Clarification of Ambiguous Problems: Effects on Problem Solving Ability and Attitude Towards Mathematics

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Introduction

Problem solving is now being encouraged to be used as the main activity in all mathematics classes. Students should be encouraged and exposed to work on problems that may take hours, days, and even weeks to solve to develop effective problem-solving ability (Mikusa, 1998). However, it has been observed that students lack the necessary skills to engage in real-life problem solving outside the school setting. This is because students are only trained to work on routine problems that have well-defined goals where all the needed information are given to be able to solve the problem. That is why it is vital to explore other types of problems that can be given as class activities to develop effective problem-solving skills and positive attitude towards mathematics.

The Merriam-Webster online dictionary defines "ambiguous" as "doubtful or uncertain especially from obscurity or indistinctness", "inexplicable", and "capable of being understood in two or more possible senses or ways." There are times that problems encountered are fragmented, contradictory, or elicit multiple meanings, which cannot be easily structured and understood. How a person reacts or deals with ambiguous situation or stimulus shows the degree of tolerance or intolerance he or she has. It can also have a profound impact on his/her educational experiences (Owen and Sweeney, 2002). Recognizing the potential positive effects of developing ambiguity tolerance to problem solving, this research utilized activities involving lateral thinking problems, riddles, and analysis of impossible figures, which encouraged students to think outside the box. As stated by Baroody (1995), solving a problem often depends on looking at the problem in a new way.

In this study, it is hypothesized that ambiguous problems will help improve the problem solving ability and attitude of students towards mathematics. Furthermore, given an ambiguous problem, students will be able to practice analysis of conditions in a problem and clarify rules and goals to come up with an array of sensible answers. Problem clarification can be one of the important skills that a student needs to develop in order to become a proficient problem solver. This hypothesized importance of problem clarification agrees with the observation of Roberts (1995) that the crucial part in problem solving is how to get started with a solution.

Since the ambiguous problems used in the study required students to come up with new or unusual ideas to answer the problems, creative thinking is also utilized. Hence, this research also regarded creativity as one of the factors that might affect the experimental treatment. When creativity is integrated with problem solving, creative problem-solving ability is developed among students, which is believed to be more effective and powerful than the ordinary problem-solving skill. Creative problem-solving process is student-centered. It is a tool for solving real-world problems.

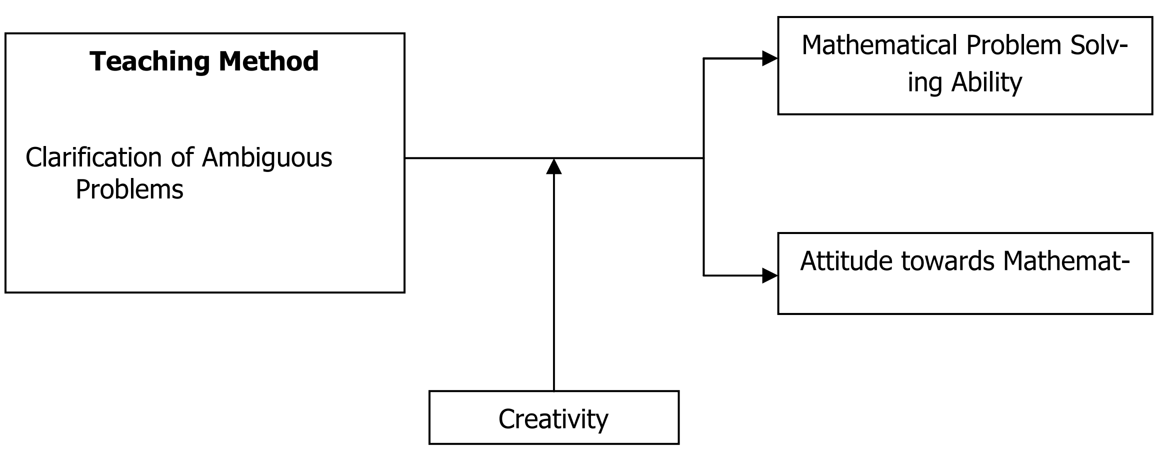
Because of this, it is also necessary to investigate how creativity could enhance students' problem-solving ability particularly in the field of mathematics.

