Mathematical Thinking and Communication Ability of Filipino and Japanese Students: A Comparative Study

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A remarkable gap between the mathematics achievement of Filipino and Japanese students, favoring the latter, is evident in the results of international comparative studies. To understand this gap and later suggest ways to improve the performance of Filipino students, this research investigated the mathematical thinking skills of Filipino and Japanese learners by scrutinizing their solutions in open-ended questions. The students' mathematical communication ability, being a fundamental tool in mathematics learning, was also studied. Furthermore, this paper also examined the relationship between mathematical thinking and mathematical communication ability.

Seventy Grade 7 Filipino students and 70 Grade 7 Japanese students participated in the study. The samples were matched based on a culture-fair intelligence test. Data analysis revealed that Japanese students outperformed Filipino students in both mathematical thinking skills and mathematical communication ability. Moreover, statistical tests prove that mathematical thinking predicts mathematical communication ability. Qualitative analysis of students' responses shows that the groups differ in the mode they represent knowledge, their flexibility in solving, and their focus when communicating their thoughts. Triangulation through analysis of textbooks, modules, and sample tests, and classroom observations was done.

Implications on mathematics teaching, learning, and assessment were discussed.

Key words: *mathematical thinking, mathematical communication, mathematics education, Japanese mathematics education, Philippine mathematics education*

Introduction

Our contemporary society in which digital culture, information overload, and a global mindset are embedded cause many institutions around the world to promote new mathematics education goals. These goals go beyond calculation and application of procedural knowledge to the development of higher order thinking skills. For instance, the United Nations Education, Scientific, and Cultural Organization (UNESCO) emphasizes that students should learn to critically assess multiple data and make reasonable choices (UNESCO, 2012). In the United States, the National Council of Mathematics Teachers (NCTM) includes adaptive reasoning and strategic competence in addition to conceptual understanding, procedural fluency, and productive disposition in the mathematics common core state standards (NCTM, 2017). In Asia, the working paper for the Southeast Asia Basic Education Standards (SEA-BES) in mathematics incorporates logical thinking, critical reasoning, and creativity in the common core regional learning standards in mathematics (Southeast Asia Ministers of Education Organization, 2016). In the Philippines, the conceptual framework in the new K to 12 basic education curriculum revolves around twin goals – critical thinking and problem solving (Department of Education, 2016). Japan, one of the leading countries in Asia in terms of mathematics education, promote reasoning and problem solving as well. These goals comprise what is currently termed as mathematical thinking.

The present study adopts Krulik and Rudnick's (1996) definition of mathematical thinking as the ability to execute mental processes pertaining to mathematics that involves recall, basic thinking, critical thinking, and creative thinking. These four are labelled as the levels of mathematical thinking. Recall is the ability to call up an essential fact or perform an algorithm; basic thinking includes the understanding of basic mathematical concepts such as the four basic operations; critical thinking involves examining, relating and evaluating all aspects of a mathematical problem; and creative thinking refers to generating new ideas.

A fundamental tool in mathematics learning that has recently received considerable attention is communication ability. Lin and Lee (2004) defined mathematical communication as the ability of a learner to express one's own mathematical concepts as well as comprehend others' mathematical thought and equations. A considerable amount of literature suggests that mathematical communication, whether written (Albert, 2000; Pugalee, 2001) or oral (Chapin, O'Connor & Anderson, 2003; Cobb, Boufi, McClain & Whitenack, 1997), predicts mathematical thinking and learning of students. However, there are no studies, to the knowledge of the researcher, which explore the converse – that mathematical thinking predicts mathematical communication ability. This exploration stemmed from observations that when teachers ask students how they solved a problem, common responses are, "I just had it in my head," or "It just makes sense to me" (Lee, 2015). Thus, the problem of the present research can be translated as, "Is it possible that a mathematics learner who can think critically and creatively is not capable of communicating his/her thoughts?" Likewise, "Is a student with poorly developed mathematical thinking able to communicate his/her conceptions?"

Focusing on Asia, there is a remarkable gap between the mathematics achievement of East and Southeast Asian students as evident in the results of the Trends in Mathematics and Science Study (TIMSS) over the years. Japan is globally recognized for its good mathematics education and has consistently and significantly scored above the international average, remaining within the top five among the participating countries in all the six TIMSS from 1995 to 2015 where 41 to 57 countries participated. Meanwhile, the Philippines is seen on the other end of the spectrum. Given this wide gap between the mathematics achievement of Filipino and Japanese students, it is material to investigate how these two groups learn mathematics, particularly focusing on their mathematical thinking and communication ability, in order to develop effective intervention on how to improve the achievement of Filipino students.

