimation

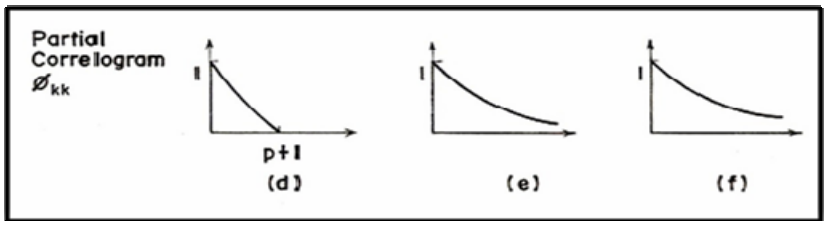
5.2.1 Preliminary Estimates of Parameters

Given a set of candidate models, parameters were first roughly estimated and then refined by using several iterative procedures. Using ρ_k as the ACF at lag k and σ_z^2 as the



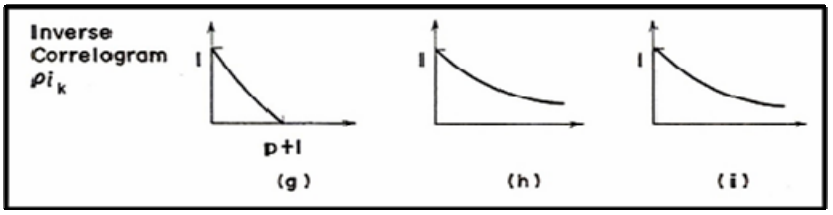
Autocorrelation Function (Correlogram)

For a MA(q) process, ρ_k cuts off and is not significantly different from zero after lag q. For an AR(p) or an ARMA (p,q) process, ρ_k damps out with shapes depending on the values of p and q and the values of the parameters. If ρ_k tails off and does not truncate, this suggest that AR terms are needed to model the time series.



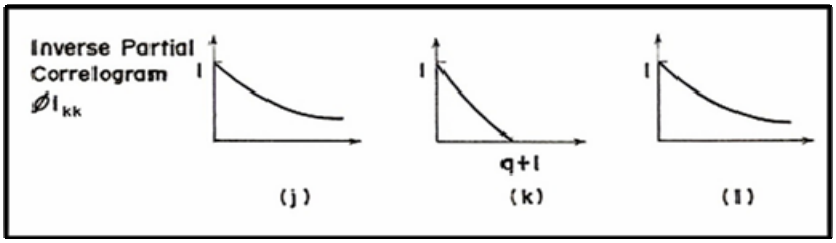
Partial Autocorrelation Function

For an AR(p) process, ϕ_{kk} truncates and is not significantly zero after lag p. If ϕ_{kk} tails off, this implies that MA terms are required.



Inverse Autocorrelation Function

For an AR(p) process, $\rho i_k = 0$ for $k > p$. . For a MA(q) or an ARMA (p,q) process, ρi_k damps out, this suggest the presence of a MA component.



Inverse Partial Autocorrelation Function

For a MA(q) process, $\phi i_{kk} = 0$ for $k > q$. If ϕi_{kk} dies off rather than cuts off, then AR terms are required.

Fig. 2 Schematic Forms of the Functions: ACF, PACF, IACF, and IPACF for ARMA Models [Source: Salas and Obeysekera (1982)]