Aside from creativity, there are other factors that affect students' performance in mathematics, particularly in solving word problems. According to McLeod and Ortega (1993), students' beliefs, attitudes, and emotions are important factors in mathematics teaching and learning. When students have positive beliefs and attitude toward mathematics, the quality of their performance also increases. The same researchers also suggested that problems in mathematics should be made into a challenging activity rather than as source of frustration among students.

Purpose and Conceptual Framework of the Study

The study aimed to determine if the deliberate use of clarified ambiguous problems is effective in developing mathematical problem-solving ability and positive attitude towards mathematics. Since solving some ambiguous problems requires new or unusual ideas, the study also aimed to determine if the students' creativity moderated the effects of the treatment on mathematical problem-solving ability and attitude.

Fig. 1. Conceptual Framework of the Study



The diagram indicates that the teaching methods used during the experimental phase are hypothesized to affect the mathematical problem-solving ability and attitude of the students. The study aims to determine whether the use of activities involving ambiguous problems improves students' ability in solving mathematical problems. The study also considered the effects of clarification of ambiguous problems to students' attitude towards mathematics. The students' creativity is considered as a moderating variable and is presumed to affect the causal relationship between the teaching methods and the students' problem-solving ability and attitude.

Methodology

The study utilized the quasi-experimental research design, specifically the pretest-posttest design, to answer the research questions. Student interview and teacher evaluation were also conducted for the qualitative aspect of the research.

The two sections who participated in the study were sections 4 and 5 of the first year students of Makati High School. The two sections were handled by the same teacher during the first two quarters. Through toss coin method, one section was assigned as the experimental group and the other section was assigned as the control group.

Research Instruments

Three instruments were used in the experiment. The first two instruments are the problem-solving ability test and the attitude rating scale. Both instruments were subjected to content validation of mathematics experts from the UP College of Education and UP Integrated School. The materials were also submitted to the teacher in-charge in Makati High School for checking and validation. The third instrument used is the Torrance Test of Creative Thinking (Verbal Form A).

Problem-Solving Ability Test. The problem-solving ability test used during the pilot testing was composed of 10 problems, 7 of which were algebra problems and 3 items dealing with ambiguous problems.

The number of items allotted for algebra problems was greater than the allotted items for the ambiguous problems since the focus of the test was to measure the mathematical problem-solving skills of the students. The ambiguous problems given in the test are problems that can be interpreted or understood in more than one way. For the experimental research, the ambiguous problems used are in the form of riddles, lateral thinking problems, impossible figures, and mathematical paradoxes.

The result of the pilot test yielded a low reliability coefficient of 0.487. Some modifications were done to increase the instrument's reliability. Initially, the rubric for algebra problems developed by Guzman (2000) was used in checking the students' answers in the test. However, since most of the students' solutions were incomplete and relied mostly on arithmetic solutions, a less detailed 4-point rubric was used to score students' solutions to the algebra problems. For the ambiguous problems, a different set of rubric was used to score students' answers. Moreover, to improve the reliability of the instrument, two items were discarded and the others were revised based on the item analysis done.

The final form of the instrument which was used for the pretest and posttest had 8 items, 6 of which were algebra problems and 2 items were ambiguous problems. Using Cronbach Alpha reliability analysis, the problem-solving ability test registered a reliability coefficient of 0.512 after it was revised. According to Fergusun and Takane (1989, p.477), "low reliability does not necessarily invalidate a technique as a devise for drawing valid references." Since reliability is a function of test length, increasing the number of items will increase the reliability of the test. However, for the study, the number of items in the test was limited to 8 items, considering the time allotment for the test, as well as the attention span of the students involved.