With the recent changes in mathematics education goals and the demand to understand the gap between high- and low-performing countries like Japan and the Philippines, respectively, this research sought to investigate the mathematical thinking and communication ability of Filipino and Japanese learners. Specifically, the present research (1) compared the mathematical thinking and mathematical communication ability of Filipino and Japanese students, and (2) determined whether mathematical thinking predicts mathematical communication ability. The study followed a comparative research design. To execute the design, a combination of quantitative and qualitative research approaches were utilized.

Methodology

There were 110 Grade 7 Filipino students and 160 Grade 7 Japanese students who originally participated in the research. Since the study focused on numbers and number sense strand of mathematics, Grade 7 students were chosen because they have already completed lessons on the real number system in this grade level. To ensure the comparability of the two groups, the Culture Fair Intelligence Test Scale 2 (CFIT2) was administered. CFIT2 is a standardized IQ test which measures an individual's intelligence such that the influence of verbal fluency and culture is reduced as much as possible (Apostol, 2016). CFIT2 results were used to match the participants based on their IQ. Students whose CFIT2 scores had no match were excluded from the data analysis. In the end, 70 Filipino students and 70 Japanese students comprised the final sample of the study.

Mathematical thinking was operationalized by the score in the Mathematical Thinking Skills Test (MTST), a researcher-made instrument. The framework was based on Joaquin's (2007) Mathematical Thinking Skills Test, which consists of multiple choice and open-ended items which assess students' mathematical thinking in terms of recall, basic thinking, critical thinking, and creative thinking. Joaquin's MTST covers topics on Measurement while in the present study, MTST covers topics on numbers and number sense.

On the other hand, the Mathematical Communication Ability Test (MCAT) measured the ability of students to express their own mathematical concepts and comprehend other's mathematical thoughts and equations. It is a researcher-made instrument which consisted of open-ended questions based on Yang, Ben, Cheng, and Tak-Wai's (2016) framework for assessing mathematical communication ability. In this framework, the questions are directed to four objectives: meaningfully use mathematical representations to solve word problems; effectively explain mathematical concepts; evaluate the correctness of other's mathematical thoughts; and use equation to express a mathematical thought. Both MTST and MCAT were originally written in English. After content-validation by Filipino and Japanese experts, they were translated to Nihongo and the translation were language-validated by Japanese professors. They were not translated to the Filipino language since the medium of instruction in teaching Mathematics in the Philippines is English and the Filipino students are used to taking Mathematics tests written in the English language. The instruments were then pilot-tested to Grade 8 Filipino and Japanese students from the same schools where the actual sample groups were taken. The pilot tests were administered to Grade 8 students because at the time of pilot testing, the Grade 7 students had not yet covered the contents of the tests. The reliability coefficient computed through the Cronbach alpha equation was .869 for MTST and .808 for MCAT.

Descriptive statistics were applied on the students' scores on MTST and MCAT. To obtain a score in the open-ended questions, a rubric to score the responses of the students from both groups was followed by two raters – the researcher and a Filipino mathematics teacher who has a degree in Japanese language. For the rating purposes of the researcher, the Japanese responses were first translated to English. The translation was validated by the other rater. The scores of the raters were subjected to the Kendall Coefficient of Concordance to ensure that there was agreement between the raters. The raters had a statistically significantly agreement in their scoring, $W = .836$, $p < .001$. The average of the scores given by the two raters was the value used in the data analysis. To determine significant group differences between the scores of the two groups, the t-test for dependent samples was applied to the MTST and MCAT scores. The study also made use of Pearson's product-moment correlation and simple linear regression to determine whether mathematical thinking predicts mathematical communication ability. Moreover, the MTST solutions and MCAT explanations gathered from both groups were scrutinized. This was done to substantiate and supplement the quantitative analyses and provide further evidence of students' thinking skills and communication ability.

