Effects of Cooperative Learning on Mathematics Problem Solving Ability and Attitudes of High School Freshmen

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The study used two matched groups of 25 students each to compare the effects of cooperative learning and traditional approaches on students' learning of mathematics problem solving. It used both qualitative and quantitative analyses of data. Although a non-significant difference in achievement was found between the two groups in the posttest, the difference in attitude toward mathematics was clearly in favor of cooperative learning. The data also gave evidence that cooperative learning fosters better communication of ideas and problem-formulating skills than the traditional approach.

Mathematics is recognized as one of the most difficult subjects, if not the most difficult, in high school since mathematical concepts and processes are often abstract and thus difficult to teach. Because of this, there is a need to produce a positive and supportive climate for mathematics learning. Teachers need to create opportunities for the students to process, explore, enjoy, and improve mathematics skills and achievement all at the same time.

What is Cooperative Learning?

Over the past 20 years, a substantial body of research at various grade levels and in numerous content areas have documented the effectiveness of cooperative learning as a method in teaching. Pioneers of cooperative learning like Johnson et al. (1988) believed that this strategy is not having students sit side-by-side to talk with each other as they do their individual task. It is not simply giving a group of students a task where only one does all the work but the others put their names on the product as well. They suggested that teachers carefully structure learning situations by implementing the five basic elements of a well-structured cooperative lesson.

The first of these basic elements is positive interdependence. Students must perceive that they "sink or swim together." This may be achieved through mutual goals, division of labor, members sharing materials and sources of information, assigning roles to students, and giving rewards.

Second, cooperative learning requires face-to-face interaction among students. The important types of verbal interchanges are oral summarizing, giving and receiving explanations, and elaborating.

Third, cooperative learning demands individual accountability; that is, each group member is responsible for learning the assigned material.

Fourth, cooperative learning requires that students appropriately use interpersonal and small-group skills.

Fifth, students must also be given the time and procedures for processing; that is, analyzing (1) how well their learning groups are functioning and (2) the extent to which they are employing their social skills.

The effectiveness of cooperative learning is shown in the studies of Sharan (1980, cited in R. Stevens 1991, p. 9) who stated that cooperative learning method produces greater academic achievement than does traditional instruction. Cooperative learning results in more positive attitude toward school and improves self-esteem and relationships among different types of students. It provides opportunities for students to engage in the same sort of teamwork, communication, effective coordination, and division of labor that characterize most real-life situations (Johnson, et al. 1988).

According to Brown and Campione (1986, cited in Stevens 1991, p. 10), "understanding is more likely to occur when a student is required to explain, elaborate, or defend his/her position to other students; the burden of explanation is often the push needed to make him or her evaluate, integrate, and elaborate knowledge in new ways."

Purposes and Method

The study sought to determine the effects of cooperative learning method on the ability of high school freshmen to solve mathematical problems. It focused mainly on its effects on such abilities as understanding mathematical problems, communicating skills in mathematics, exploring different problem solving strategies, and formulating mathematical problems. It also studied students' attitude towards mathematical problems.

The study utilized the pretest-posttest control group design. Two intact classes were randomly assigned to the experimental class which underwent the method of cooperative learning, and the control class which went through the traditional method of teaching. The students in the experimental class were divided into five groups of 5 to 6 members using stratified random assignment. A mathematics achievement test and an attitude rating scale were administered to both classes before and after the experiment.

Each class had heterogeneous composition of students. Twenty-five students (five groups) in the experimental class were randomly selected and were matched with the same number in the control class based on scores in the achievement test given by the school, grades in mathematics in the first grading period, age, and gender.

The study was conducted for a period of eight weeks during the second grading period of the school year 1998-1999 with the researcher handling both classes. Before the start of the experiment, the researcher and the math teachers of the two classes discussed the objectives, strategies and procedures of the study. The teacher sat in and observed the classes to determine differences in procedures and instruction.