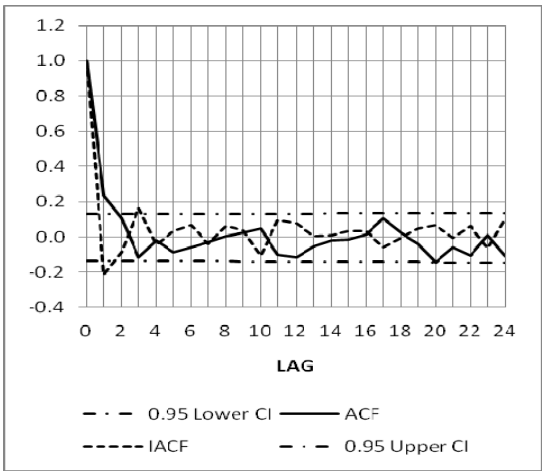


Fig. 3 ACF and IACF of Standardized Series

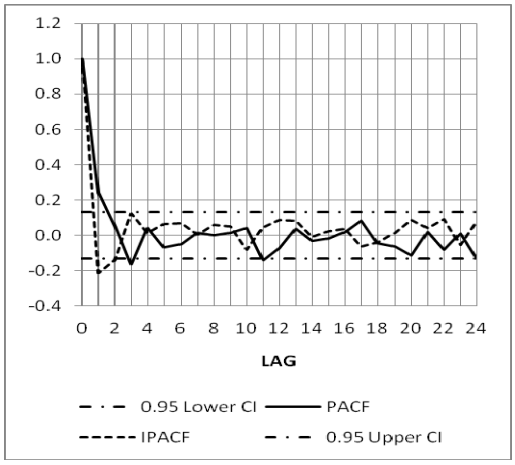


Fig. 4 PACF and IPACF of Standardized Series

variance of sample, the preliminary estimates of model parameters for simple ARMA models are as follows:

AR(1) Model

$$\phi_1 = \rho_1 \quad (9)$$

$$\sigma_z^2 = \sigma_a^2/(1-\rho_1^2) \quad (10)$$

AR(2) Model

$$\phi_1 = (\rho_1 - \rho_1\rho_2)/(1-\rho_1^2) \quad (11)$$

$$\phi_2 = (\rho_2 - \rho_1^2)/(1-\rho_1^2) \quad (12)$$

$\sigma_z^2 = \sigma_a^2/(1-\rho_1\theta_1-\rho_2\theta_2)$ (13)

AR(3) Model

$\phi_1 = (\rho_1 - \rho_1\rho_2)/(1-\rho_1^2)$ (14)

$\phi_2 = (\rho_2 - \rho_1^2)/(1-\rho_1^2)$ (15)

$\phi_3 = 0.0$ (16)

$\sigma_z^2 = \sigma_a^2/(1-\rho_1\theta_1-\rho_2\theta_2)$ (17)

ARMA(1,1)

$\phi_1 = \rho_2/\rho_1$ (18)

$\rho_1 = [(1-\theta_1\phi_1)(\phi_1-\theta_1)]/(1+\theta_1^2-2\phi_1\theta_1)$ (19)

$\sigma_z^2 = \sigma_a^2(1-\theta_1^2-2\phi_1\theta_1)/(1-\phi_1^2)$ (20)

Using equations (9) to (20), the preliminary estimates of the autoregressive ϕ and the moving-average θ coefficients, white noise or residual variance σ_a^2 of the five (5) candidate models were computed and shown in Table 2.

Table 2 Preliminary Estimates of Parameters

Model	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	σ_a^2
AR(1)	0.2395					0.8392
AR(2)	0.2263	0.0551				0.8341
AR(3)	0.2263	0.0551	0.0			0.8341
ARMA(1,1)	0.4563			-0.230		0.5856
ARMA(1,2)	0.3424			0.1117	-0.085	0.8878

5.2.2 Maximum Likelihood Estimation of Parameters

After obtaining the preliminary estimates of the p AR coefficients ϕ, q MA coefficients θ, and the white noise variance σ_a^2, the maximum likelihood estimates of the parameters were determined. These estimates will be used in forecasting, if the candidate models pass a series of tests under model verification. To get likelihood estimates, the residuals were computed from the difference equation of the ARMA(p,q) process as

$$a_t = z_t - \phi_1 z_{t-1} - \ldots - \phi_p z_{t-p} + \theta_1 a_{t-1} + \ldots + \theta_q a_{t-q} \quad (21)$$

The sum of the squares of the residuals $S(\phi,\theta)$ was then computed for several sets of (ϕ,θ) and the sum $S(\phi,\theta)$ was minimized using the modified steepest descent method. The resulting coefficients ϕ and θ are called the maximum likelihood estimates. The computed parameters for the candidate models were as follows,