For triangulation purposes, relevant documents such as teacher-made assessments in mathematics and mathematics textbook and module excerpts were collected and analyzed. The textbook and module chosen for analysis are the ones that are used as primary reference materials of the students in the schools where the sample came from. While the version of the textbooks from Japan (published 2015) that were cited in this study were released years later than the modules from the Philippines (published 2004 and 2006), note that the original publication of the Japanese textbooks was in 2005 and there are no major differences in terms of lesson development between the 2005 and 2015 versions of the Japanese textbooks. Moreover, even though Grade 7 students were tested, the textbooks and modules that were analyzed were from the previous grades when the students were expected to develop number sense. The textbook analysis was delimited on the lessons which are related to fractions, a big part of the numbers and number sense spectrum.

To supplement the analysis of data, two Grade 7 mathematics classes in the Philippines and in Japan were observed. The observation focused on teacher-student and student-student mathematical discourse. The classes were videotaped, transcribed, and translated for accurate referencing during data analysis.

Results and Discussion

Mathematical Thinking

While there are four levels in the definition of mathematical thinking adopted in the study, this publication focuses on the analysis of the students' critical and creative thinking levels.

Table 1 presents the mean and standard deviation of the results of MTST.

Table 1
Descriptive Statistics for MTST

	N	Mean	Std. Deviation	Std. Error Mean
Filipino	70	41.8286	10.80924	1.29195
Japanese	70	54.6286	7.74442	.92563

The highest possible score is 69 and the mean score of all samples is 48.221 ($SD = 11.349$). Similar to the results in international comparative studies like TIMSS, the Filipino students scored below the average while the Japanese students scored above the average. Moreover, the scores of the Japanese students are less varied ($SD = 7.744$) than the scores of the Filipino students ($M = 41.829$, $SD = 10.809$). Dependent samples t-test was run to determine whether this difference is significant. Table 2 shows the result of the t-test.

Table 2
Dependent Samples t-test for MTST

Paired Differences					t	df	Sig. (2-tailed)
Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper			
-12.78571	12.31341	1.47173	-15.72174	-9.84969	-8.688	69	.000

At .05 level of significance, there is a statistically significant difference in the mathematical thinking skills of Filipino and Japanese students, $t(69) = 12.786$, $p < .001$. The score of the Japanese students is 12.786 ($SE = 1.47$) higher than the score of the Filipino students.

Analysis of the solutions reveals themes in the mathematical thinking of the Filipino and Japanese students. These themes are discussed in the following sections.

Mode of representing mathematical knowledge

An open-ended item in MTST, the results, and the rubric for scoring are presented in Figure 1.

Dan, Kim, and Seth joined a swimming competition. Kim took 9 minutes faster than Dan, and Seth took 3 minutes longer than Dan. Who among the three finished second place?

	no point	1 point	2 points
Filipino (N = 70)	33%	1%	66%
Japanese (N = 70)	14%	0%	86%

Description	Points
The decision is correct and the answer and solution/explanation is entirely correct.	2
The decision is correct but some parts of the solution/explanation are wrong. OR The decision and answer are correct but there are NO solutions. OR The decision is correct but there are NO explanations.	1
The decision is correct but the answer/explanation is entirely wrong. OR The decision is correct but there is NO answer. OR The decision is wrong.	0

Figure 1. MTST Item II-4, the results, and the rubric for scoring

The item in Figure 1 requires students to decide whether the problem is solvable or not. If it is, they are to show their solution; otherwise, they must explain why they don't think so. The problem in Figure 1 is solvable, and the correct answer is Dan. Majority of the students from both groups correctly assessed the solvability of the problem. The analysis of this item focused on the notable difference between how the Filipino and Japanese students approached this problem. For the sample solutions, the following codes were used: F - written by a Filipino student, J - written by a Japanese student. Figure 2 shows sample responses from the Japanese group.