The students' regular schedule (40 minutes a day, five days a week for eight weeks) was observed during the study. On the first day, the attitude scale and the researcher-made mathematics test on ratio, percent, and proportion with emphasis on problem solving were administered to both classes. For the experimental class, the rest of the period was spent in explaining the basic concepts of cooperative learning method.

The next two meetings were spent in giving the experimental class and the control class some mathematical puzzles. This was done to build rapport between the researcher and the students in both classes. For the experimental class, it was also done to establish camaraderie among the group members and for them to start working together on their assigned roles. Each member of the was given a specific role to perform to foster positive interdependence and to acquire collaborative skills. Roles were rotated in every lesson to make sure that each member would be able to practice a role and there would be equal participation among members. The five roles were (a) s*ummarizer/ recorder* who summarized and recorded the group's discussion; (b) *observer/checker/sitting arranger* who ensured that all members could explicitly explain how to arrive at an answer and who arranged the chairs of the group; (c) r*esearcher/runner* who got needed materials for the group and communicated with other learning groups and the teacher; (d) *encourager/noise monitor/ timekeeper* who made sure that everyone participated and did not make noise during discussion and reminded the members of the remaining time; and (e) *reader/energizer* who read the problem aloud to the group and lightened the mood of the group.

In addition, all members were required to know and understand the correct solution to each problem because the researcher randomly picked one member in each group to present the solution during the discussion. The performance of the group was rated on the presentation as another way to ensure individual

accountability.

In the succeeding days, both classes were given exactly the same mathematics instruction, activities, seatwork and homework. The two classes differed only on the method of instruction.

The control class underwent the traditional learning method where the teacher was the only source of knowledge and did not explore the learning-teaching potential in group processes or interaction. Discussions of every solution were tape-recorded to see how each member of the group in the experimental class communicated his/her mathematical ideas with other members.

Instrumentation

Two instruments were used to get the needed data: the mathematics achievement test and an attitude rating scale.

Achievement Test. A mathematics achievement test was developed by the researcher to measure the achievement of the students in mathematics. This test was administered as pretest and posttest. The 40-minute test has two parts: the first part consists of six verbal problems, some of which are of the open-ended and some are of the closed type. The second part presents a situation about which the students are asked to formulate a verbal problem based on given facts. The Focused Holistic Scoring Point Scale (Shaw, et al. 1997, p. 35) was used to score the responses.

Initially, 10 mathematics verbal problems were constructed for the first part and a given situation for the second part. These were content validated by four mathematics experts for appropriateness to the level of the students, for the 40-minute time allocation, and for clarity. As a result, four problems were deleted, leaving only six problems. The test was pilot tested among 90 second year high school students of the same school. The pilot test indicated a low reliability and the test was revised again, eventually yielding a reliability coefficient of 0.4965.

Attitude Rating Scale. This instrument contains 20 items that measure attitude toward mathematical problems. The scale was patterned after a scale by Beltran (1995). Eleven of the statements are positively oriented and nine negatively oriented along a 5-point scale. The attitude scale has a reliability coefficient alpha of 0.8620.

SPSS for Windows, version 10.0 was used in all the statistical analyses of the data. The posttest mean scores of the 25 matched pairs were subjected to the t-test of difference for paired samples. Simple frequencies were utilized in the analyses of solutions to the problem.

Discussion of Results

The students of the two classes were found to be initially comparable with respect to knowledge in general mathematics ($t = -1.77$, $p > .05$) and attitude toward solving mathematics problems ($t = -0.33$, $p > .05$).

Achievement Posttest Scores

The t-test for paired-samples applied to the mean posttest scores in the achievement test showed no significant difference ($t = 0.47$, $p > .05$). This result could be due to the following: (1) the eight-week duration

of the study may not be enough; (2) classes were suspended for almost two weeks because of floods during the rainy season in November and this meant a major drawback in the pacing of the lessons; (3) during the latter part of the experiment, the classes were shortened because of the upcoming second periodic test; and (4) the control class was given more exercises in order to balance the time spent in the experimental class.