Table 3 Autoregressive and Moving-Average Coefficients of Candidate Models

	AR(1)	AR(2)	AR(3)	ARMA (1,1)	ARMA (1,2)
ϕ_1	0.2401	0.2248	0.2337	0.3424	0.0967
ϕ_2		0.0586	0.0907		
ϕ_3			-0.1671		
θ_1				0.1086	-0.1408
θ_2					-0.1331
σ_a^2	0.8838	0.8805	0.8561	0.8823	0.8722

5.2.3 *Visual Comparison of Sample and Population ACF and PACF*

The population ACF and PACF for the different candidate models were calculated and plotted. Based on the comparison between the sample and population ACFs and PACFs, shown in Fig. 5, the AR(3) model can best represent the sample. For lags 1 to 10, the plots of ACF and PACF of the AR(3) model followed the shapes the sample ACF and PACF.

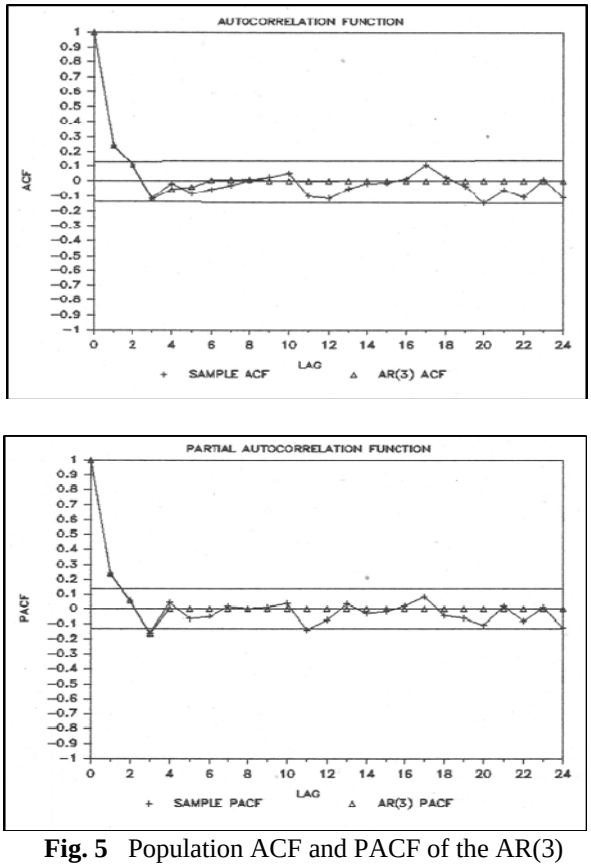


Fig. 5 Population ACF and PACF of the AR(3)
[Source: Dizon (1987)]

5.3 Step 3 Model Checking

In order to test the adequacy of the candidate models identified, the models underwent the following diagnostic checks; (1) Akaike Information Criterion (AIC) Test, (2)Test for stationarity of AR parameters, (3) Test for invertibility of MA parameters, (4) Whittle Overfitting Test, and (5) Test for independence and normality of residuals.

5.3.1 *Akaike Information Criterion Test*

It is very difficult to choose among candidate models on the basis of the sample data when the more than one model being tested fit the data equally well. A way of selecting between candidate models is by applying the principle of parsimony, which requires a model with the smallest number of parameters. A formula which considers this principle is the Akaike Information Criterion (AIC) of an ARMA(p,q) model and is defined as ,

$$AIC(p,q) = N \ln \sigma_a^2 + 2(p+q) \quad (22)$$

where N is the sample size

Ozaki [1977] presented another definition of the AIC that usually gives a positive value,

$$AIC(p,q) = N \log \sigma_a^2 + 2(p+q+2) + N \log 2\pi + N \quad (23)$$

In order to pass the AIC test, the AIC of the candidate model ARMA(p,q) must have the lowest value compared to the AICs of ARMA(p+1,q) and ARMA(p,q+1). Using Eq. 23 defined by Ozaki, the following table summarized the results of the AIC test for the different candidate models,