Attitude Rating Scale. The 5-point attitude rating scale used for the pilot testing consists of 20 statements, 12 were positively stated while 8 statements were negatively stated.

The pilot test results on attitude rating scale yielded a reliability coefficient of 0.649. This result is quite low for an attitude rating scale. Further analysis of students' responses revealed that many students answered "undecided" on most of the statements. Because of this, the overall rating was neutral. With these results, the researcher opted to utilize a 4-point attitude inventory using the same statements. The modified attitude inventory had a reliability coefficient of 0.816.

Torrance Test of Creative Thinking. The Torrance Test of Creative Thinking has two forms – verbal and figural creativity. To measure the students' creativity level, the researcher used only the test for verbal creativity for the study since it s the type of creativity closely related to solving ambiguous problems.

The creativity test was given a day after the pretest on problem-solving ability and attitude rating scale was given. The researcher followed exactly all the instructions given in the directions manual accompanying the test booklets. The test has 7 subtests and each subtest is separately timed. Two subtests were timed for 10 minutes and the remaining 5 subtests are timed for 5 minutes. The test took one hour to finish including all the instructions per subtest.

As emphasized in the manual, a non-test atmosphere should be practiced during the session. The session must be game-like and must have a thinking or problem-solving atmosphere. The psychological climate during the session is very important so that students would not associate the activity to testing. The students should feel comfortable and they must be invited to have fun and enjoy each activity in order to stimulate their creativity.

Since the creativity test is standardized and its reliability has been established, the test was given directly to the students of the study without undergoing the pilot testing procedure. Also, as stated in the scoring manual by Torrance (1967), findings suggest that no special training was necessary in scoring the test to assure reliable results.

The important requirement was for the researcher to read and follow the scoring guide as precisely as possible. The researcher herself scored all the students' answers in the creativity test. Students' score had three components – fluency, flexibility, and originality. As mentioned in the scoring manual, elaboration score was optional since scoring for the elaboration part of students' answers tend to be subjective and yields low reliability. Based on subsequent studies conducted by Torrance, it was assured that omitting the score for elaboration does not affect the quality of measurement resulting from only three components of the creativity test.

After careful checking and analyzing students' answers in the creativity test, raw scores were computed and summarized. Then raw scores for each component of creativity were converted to t-scores. In converting raw scores to t-scores, standards for 5th grade students were used as suggested in the manual. As mentioned in the manual, the standards for 5th grade students will suffice in converting elementary and high school students' raw scores. The resulting t-scores for each component yielded a reliability coefficient of 0.943 for 56 samples. The creativity test was given only before the start of the experimental period. To determine the level of creativity of the students, they were classified as "high" if their overall creativity t-score is 50 or higher while those who got t-scores below 50 were placed in the "low" classification. In the control group, 15 students were classified in the "high" group while 13 students belonged to the "low" group. In the experimental group, 12 students were in the "high" group and 16 students were in the "low" group.

For the qualitative analyses, observation checklists were used to evaluate the researcher's teaching capacity and to compare the two teaching methods used in the study. Student interviews were also conducted. The interview aimed to solicit students' reactions, comments, and ideas about the ambiguous problems and the activities given to the class. The interview session with the group also gave the researcher a chance to clarify some misconceptions about the experimental treatment given to the group.

Students' outputs from each activity were also compiled and analyzed as evidences of improvement of students' thinking and problem solving skills.

Experimental Treatment

The experimental treatment was given to the subjects for a total of 25 one-hour class meetings. Prior to the actual exposure to the two teaching methods, the clasess were already handled by the researcher to establish rapport with the students and to be able to plan grouping procedures for the experimental group in lieu of the group activities for the class in terms of students' creativity level.

The skills that the students learned in clarifying and answering ambiguous problems are expected to materialize during the first two steps on the problem solving process (read and think/ explore and learn). Creativity, a crucial skill during the end part of the process (reflect and extend), is also expected to materialize.