Attitude Posttest Scores

A t-value of 2.515 ($p < .02$) was obtained from the comparison of the attitude posttest scores. This means that cooperative learning method reflected a significant difference in the attitude towards mathematical problems in the two classes implying that the cooperative learning approach improved the students' attitude toward mathematics problems. There were significant differences between the two classes in five out of 20 items in the attitude rating scale as reflected in the following statements:

"I find solving mathematics problems interesting."

"I try solving mathematics problems more than what is expected of me."

"I really enjoy solving mathematics problems."

"I enjoy the company of those who are good in solving mathematics problems."

"Mathematics problems often scare me."

Analysis of Students' Solutions

Mathematics Problem Solving Strategies. The solutions made by the students from both classes showed eight different strategies. Three of these are commonly used in mathematical problem solving: *Pattern Recognition,* in which the students try to recognize a pattern; *Guess and Test,* where the students think of a possible answer then check if it fits in the solution; and *Write an Equation,* in which the students form a number sentence then substitute all the given information.

The five other strategies identified by the researcher are: *Identification of the Given and Needed Information,* in which the students sort out the given information that are necessary to the solution; *Computational Solution,* in which the students directly show the computations; *Concept Definition,* in which the students use the meaning of the concept involved in the problem; *Indirect Solution,* in which the students use a different operation, not the expected one, to get a partial answer that will be used to arrive at the final answer; and *Word Solution,* in which the students simply explain in their own words how they got the final answer without showing the solution.

Table 1 shows the percentage of students who used each of the eight strategies.

In almost every problem, significantly more students in the experimental class showed different solutions. Moreover, eight different strategies in solving the mathematical word problems were noted in the solutions of the students in the cooperative learning class. On the other hand, the students in the traditional learning class used only five strategies and failed to utilize *Guess and Test, Indirect Solution*

Table 1. Percent of Students Using Different Problem Solving Strategies in Each Class as Reflected in the Posttest

Problem	1		2		3		4		5		6	
Strategy / Class	**Exp**	Con	**Exp**	Con	**Exp**	Con	**Exp**	Con	**Exp**	Con	**Exp**	Con
1. Pattern Recognition	**40**	44	**12**	0							**4**	36
2. Identification of the Given and Needed Information			**48**	16	**32**	16	**8**	0	**28**	12		
3. Write an Equation			**16**	4			**8**	4	**16**	12	**56**	32
4. Computational Solution			**4**	24			**16**	28	**12**	24		
5. Guess and Test	**16**	0										
6. Concept Definition	**20**	4										
7. Indirect Solution					**20**	0						
8. Word Solution							**4**	0	**4**	0		
TOTAL	**76**	48	**80**	44	**52**	16	**36**	32	**60**	48	**60**	68

Ability to Communicate Mathematical Ideas. The students' ability to communicate mathematical ideas was examined in terms of two distinct perspectives: quality of communication, which involves clarity and correctness of written communication; and presentation of mathematical communication, which involves the modes used in finding the answers.

In the first open-ended problem, although the answers of the students in the traditional learning group were correct, they were not clearly justified. The explanations showed some vagueness; the reasons advanced were insufficient, unreasonable, or unrealistic. The solutions and explanations of the students exposed to conventional teaching method showed their difficulty in communicating mathematical ideas. On the other hand, the solutions of the students under the cooperative learning method were found to be more complete and better explained. The students had a clear description of the definition of the mathematical concepts involved as reflected in their explanations.

Here are some responses of the students from both groups:

Problem:
The ratio of boys to girls in Annie's class is 2 : 5. Give a possible number of boys and girls in her section. Explain your answer briefly.

CS5 (Control Class Student # 5)

1) boys are: 20 girls are: 50 because it is 2:5

ES5 (Experimental Class, Student # 5)

1) 10:25, when you multiply the means and the extremes of 2:5 and 10:25, the answer is 50.