Table 4 AIC Test on Candidate Models

Candidate Model ARMA(p,q)		ARMA(p+1,q)	ARMA(p,q+1)	AIC Test
Model	**AR(1)**	AR(2)	ARMA(1,1)	PASSED
AIC	382.82	384.47	384.66	
Model	**AR(2)**	AR(3)	ARMA(2,1)	**FAILED**
AIC	384.47	383.83	386.62	
Model	**AR(3)**	AR(4)	ARMA(3,1)	PASSED
AIC	383.83	385.60	388.73	
Model	**ARMA(1,1)**	ARMA(2,1)	ARMA(1,2)	PASSED
AIC	384.66	386.61	385.58	
Model	**ARMA(1,2)**	ARMA(2,2)	ARMA(1,3)	PASSED
AIC	385.58	387.28	385.66	

All candidate models, except AR(2), were adequate based on the AIC test. The AR(2) model was eliminated, reducing the candidate models to only four. Based on the values of the AIC, the AR(1) had the lowest followed by AR(3).

5.3.2 Test for Stationarity of Autoregressive Parameters

The AR(p) model using the backward shift operator B,

$$(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)\, z_t = a_t$$

is stationary if the set of AR parameters $\phi_1, \ldots \phi_p$ satisfy the so called stationary conditions. These conditions are satisfied if the roots of the equation for variable B,

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p = 0 \qquad (24)$$

lie outside the unit circle.

All four candidate models met the stationarity condition of the autoregressive parameters.

5.3.3 Test for Invertibility of Moving-Average Parameters

Box and Jenkins [1976] derived the conditions which the parameters $\theta_1, \ldots, \theta_q$ must satisfy to ensure the invertibility of the MA(q) process,

$$z_t = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)\, a_t .$$

The invertibility condition is satisfied when the roots of the equation for B,

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q = 0 \qquad (25)$$

lie outside the unit circle.

The two candidate models containing moving-average parameters, ARMA(1,1) and ARMA(1,2), met the invertibility condition of the parameters.

5.3.4 Whittle Overfitting Test

Once the stationarity and invertibility conditions of the parameters have been checked, the next step is to test the goodness of fit of the model selected. One way of accomplishing this is by overfitting. Overfitting involves fitting a more complicated model to see if adding more parameters improves the fit, that is, the added parameters are not significantly different from zero.

A method of checking the model adequacy is by the Whittle Overfitting Test done by fitting a high-order AR model of order r. Suppose the model being tested has k parameters where k=p+q and residual variance $\sigma_a^2(k)$, Hipel et. al.[1977] stated that,

$$\chi^2(r-k) = N \ln[\sigma_a^2(k)/ \sigma_a^2(r)] \qquad (26)$$

where N is the size of the series.

If the value of $\chi^2(r-k)$ is greater than $\chi^2(r-k)$ from Chi-square tables at a chosen confidence level, then a model with more parameters is needed.

The selected high-order AR model that was overfitted to test the adequacy of the candidate models is of the order 10 (r = 10). The computed value of the likelihood statistics given by Eq. 26 was compared to the Chi-square value at 95% confidence level, and all candidate models passed the Whittle Overfitting Test.

5.3.5 Residual Checks: Test for Independence and Normality

Two tests are commonly applied to check the independence of model residuals. One procedure is to examine the residual autocorrelation function (RACF) and another is to apply the Portemanteau Lack of Fit test.

If some of the values of the RACF are significantly different from zero or are outside the confidence limits, then the selected model is inadequate.

A second but less sensitive procedure to test the independence of the residuals is to compute the portmanteau statistic Q and comparing its value with the Chi-square value X^2(L-p-q) of a given confidence level. The portmanteau statistic Q is defined as,

$$Q = N \sum_{k=1}^{L} r_k^2(a) \qquad (27)$$

where N is the sample size, L=N/4, $r_k^2(a)$ is the square of RACF. If the statistic $Q < \chi^2(L\text{-}p\text{-}q)$, the residuals are independent and the model is adequate.

The RACF for the different models were plotted and shown in Figure 6.