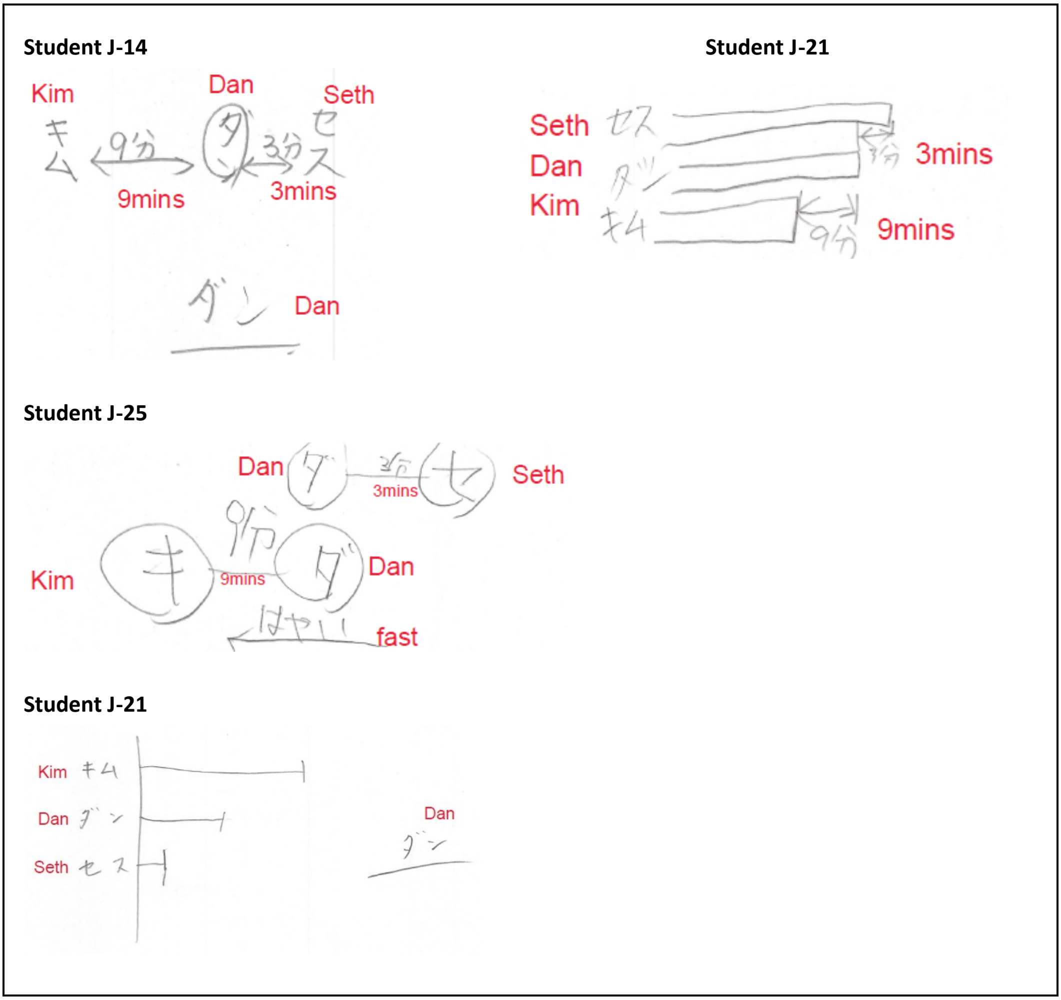


Figure 2. Sample responses from the Japanese Group in MTST Item II-4

The responses from the Japanese students show the use of lines, arrows, circles, and bars. Interestingly, 39 among the 47 Japanese students who wrote a solution in this item, used such visualizations as their solution. This demonstrates that Japanese students tend to think visually. Figure 3 presents sample responses from the Filipino group.