CS24 (Control Class Student # 24)

1) 10:25 $\frac{2}{5} x \frac{5}{5} = \frac{10}{25}$

ES24 (Experimental Class Student # 24)

1) 2:5 = 12:30
We get a number that is equal to the ratio. We multiply the extremes & the means and we get the same answer.

In the second open-ended problem, the solutions and explanations of the students in the traditional learning class revealed little understanding of the problem. They used the given to get an answer and used this as the basis to explain the final answer, which should not be the case. On the other hand, the solutions and explanations of students in the cooperative learning group were clearly written and complete. Illustrating a thorough understanding of the mathematical concept, some students clearly represented all the given information and unknown, some showed computational skills and others started with the formula or equation, which is a better way to start the solution to a problem.

Here are some responses of the students from both groups:

Problem:
This time, Annie needed assistance at shopping. Her friends, Marj and Raquel, were with her at Pure Gold. They found a pair of Levi's pants worth P1,250 with a 15% discount and a pair of Guess pants worth P1,150 with a 10% discount. Which one would be cheaper? Explain your answer.

CS4 (Control Class, Student #4)

4) P1,250 w/ a 15%
P = B (R)
= 1250(.15)
P = 137.50
CHEAPER

P1,150 w/ a 10%
P = B (R)
= (1,150).10
P = P125

ES4 (Experimental Class, Student #4)

4) LEVI's P1,250

x .15 – DR

6250

1250

P187.50 – D

P1,250.00

- 187.50

P1 062.50

GUESS P1,150

x .10 – DR

P115.00 – D

P1,150.00

- 115.00

P1 035.00 – cheaper

ES22 (Experimental Class, Student #22)

4) GUESS

P = ? P = R(B)

R = 10% P=10%(1,150) SP=P1,035

B = 1,150 P = 115.00

SP = ? SP = 1150 – 115

LEVI's

P = ? P = R(B)

R =15% P =15%(1,250) SP= P1062.50

B = 1250 P = 187.50

SP = ? SP = 1250 – 187.50

* Guess will be cheaper because the SP of Guess is P1,015 while Levi's is = P1062.50

CS22 (Control Class, Student #22)

4) The pair of Guess pants worth P1,150 w/a discount 10%

$$10\overline{)1150}\quad 115$$

10

15

10

50

50

x

Group discussions were also recorded to find out if students in the cooperative learning method could really communicate mathematically. Each group was given at least five problems to work on per session. The students used three languages in the discussion: English, Filipino and the vernacular of the area (Kapampangan). To maximize the time and their mathematical abilities, the group was encouraged to participate as actively as possible and accomplish their task in 20 minutes.

Below are excerpts from the tape-recorded discussions, showing the students' mathematical communication ability:

PROBLEM ON PERCENT

Lolo Kiko's medicine is worth Php80.00. But because he is a Senior Citizen, he paid only Php64.00 for it. What is the discount rate given to Lolo Kiko?

Armi: (After reading the problem) Marked price...

Faye: Php 80...

Armi: Php 80

Faye: 'Yung selling price...(the selling price...)
Armi: 'Yung selling price...(the selling price...)
Faye: Php 64.
Armi: Php 64. Ang tanong dito, what is the book's original price? Ayy...mali, no.1 pala 'yon. (Php 64. The question here, what is the book's original price? Ohh... wrong, that's no. 1.)
Faye: Ito, ito. (This one, this one)
Armi: Bale, 'yung... (the...)
Faye: 'Yung discount... (the discount...)
Armi, Faye: Discount rate...
Armi: 'Yung tinatanong. (That's the unknown.)
Faye: 'Yung binigay niya. (That's the given.)
Armi: Discount rate...
Cherry: Is unknown.
Armi: Tapos, ano n'yan? (Then, what's next?)
Jane: Kunin muna discount. Kailangan 'to para makuha ang discount rate. 80 pesos minus 64 pesos ay 16 pesos. (Get the discount first. This is needed to get the discount rate. 80 pesos minus 64 pesos is 16 pesos.)