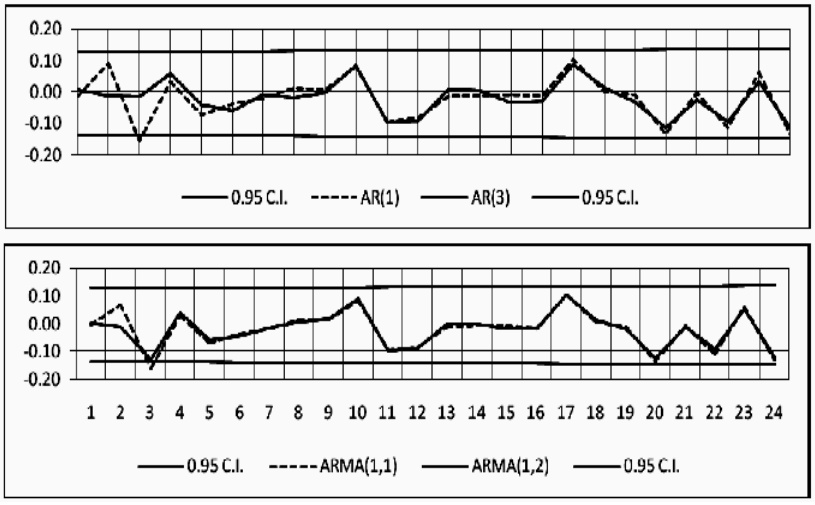


Fig. 6 Residual Autocorrelation Function of Candidate Models

An inspection of the RACF of AR(1) and ARMA(1,1) showed that at lag 3, the RACF was outside the 95% confidence limits. For all other lags, the RACF of the models were within the confidence limits. For the AR(3) and ARMA(1,2), the RACF at all lags were inside the confidence limits. Based on these observations, the AR(3) was the best, followed by the ARMA(1,2). For the other two models, it can be concluded that residuals were independent since most of their RACF were within the confidence limits. Computing the portmanteau statistic Q for all models and comparing with χ^2(L-p-q) resulted in,

AR(1) : Q = 58.53 $\chi^2(216/4 - 1) = 70.98$
residuals were independent
AR(3) : Q = 46.74 $\chi^2(216/4 - 3) = 68.66$
residuals were independent
ARMA(1,1) : Q = 57.18 $\chi^2(216/4 -1- 1) = 69.82$
residuals were independent
ARMA(1,2) : Q = 51.99 $\chi^2(216/4 -1- 2) = 68.66$
residuals were independent

The normality of the residuals was checked using the skewness coefficient test. For a confidence level of 95%, the computed skewness coefficients of the residuals for AR(1), AR(3), ARMA(1,1), and ARMA(1,2) are 0.100, 0.103, 0.102, 0.110 respectively. Salas et. al. [1980] compared the skewness coefficient to a tabulated value $G_{1-\alpha/2}(N)$. If the skewness coefficient $G < G_{1-\alpha/2}(N)$, the hypothesis of normality is accepted. The tabulated $G_{0.95}$ is 0.389, the model residuals for all candidate models were normal.

5.3.6 ARMA Candidate Models

The following candidate models passed all diagnostic checks and proceeded to the next phase which was forecasting:

AR(1) Model : $z_t = 0.2401\, z_{t-1} + a_t$
AR(3) Model : $z_t = 0.2337\, z_{t-1} + 0.0907\, z_{t-2} - 0.1671\, z_{t-3} + a_t$
ARMA(1,1) Model : $z_t = 0.3424\, z_{t-1} - 0.1086\, a_{t-1} + a_t$
ARMA(1,2) Model : $z_t = 0.0967\, z_{t-1} + 0.1408\, a_{t-1} + 0.1331\, a_{t-2} + a_t$

5.4 Step 4 Forecasting

After the three phases of model construction – model identification, parameter estimation, and diagnostic checks – the identified candidate models were used to make minimum mean square error forecasts for any lead time L. The one-step-ahead forecast (L=1) at time t, $z_t(1)$ is

$$z_t(1) = \phi_1 z_t + \phi_2 z_{t-1} + \ldots + \phi_p z_{t+1-p} - \theta_1 a_t - \theta_2 a_{t-1} - \ldots - \theta_q a_{t+1-q} \quad (28)$$

where $a_{t-j} = z_{t-j} - z_{t-j-1}(1)$ $\quad j = 0,1,2, \ldots$

The following were the 1-step-ahead forecast (lead-1) functions:

AR(1) Model : $z_t(1) = 0.2401\ z_t$

AR(3) Model : $z_t(1) = 0.2337\ z_t + 0.0907\ z_{t-1} - 0.1671\ z_{t-2}$

ARMA(1,1) Model : $z_t(1) = 0.3424\ z_t - 0.1086\ [z_t - z_{t-1}(1)]$

ARMA(1,2) Model : $z_t(1) = 0.0967\ z_t + 0.1408\ [z_t - z_{t-1}(1)] + 0.1331\ [z_{t-1} - z_{t-2}(1)]$

The verification data from January 1968 to December 1985 was periodically standardized using the monthly means and standard deviations of Table 1 . For the one-step-ahead(lead-1) forecast, the start of forecast was February 1946 (t=2) and ended on December 1963 (t=216) for the calibration data. For the verification data, the start of forecast was January 1968 (t=217) and ended on December 1985 (t=432).

After forecasting, the z_t series was converted into the unstandardized y_t series, then to the original untransformed x_t series. The results of the forecasts of the different candidate models were compared based on statistical properties: mean, standard deviation, correlation coefficient, root mean square error, and peak forecast error.

Analyzing the overall monthly statistics (Table 5), it can be seen that the AR(3) model had the closest value to the measurement with respect to the mean and standard deviation. Basically in all categories, the AR(3) model is the most appropriate model. It had the smallest root mean square error, peak forecast error and the highest correlation coefficient between forecasted and measured values.

Table 5 Overall Comparison of Candidate Models

Inflows : 1946 - 1985				
Parameter	**Model Forecast**			
(Measured Value)	**AR(1)**	**AR(3)**	**ARMA(1,1)**	**ARMA(1,2)**
Mean (175.50 MCM)	160.37	162.82	160.03	160.20
Error of Forecast (%)	8.62%	**7.23%**	8.82%	8.72%
Standard Deviation (163.87 MCM)	96.68	100.19	96.62	98.18
Error of Forecast	41.00%	**38.86%**	41.04%	40.08%
Root Mean Square Error (MCM)	129.49	**128.61**	129.47	129.19
Correlation Coefficient	0.6192	**0.6234**	0.6198	0.6215
Mean of Peak Forecast Error (%)	47.24%	**45.96%**	47.62%	47.99%

Therefore, judging from the comparative statistics and the results of the diagnostic checks, it can be concluded that the AR(3) model is the best among the candidate models to forecast the Angat Reservoir monthly inflows.

6. VALIDATING THE AR(3) MODEL OF THE ANGAT RESERVOIR INFLOWS

The adequacy of the AR(3) model to predict the monthly inflows of the Angat Reservoir is validated by testing the goodness of fit of the selected forecast model to the new series of monthly inflows from January 1986 to April 2008. Table 6 gives the overall monthly comparison between the old and new series. As seen in Table 6, there was a marked decrease in the overall mean of 158.58 MCM from 1986 to 2008 compared to the mean of 175.50 MCM from 1946 to 1985. This may be due to the effects of climate change, in particular, the episodes of the El Niño phenomenon that occurred during this period.

Table 6 Overall Monthly Comparison

	Inflows : 1946 - 1985		
Parameter		**AR(3)**	**Error**
Mean (MCM)	175.50	162.82	7.23%
Standard Deviation (MCM)	163.87	100.19	38.86%
Root Mean Square Error (MCM)		128.61	
Correlation Coefficient		0.6234	
Peak Forecast Error (%)		45.96%	
	Inflows : 1986 - 2008		
Parameter		**AR(3)**	**Error**
Mean (MCM)	158.48	159.53	**0.66%**
Standard Deviation (MCM)	137.88	97.71	**29.13%**
Root Mean Square Error (MCM)		**106.47**	
Correlation Coefficient		**0.6376**	
Peak Forecast Error (%)		**45.70%**	

Similar to the model verification data, the new series was logarithmically transformed and periodically standardized using means and standard deviations of Table 1 and equations (7) and (8). Using the one-step-ahead forecast function of the AR(3) model, the forecast was extended starting with the initial inflow of the new series (Jan. 1986, t=433) up to the last measured inflow of April 2008 (t=700). The results were summarized and shown in Table 6.