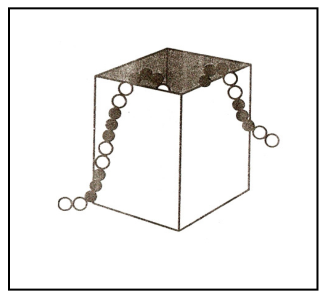
The ambiguous problems that were given in the experimental group were in the form of lateral thinking problems developed by de Bono (1967). In addition, riddles, impossible figures and mathematical paradoxes were also given as ambiguous problems. Each problem was evaluated based on Norton's categories of ambiguity. These were: 1) multiple meanings; 2) vagueness, incompleteness, fragmented; 3) unstructured; 4) lack of information; and 5) inconsistencies, contradictions, contraries. These categories directly focus on the structure, conditions, details, and students' possible solution on each ambiguous problem.

The ambiguous problems were given as a lesson motivation or warm up activity, for individual work, pair work and group project. This is in consonance with Treffinger's (1980, cited in Sorenson, et al., 1996) idea and suggestion that, "warm up activities allow one to get the creative juices flowing. Creative thinkers must ready themselves by "stretching the mind" and opening the senses to original and imaginative thought (pp. 146-147)." On the other hand, ambiguous

Problems that can be directly related to the lessons were given at the start of every new lesson. The riddles were given as homework while lateral thinking problems were given as pair work before the lesson proper. Problems that require more time for analysis were given as a group project. Ambiguous problems are problems that can be interpreted or understood in more than one way, and below are examples of ambiguous problems used during the experiment:

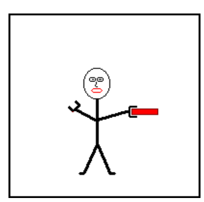
A log was cut into 4 pieces in 12 minutes. How many minutes will it take to cut a log into 5 pieces?

How many beads are in the chain?



(Source: The New Sourcebook for Teaching Reasoning and Problem Solving in Junior and Senior High School by Krulik and Rudnick, © 1996)

Have a look at the picture below. It shows a person holding a block of wood. Now, what will happen to the piece of wood when the person lets go of it?



(Source:http://www.brainstorming.co.uk/puzzles/dropblock.html)

The clarification of ambiguous problems was done using one or a combination of formulation of questions in order to identify rules or conditions applicable to each problem and/or identification of an ambiguous term, phrase, or statement in the given problem.

Aside from thinking skills, positive attitudes such as perseverance and creativity were practiced and emphasized every class meeting. After answering ambiguous problems, the students in the experimental group were encouraged to apply the thinking skills or the positive attitude that was identified during the motivation activity.

Each ambiguous problem given to the experimental group was expected to improve students' thinking skills by requiring them to analyze and formulate as many reasonable answers in a stimulating and fun class atmosphere. It was also expected that clarification of ambiguous problems would eventually develop positive attitude towards mathematics. This is because some problems involved humor, fun, and strategic thinking. Having these factors, the classroom atmosphere is expected to help students become risk-takers when it comes to giving out solutions and answers to problems, which is the very essence of creativity. Moreover, the thinking skills that were developed through the aid of ambiguous problems would be utilized by the students when asked to solve mathematical word problems

The results of the posttest on problem-solving ability and attitude inventory were used to verify if the expectations were achieved. Moreover, the qualitative data based on student interview and teacher evaluation were used to further explain the result of the experimental treatment.

Data Gathering Procedure

The researcher utilized both quantitative and qualitative analyses of data. Quantitative analysis was used to determine students' comparability, significance of difference between the experimental and control group, and significance of interaction effect of creativity on the teaching method. For qualitative analysis, a teacher evaluation guide was used to help the researcher during the experimental phase of the study. The evaluation guide was accomplished by the teacher-in charge who acted as observer during the course of the experiment. Samples of students' work were also included to point out some relevant findings during class activities. Lastly, interview responses of students were also included to create a better picture of the effect of the intervention to the experimental group.