At this point, the members of the group displayed an understanding of the problem. They identified the given information, Php 80.00 as the marked price and Php 64.00 as the selling price; the unknown is the discount rate. One member, Armi, encouraged the group to proceed with the discussion by asking "Tapos, ano na 'yan?" (Then what's next?). Another member, Jane, explained the need to compute for the discount first, although this was not directly asked in the problem, but was necessary to get the final answer.

This is illustrated in the continuation of the discussion:

Faye: 16 pesos minus 80 pesos, makukuha mo 'yung discount rate. (16 pesos minus 80 pesos, you can get the discount rate.)
Cherry: Hindi! (No!)
Faye: 16 pesos
Armi: Minus...
Faye: Divide...
Armi: Ahh, divided by, 16 divided by...
Faye: 80
Armi: 80
Faye: Point...(while solving)
Armi: Point...
Faye: 0...0...2...
Armi: Ah, 20%.
Faye: Oo, 20%. Kasi imo-move mo 'yung ano... (Because you will move the...)
Armi: Two places to the right 'yung decimal point. Bale ang formula ng discount rate dito MP...MP... (Two places to the right the decimal point. The formula of discount rate here MP...MP...)

Cherry: *Anong discount rate? (What is the discount rate?)*
Armi: *Minus selling price equals divided by...divided by...marked price...discount!*
Cherry: *Marked price, kasi 'di ba 'yung makukuha mo dito 'yung discount? Kapag minaynus mo, 'yung makukuha mo 'yung discount... (Marked price, because isn't it, you can get here the discount? If you subtracted, you get the discount...)*
Armi: *Hindi ba nakuha mo 'yung discount, no? (Isn't it you get the discount?)*
Cherry: *Oh, oh...kapag inayos mo sila... (Oh, oh...If you arrange them...)*
Armi: *Hindi ba discount? 'Yung 16 pesos, iyon 'yung... (Is it not the discount? The 16 pesos, this is the...)*
Armi,
Cherry: *Discount!*
Armi: *Divided by...*
Cherry: *MP.*
Armi: *MP and SP.*
Cherry: *Hindi! (No!)*
Armi: *Divided by the marked price.*
Cherry: *Oo. Kaya kasi dapat nilagay mo dito equals D tapos, D... (Yes. You should have put here equals D then, D...)*
Armi: *Equals D divided by ...MP*
Jane: *Mali na naman...ay tama! (It's wrong again...oh it's correct!)*
Cherry: *20% ang sagot! No. 3. Basahin mo na. (20% is the answer! No.3. You read.)*
Jane: *20% 'yung sagot. (20% is the answer.)*

The exchanges among members in getting the discount rate were indicative of communication. Two students, Faye and Cherry, displayed an incomplete comprehension on how to go about the computation. They asked questions and sought elaboration. The other members showed support by patiently explaining the procedure to them.

Mathematics educators agree that communication is a component that is essential in learning, doing and understanding mathematics since "communication in mathematics means that one is able to use its vocabulary, notation, and structure to express and understand ideas and relationship" (Cai, et al. 1996). Communication is especially important that students are able to express their thinking and problem-solving processes in both written and oral formats. In evaluating students' responses, teachers should pay attention to the nature of the communication mathematically rather than linguistically.

Cooperative learning provides the students an opportunity to communicate their mathematical ideas through the interaction among the members of the group. They are free to express and share their solutions with other members, which the students in the traditional learning method do not experience.

Skill in Formulating Mathematics Problems. In the second part of the achievement test, the students were asked to formulate a problem on ratio, proportion or percent. This part sought to determine which concept is predominant in the student-formulated problems and whether the two classes differed in the number of formulation attempts and in the types of problems formulated in terms of complexity/number of steps in solving the problem, creativity/additional information supplied, and sufficiency of data.