For the new series, the overall mean of the forecast and measured inflows were almost identical resulting in a 0.66% error. There is also a decrease in the root mean square error and an increase in the correlation coefficient between forecasts and measured inflows for the new series.

Based on the monthly statistics between the forecasts and measured inflows of the old and new series , it can be deduced that the performance of the AR(3) model was relatively the same in all comparative statistics for both sets. The monthly errors ranged from 0.78% to 29.64% for the old series and 1.28% to 32.63% for the new series. The monthly root mean square errors (RMSE) for the new series were slightly smaller as shown in Fig. 7. A sample plot of the AR(3) forecasts

against the measured inflows for 1998 to 2008 are shown in Fig. 8. The overall shape of AR(3) inflows were acceptable, except for the predicted peak flows.

The overall peak forecast error of 47% for both series showed the inability of the model to predict peak flows. This inability to forecast maximum flows is a poor property of ARMA models. But looking at the monthly means of the observed and forecasted inflows of Fig. 9, it was evident that the AR(3) model predicted recession flows or the inflows during the dry season.

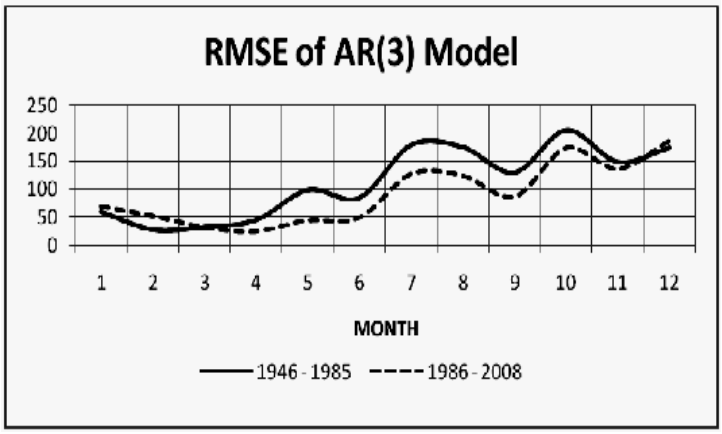


Fig. 7 Monthly Root Mean Square Error

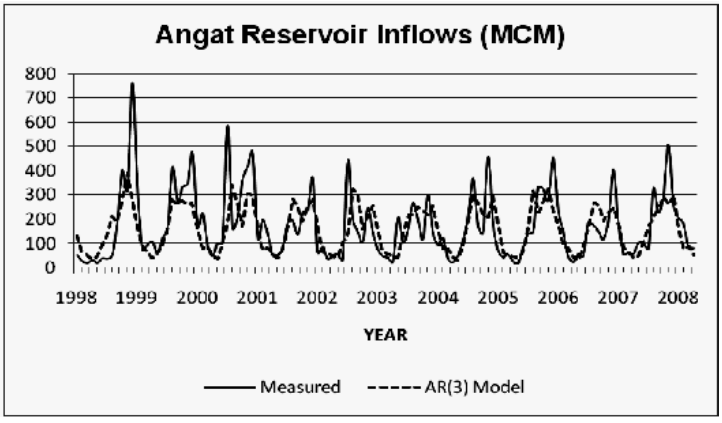


Fig. 8 AR(3) Monthly Inflows for New Series of AR(3) Model

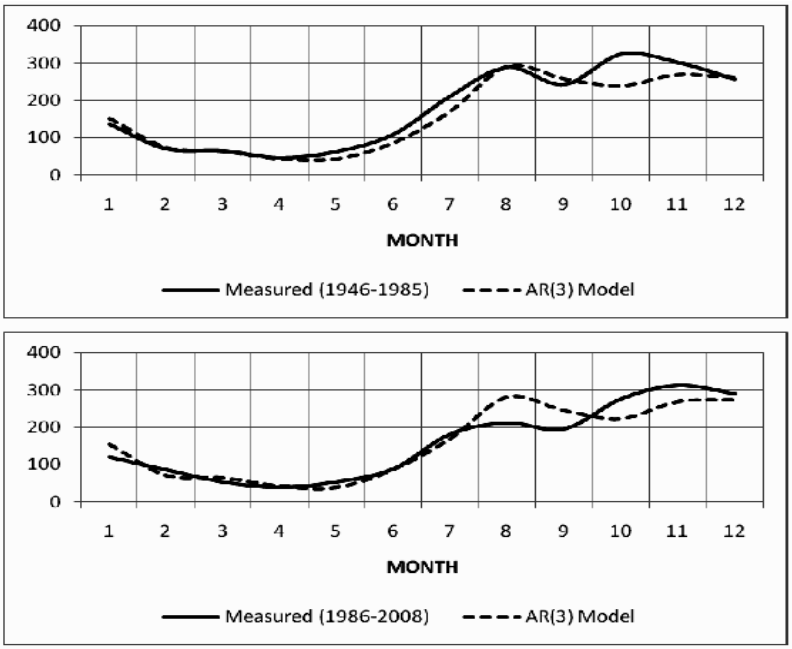


Fig. 9 Monthly Mean of Model and Measured Inflows

For both sets of monthly inflows, the forecast model could predict the inflows for the months of December to May, while the forecast for June to September for the old series and the months of June and July for the new series were acceptable.

Based on the analysis of the results, the autoregressive model AR(3) is valid and adequate to forecast the monthly inflows of the Angat Reservoir.

7. CONCLUSION

The major problem usually encountered in the operation of the Angat Reservoir is to predict the inflows during the dry season. It was clearly shown that the AR(3) model, for the monthly inflows from 1946 to 2008, was adequate and predicted low flows. A reliable forecast model is needed in the operation of the Angat Reservoir to help decision-makers evaluate different alternatives. The optimal management of the scarce water resource during the dry season would benefit users for water supply, irrigation, and power generation.