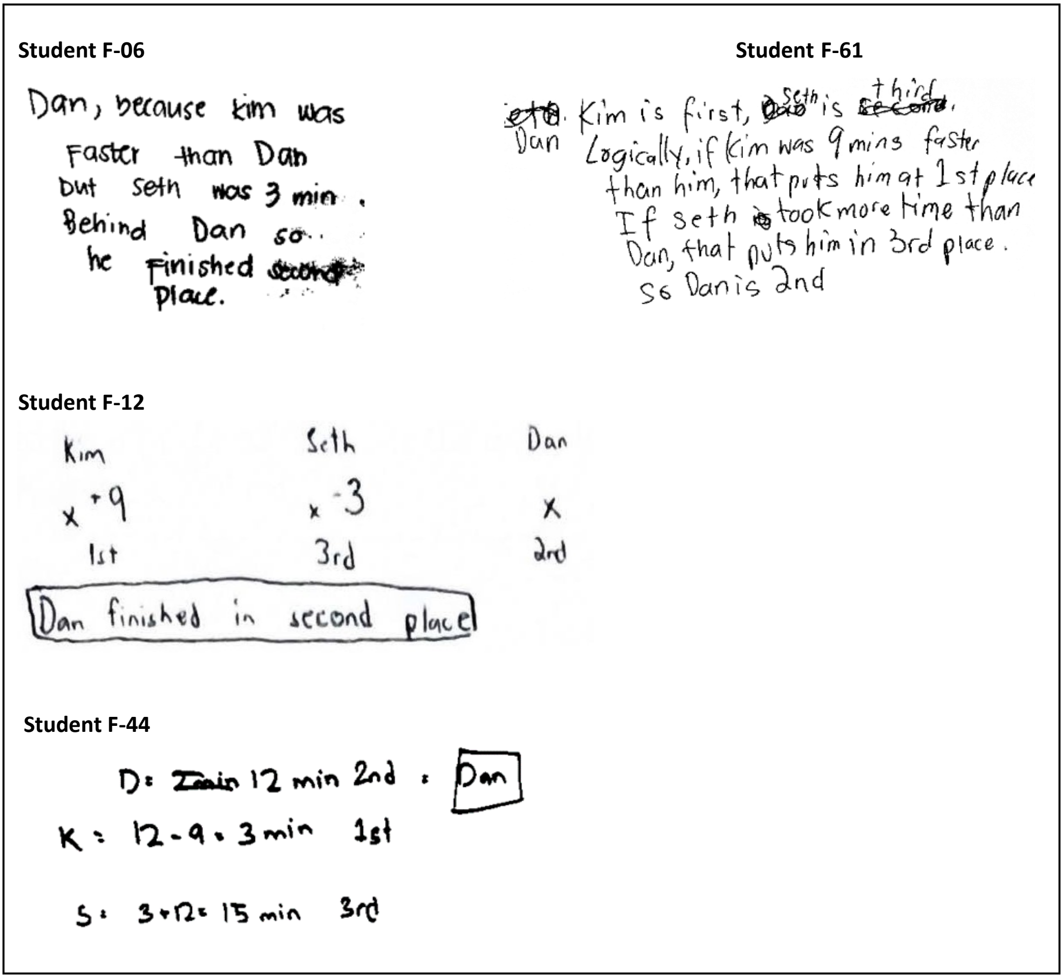
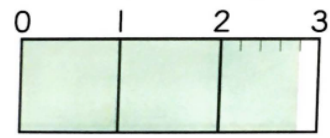


Figure 3. Sample responses from the Filipino Group in MTST Item II-4

The responses from the 48 Filipino students show that 20 solved algebraically and 19 solved by giving worded explanations. This illustrates that Filipino students tend to think symbolically.

This contrast on how the two groups represented mathematical knowledge is evident not only in their responses on the item presented in Figure 1 but also in their responses in many other items in the MTST. However, this does not mean that symbolic solutions cannot be found in the Japanese group and visual solutions cannot be found in the Filipino group. In fact, there were five Japanese students who solved using words and symbols and nine Filipino students who solved using visualization. The expounded observation only means that visual solutions are more evident in the Japanese group, i.e., Japanese Grade 7 students prefer representing knowledge through images and models to representing knowledge through worded text. In the same way, worded and symbolic solutions are more evident in the Filipino group, i.e., Filipino Grade 7 students are more accustomed to worded explanations and algebraic proofs than visual representations.

The tendency of the Japanese students to think visually may be explained by their textbooks. Japanese textbooks are well-illustrated; these illustrations serve various purposes, one of which is to use illustrations as part of the solution, if not the solution themselves. Figure 4 exhibits an example of a mathematical task for Grade 3 students that may be completed by illustration. The task is to change $2\,^4/_5$ to an improper fraction and the students are encouraged to visually interpret and complete the task by marking on the given figure.



6 Let's change $2\frac{4}{5}$ to improper fraction.

1 Let's think by marking the figure below.

2 Fill in the ☐ with numbers.

Thinking about fractions with denominator of 5, $2\frac{4}{5}$ is $\frac{5}{5}$, $\frac{5}{5}$ and $\frac{4}{5}$.