Table 1. Summary of the t-test of Initial Comparability of the Two Sections

Variable	Group	N	Mean	Std. Deviation	t	p-value (2-tailed)
Average Mathematics Grade for the first two quarters	Exp	28	79.75	5.19	1.88	.071
	Con	28	77.50	3.45		
Problem-Solving Ability[a]	Exp	28	5.68	2.83	.69	.495
	Con	28	5.11	2.88		
Attitude Towards Mathematics[b]	Exp	28	2.89	0.39	.31	.760
	Con	28	2.86	0.36		
Verbal Creativity[c]	Exp	28	47.68	6.35	-.68	.500
	Con	28	49.36	10.00		

a. highest possible score = 32 b. from a 4-point scale c. raw scores converted to T-score (mean = 50, SD = 10)

Data Analysis and Findings

The comparability of the two groups was tested in terms of the students' pre-test scores in problem-solving ability, attitude, and creative thinking ability (verbal), and mathematics average grade for the first two quarters. Twenty-eight matched pairs for the experimental and control group were considered in the analyses of data. The selection of paired samples was based on student's completion of all the requirements for the study which were as follows: (1) has taken both pre and posttests of the problem-solving ability test and attitude rating scale; (2) has taken the creativity test; and (3) his/her class attendance not lower than 90% of the total number of class meetings. The pairs were also matched based on their average mathematics grade in the first two quarters and in terms of gender.

To determine the significance of the difference between the mean scores of the experimental and the control group in the problem-solving ability test and the attitude inventory, the t-test for paired samples was used.

Since all the p-value in each variable is greater than .05, the statistical results showed that the two groups were equivalent on the onset of the experimental treatment.

The two groups used in the study were equated as nearly as possible to avoid getting effects from factors other than the treatment given to each group. With the statistical results on the pretests, it can be said that whatever differences the two groups had at the end of the study can be attributed to the teaching methods used in the study.

After five weeks, the problem solving ability test and attitude inventory were given to the students as posttests. Table 2 shows the t-test of the posttest mean scores of the matched pairs in the problem-solving ability test. The experimental group has a mean score of 14.89, which is higher than the mean score of 10.79 of the control group. The t-value of 2.68 with p = .012 indicates that there was a significant difference between the experimental and the control group. The experimental group scored significantly better in problem solving. However, both groups still have a mean score below the passing mark of 16 points.

Table 2. Summary of the t-test of Difference between Posttest Means

Dependent Variable	Group	N	Mean	Std. Deviation	t	p-value (2-tailed)
Problem-Solving Ability	Exp	28	14.89	4.53	2.68	.012
	Con	28	10.79	7.17		
Attitude towards Mathematics	Exp	28	3.09	0.37	2.55	.017
	Con	28	2.79	0.48		

Further analysis of the samples' problem-solving ability test scores revealed that the mean gain score of the experimental group is 9.21 while the control group only has a 5.68 mean gain score. The mean gain score of the experimental group is significantly different from the mean gain score of the control group as indicated by the t-value of 2.615 (p = .014).

The posttest scores on the attitude inventory revealed a significant difference between the experimental and the control groups. As shown in Table 2, the mean score of 3.09 of the experimental group is significantly higher than the mean score of 2.79 of the control group as indicated in the t-value of 2.548 (p = .017). Students' responses per item were also analyzed.

Based on the results, the mean scores of the two groups yielded four statements that are significantly different. Table 3 shows the four statements.