The data showed a high frequency of use of percent in formulating problems. This outcome can be attributed to a number of reasons. One is the concept most familiar to the students. At this stage, the students are already interested in shopping. The grading system of schools also gives them the occasion to learn the concept of percent.

However, studies show that students do not perform well on questions dealing with percent (Gay, et al. 1997). This difficulty does not totally hinder students' comprehension on the subject. This is manifested in the student formulated problems in which complexity and creativity are evident. Some problems made by the students in the experimental class used realistic situations such as shopping and eating.

Many of the problems made by the students in the control group were on the use of basic operations (addition, subtraction, multiplication and division), although these were not among the given options. They entailed operations/steps that did not require additional concepts in problem solving. The infrequent use of ratio and proportion appears to be due to the students' failure to see the direct application of these concepts in their daily lives.

The study also tried to find out how students would attempt to formulate a problem out of a given situation. The pretest data showed that more students in the control class (88%) than in the experimental class (64%) attempted to formulate a problem. Four percent of the experimental class simply copied the given situation, which none from the control class did. The rest of the students in both classes made no attempts. After the treatment, all the students in the experimental class attempted to formulate a problem as compared to 96% of the control class (4% just copied the given situation) who did so. It should be noted though that 8% of the experimental class and 17% of the control class formulated problems with insufficient data.

The difference between the numbers of students in the experimental and control classes who attempted problem formulation could be attributed to the use of cooperative learning in the experimental class. In general, most of the features of cooperative learning, as discussed by Johnson, et al. (1988), were observed in this class. The students in small heterogeneous groups worked on learning tasks that required collaborative and mutual support. The exchange of ideas among members of the group helped them to analyze and solve problems thoughtfully, and to examine and refine tentative ideas and solutions. It taught them to make and offer constructive criticism. It encouraged them to accept everyone's thinking and to view errors as a part of learning, eventually leading to autonomy and persistence.

Problem formulation entails the use of several skills and sub-skills such as noting important details, making inference, forming relationships, applying mathematical concepts, and using verbal skills. With the use of cooperative learning in the experimental class, the students were able to attack such tasks with ease and novelty. They had the confidence to explore all the possibilities and make creative connections, which the students who were exposed to the traditional method could not do as well. Indeed, students in many traditional classes may have the mathematical skills to accomplish the task but may not have the right motivation to do it.

Another purpose of this research was to find out the impact of cooperative learning on the types of problems formulated by the students whether routine or non-routine problems. A routine problem entails a one to two step solution process; it can be solved directly by translating the wording in a concrete model and/or a symbolic expression (number sentence). Thus, solving routine problems may not be an adequate indication of problem solving skills (Baroody 1993). On the other hand, a non-routine problem entails the application of one operation or a combination of several different operations. It may include extra information which is not needed to arrive at the final answer. It therefore provides a greater challenge because it requires a thoughtful analysis of the unknown, the data, and the solution method than do routine problems. This type is more likely to constitute a genuine problem for students.

Analysis of the problems formulated showed that 88% of the experimental class were able to formulate non-routine problems, compared to only 28% of the control class doing the same. Only 4% of the experimental class formulated routine type problems compared to 35% of the control class. Furthermore, the mathematical problems made by the students in the cooperative learning class were more complex. Again, such achievement could have been possible because the students verbally interacted with each other freely, processed their thoughts together intelligently, and helped each other to achieve a mutual goal.

Conclusion

This study showed the many benefits of cooperative learning as a strategy in teaching Mathematics, specifically problem solving, to high school students.

Although the quantitative analysis failed to show a significantly higher mathematics achievement for students exposed to the cooperative learning method than those taught in the traditional method, it revealed that cooperative learning enhanced the attitude toward problem solving in Mathematics. The qualitative analysis of data showed that the cooperative learning method facilitates a higher level of learning, which includes mathematical communication. It enhances the students' ability to learn, explore and apply different strategies in solving mathematical problems, including formulating their own mathematics problems.

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