Considering $\frac{1}{5}$ as the unit, we get ☐ sets of $\frac{1}{5}$ calculating $5 \times 2 + 4$

$$2\frac{4}{5} = \frac{\square}{5}$$

Figure 4. Illustration as Solution in Grade 3 Mathematics Japan Textbook (p. 81) by Study with Your Friends Mathematics Textbook for Elementary School, 2015, Japan: Gakko Tosho

While Filipino modules also have illustrations, such are usually present in lesson introductions and not commonly considered as solutions to mathematical problems. Solutions through computations and algorithms are given more importance as illustrated in the lesson on mixed number to improper fraction for grade 3 students in Figure 5.

Mixed Number to Improper Fraction

Ang mga mixed number naman ay maaari ding isalin sa pormang improper fraction. Ngayon naman ay multiplication at addition ang gagamiting operation para maisalin ang isang mixed number sa improper fraction.

Mixed numbers can also be easily transformed to improper fraction form. This time, multiplication and addition are to be used.

Examples:

Mga Halimbawa:

1. Ano ang katumbas na improper fraction ng $4\,\frac{1}{7}$?

What in improper fraction for $4\,\frac{1}{7}$?

$+\;4\,\frac{1}{7}\;\times$ Sagot: $\frac{7 \times 4 + 1}{7} = \frac{29}{7}$

Answer:

Pansinin na ang denominator ng improper fraction ay parehas lamang sa denominator ng fraction sa mixed number.

Note that the denominator of the improper fraction is just the same as the denominator of the mixed number.

Figure 5. Improper fractions and mixed numbers in Grade 3 Mathematics module (p. 229) by L. Sibbaluca, 2006, Quezon City: UP Integrated School

According to Bruner (1965), iconic representations are necessary for students to have a firm sense of mathematical abstractions. As explained, Japanese textbooks are loaded with images and models, and Japanese students are exposed to and trained to represent their thinking through visual representations mathematics classes. This gives them the advantage of having deeper understanding of mathematical concepts. This explains the significantly higher scores of the Japanese students than the Filipino students in MTST.

Flexibility in solving word problems

An item in MTST required the students to write two or more different ways to solve a given problem. This item, shown in figure 6, tests the students' flexibility, a component in mathematical creative thinking. Flexibility is the ability to give multiple solutions to a problem, and these solutions should come from different perspectives (Herlina, 2015).

In a laboratory, one bacteria sample was cooled to a temperature of –51°C while another to –76°C. What was the temperature difference between the two samples? Show as **many different solutions** to this problem as you can think of. You may use illustrations, if necessary. *If possible*, write more than 2 solutions.

Figure 6. MTST Item III-4

Table 3 presents the percent of students from both groups who were able to give one solution, two solutions, and more than two solutions, regardless of computational errors.

Table 3
Results of MTST Item III-4

	more than 2 solutions (4 or 3 points)	2 solutions (2 or 1 point)	1 solution (no point)	no/wrong solution (no point)
Filipino	7%	31%	19%	43%
Japan	47%	33%	17%	3%

Almost half (47%) of the Japanese group is able to generate more than two solutions, while only 7% of the Filipino group does so. Common solutions that were given by both Filipino and Japanese students are presented in Figure 7.

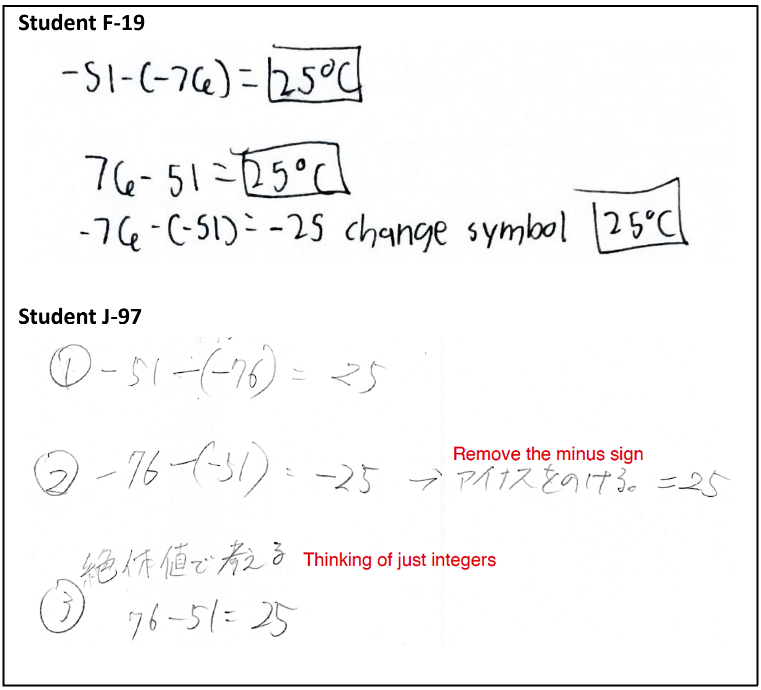


Figure 7. Solutions in MTST Item III-4 that are common to both Filipino and Japanese groups