Table 3. T-test of Difference between Means of 4 items on the Attitude Inventory Posttest

Statement	Mean		t	p-value (2-tailed)
	Exp	Con		
1. I find mathematics interesting.	3.52	3.07	2.943	.005*
2. My mind goes blank when I am solving mathematics word problems.	2.96	2.32	2.875	.006*
8. I enjoy attending my math class.	3.56	3.07	2.465	.017*
20. I easily give up when solving math problems.	3.30	2.64	3.006	.004*

*significantly different

The significant differences on these four statements were in favor of the experimental group, which can be attributed to the treatment given to the group. This was because students enjoyed solving the ambiguous problems given as a motivation or as homework, which can be the reason for having significant differences on the mean scores of the two groups for statements 1 and 8. Since problem clarification was also emphasized during problem solving in class, the students may have developed the attitude of not giving up easily when faced with difficulties in solving a problem. Furthermore, since they were also encouraged to analyze a problem from a different perspective in order to understand and solve problems better, this could have prevented students from feeling or experiencing mental block when faced with word problems, as exhibited by the significant difference in the mean scores for statement 2 in the attitude inventory.

The results indicate that students exposed to clarification of ambiguous problems performed better in the problem-solving ability test and exhibited a more positive attitude towards mathematics based on the students' self-ratings in the attitude inventory. These indicate that the use of ambiguous problems is effective in improving students' problem solving skills because it trains them to analyze problems through problem clarification. The goal of problem clarification was to develop creative thinking processes, which students may eventually use in solving mathematical word problems.

The use of problem clarification in class also aimed to help students develop effective and efficient thinking processes, which can be used in solving any kind of problem that they would encounter. Moreover, the use of ambiguous problems as classroom activity provided an enormous impact in cultivating positive attitude towards mathematics particularly in problem solving because such activities it made the lessons both interesting and challenging.

Solving ambiguous problems, to some extent, requires creative thinking skills particularly verbal creative thinking ability. Because of this, it is important to determine if the verbal creative thinking ability of the students moderates the effect of the treatment. Students' raw scores in the creativity test were converted to its corresponding t-score wherein the highest possible t-score was 100. To be able to group the students for the two-way analysis of variance, students were classified as "high" if their overall creativity t-score is 50 or higher while those who got t-scores below 50 were placed in the "low" classification. Table 4 shows the summary of posttest mean scores on problem-solving ability and attitude inventory, grouped according to creativity level. Out of 56 students used for the posttest analysis, 27 scored at least 50 in the creativity test while 29 students scored lower than 50. Analysis of the mean scores for both experimental and control group reveals that students with higher levels of creativity scored better in the problem-solving ability test than those students with creativity scores below 50.

Table 4
Summary of Posttest Mean Scores by Creativity Level

Dependent Variable	Creativity Level	Group	Mean	Std. Deviation	N
Problem-Solving Ability	High	Exp	16.79	4.69	12
		Con	12.07	7.06	15
	Low	Exp	13.47	3.96	16
		Con	9.31	7.22	13
Attitude towards Mathematics	High	Exp	3.19	0.43	12
		Con	2.77	0.43	15
	Low	Exp	3.02	0.30	16
		Con	2.81	0.55	13

The two-way analysis of variance was used to find out if the effect of the teaching method was moderated by the level of creativity of the students. The interaction between the teaching method and level of creativity was measured in terms of students' problem-solving ability and attitude towards mathematics. The presence of interaction between the teaching method and creativity measures the effect of creativity as a moderator variable. According to Keppel (1991, in Graham, 2000), "an interaction is present when the effects of one independent variable on behavior change at the different levels of the second independent variable."

Table 5 shows the results for the two-way analysis of variance for the effects of the two independent variables in the study: the teaching method and the levels of creativity of the students. The statistical result of .032 F-ratios has a p-value of .859. This means that there is no significant interaction between the two variables. Therefore, creativity did not moderate the effect of the teaching method on students' problem solving ability.