When water is abundant, the resulting excess storage is just released to the river system giving minimal benefits to the various stakeholders or users of the water resource. The overall peak forecast error of 47% showed the inability of the model to predict peak flows. This inability to forecast maximum flows is a poor property of ARMA models. Peak flows are usually due to extreme storm events and are difficult to predict.

In general, ARMA models are only capable of predicting low flows satisfactorily as seen from the results of the AR(3) forecast model fitted to the Angat monthly inflows. Possible refinements of the ARMA model can be achieved by including rainfall as an additional input following the ARMAX formulation. However, benefits from predicting the low inflows of the Angat Reservoir clearly outweighed the weakness of the AR(3) to predict peak flows.

REFERENCES

1. Box, G. E. P. and Jenkins, G. M., 1966, Some Recent Advances in Forecasting and Control, Paper presented at the European Meeting of Statisticians, Imperial College, London.
2. Dizon, C. Q., 1987, ARMA Modeling and Forecasting of Monthly Streamflows: With application to the Angat Reservoir inflows, M.S. in Water Resources Thesis, College of Engineering, University of the Philippines, Diliman, Quezon City.
3. Hipel, K. W., et. al., 1977, Advances in Box-Jenkins Modeling: Model Construction, Water Resources Research, Volume 13, No. 3, pp.567 – 576.
4. Ozaki, T., 1977, On the Order of Determination of ARIMA Models, Applied Statistics, 26, No. 3, pp. 290 – 301.
5. Salas, J. D., et. al., 1980, Applied Modeling of Hydrologic Time Series, Water Resources Publications, Littleton, Colorado.
6. Salas, J. D. and Obeysekera, J. T. B., 1982, ARMA Model Identification of Hydrologic Time Series, Water Resources Research, Volume 18, No. 4, pp.1011 – 1021.
7. Salas, J. D. and Smith, R. A., 1978, Analysis of Hydrologic Time Series with ARIMA(p,d,q) Models, Paper presented at the Computer Workshop in Statistical Hydrology, Colorado State University.