The first solution in Figure 7 applies the concept of subtracting the smaller number from the bigger number in order to get the difference. In the second solution, the negative signs are "removed"; since the numbers being dealt with are both negative, getting their difference is the same as getting the difference of their absolute values. In the third solution, the numbers are subtracted regardless of whether the minuend is bigger or smaller than the subtrahend. The sign is removed since temperature difference requires an answer without sign. This solution recognizes that the absolute difference between two numbers does not change even when the minuend and subtrahend are interchanged. Being able to provide these meaningful solutions means that the student has a good grasp of knowledge and skills on integers, subtraction, and absolute value. Moreover, these solutions which demonstrate different thinking processes exhibit creativity in terms of flexible thinking.

Another solution common to both groups is drawing a number line and then manually counting the units between -76 and -51. Examples are shown in Figure 8.

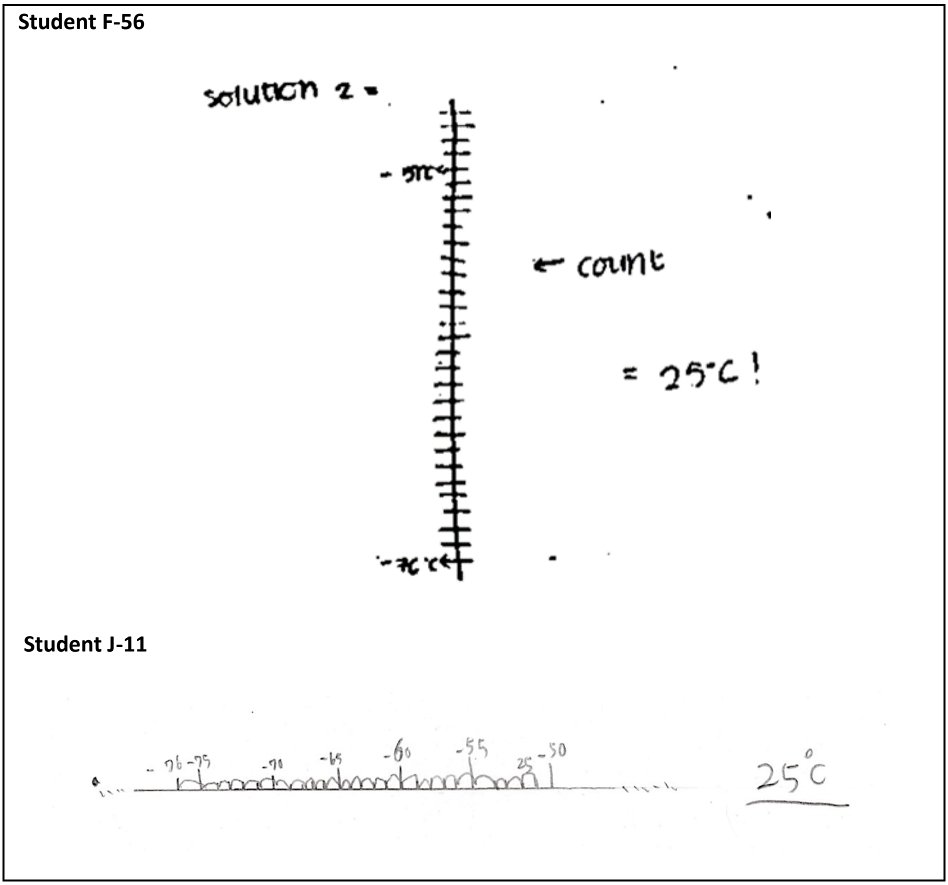


Figure 8. Number line solution for MTST Item III-4 that is common to both Filipino and Japanese groups

There are some solutions that can only be found in the Japanese group. One of these is the use of reference numbers – numbers which can be used instead of the original given to make calculations easier. Examples of the use of reference number are shown in Figure 9.