Table 5
Summary of Tests of Between-Subjects Effects
Dependent Variable: Problem-Solving Ability Posttest

Source	Type III Sum of Squares	df	Mean Square	F	p-value
Corrected Model	364.887(a)	3	121.629	3.510	.021
Intercept	9211.947	1	9211.947	265.877	.000
Creativity level	127.804	1	127.804	3.689	.060
Teaching Method (TM)	272.826	1	272.826	7.874	.007
TM*Creativity level	1.099	1	1.099	.032	.859
Error	1801.666	52	34.647		
Total	11398.000	56			
Corrected Total	2166.554	55			

R Squared = .168 (Adjusted R Squared = .120)

Table 6 presents the result of the two-way ANOVA for the two independent variables with post-test attitude means as dependent variable. The F-ratio of 0.878 with p-value of .353, which is greater than alpha = .05 signifies acceptance of the third null hypothesis, and it can be concluded that there is no significant interaction between students' level of creativity and the teaching method used in terms of influencing students' attitude towards mathematics. It can also be claimed that regardless of the level of creativity of the student in the experimental group, the students' attitude improved with the help of the experimental treatment in the study.

Table 6. Summary of Tests of Between-Subjects Effects
(Dependent Variable: Attitude Inventory Posttest)

Source	Type III Sum of Squares	df	Mean Square	F	p-value
Corrected Model	1.524(a)	3	.508	2.722	.054
Intercept	480.168	1	480.168	2572.068	.000
Teaching Method (TM)	1.381	1	1.381	7.397	.009
Creativity Level	.057	1	.057	.303	.584
TM* Creativity Level	.164	1	.164	.878	.353
Error	9.708	52	.187		
Total	495.332	56			
Corrected Total	11.232	55			

a. R Squared = .136 (Adjusted R Squared = .086)

These results indicate that students' creativity level did not interact with the method to produce the effect of the treatment, especially in the experimental group.

Conclusion

Based on the outcome of the study, the higher scores of the students from the experimental group in the problem-solving ability test provide concrete evidence that clarification of ambiguous problems is an effective tool in enhancing thinking and problem solving skills of students. Ambiguous problem, complemented by a non-threatening atmosphere it induces in the classroom, is effective in stimulating students' thinking and interest in class.

Solving ambiguous problems is helpful in motivating students, particularly those who are not inclined in solving mathematical problems. The deliberate use of ambiguous problems can develop and nurture positive attitude towards mathematics particularly in problem solving among students.

Though solving ambiguous problems do not directly teach the students the step-by-step process of solving algebraic problems, it can help students acquire necessary thinking skills that can be used in order to solve algebraic problems effectively.

The statistical results shows that students' creativity level does not exhibit a significant interaction effect on the treatment used in the study, activities involving ambiguous problems can be used with any level of students' creativity. This implies that students can benefit from the use of ambiguous problems in class, if they keep an open mind and their interests are sustained.

Recommendations

The positive result on the emphasis on problem clarification and the use of ambiguous problems as part of a mathematics class serve as an impetus to analyze other factors that could refine and substantiate the findings of the study.

Mathematics teachers should consider including ambiguous problems during class activities to serve as a tool in developing thinking skills and positive attitude towards mathematics among students. As pointed out by Janzen (2005), problem-solving activities should go beyond algorithmic and routine activities. A seminar workshop exposing teachers to ambiguous problems should be conducted. This will enable the teachers to use ambiguous problems effectively as part of mathematics lessons by acquiring the necessary thinking skills and problem-solving ability.

Further studies involving measurement of students' level of ambiguity tolerance in a mathematics classroom and its effects on students' performance in class particularly in problem-solving activities can be made. Because every individual has different levels of ambiguity tolerance, the levels of tolerance may serve as a determinant for students' success or better performance in mathematical problem solving.

Finally, the results of the study can be subjected to further analysis and investigation to determine how each category of creativity (fluency, flexibility, originality, and elaboration) directly affects students' problem-solving ability. The knowledge on what each aspect of creativity contributes to the problem solving ability of a student will be helpful in planning classroom activities.

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