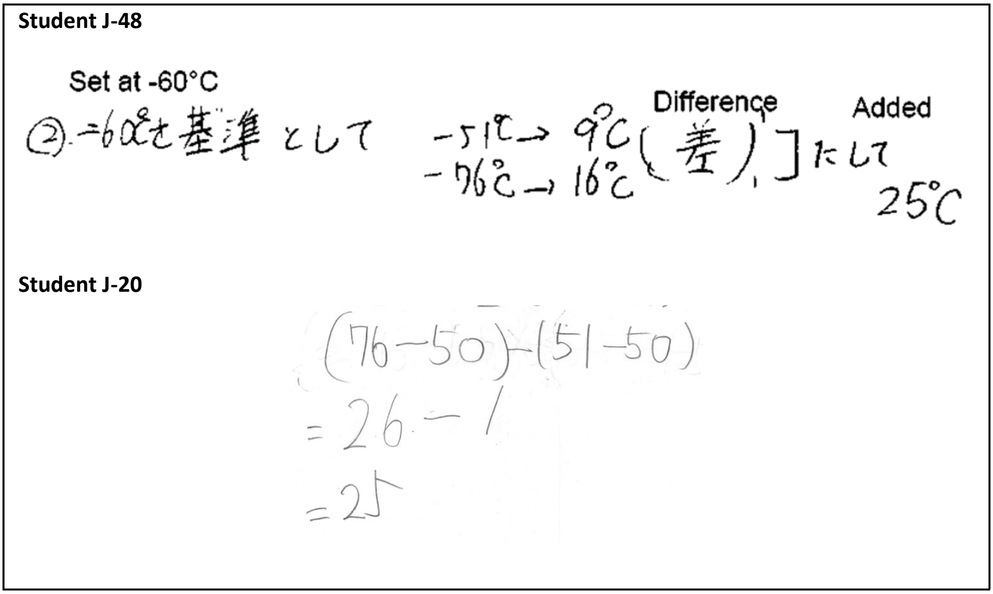


Figure 9. Examples of using reference numbers to solve MTST Item III-4

In the solution of student J-48, 60 is set as the reference number. Sixty is conveniently chosen; because it is between 51 and 76, and the difference between 60 and 51, and between 60 and 76 can be easily be computed. Afterwards, the computed differences (9 and 16) are added. An even more convenient reference number is chosen by student J-20. The student used 50 as reference and ended up subtracting 26 and 1.

Another solution unique to the Japanese group is decomposing the numbers according to place value (Figure 10) to make calculations easier. In the solution, -51 is decomposed into -50 and -1, and -76 is decomposed to -70 and -6. The difference between -50 and -70, and the difference between -1 and -6 are then calculated. Afterwards, the two differences (20 and 5, respectively) are added resulting in 25.

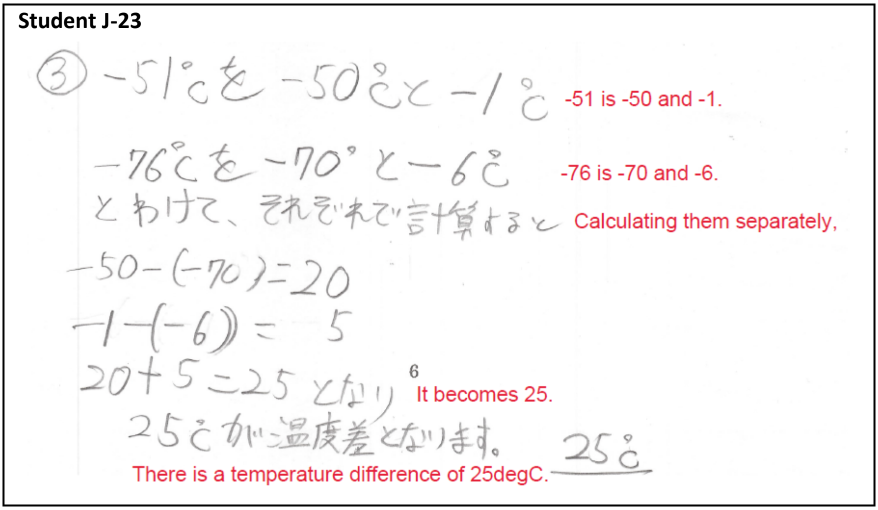
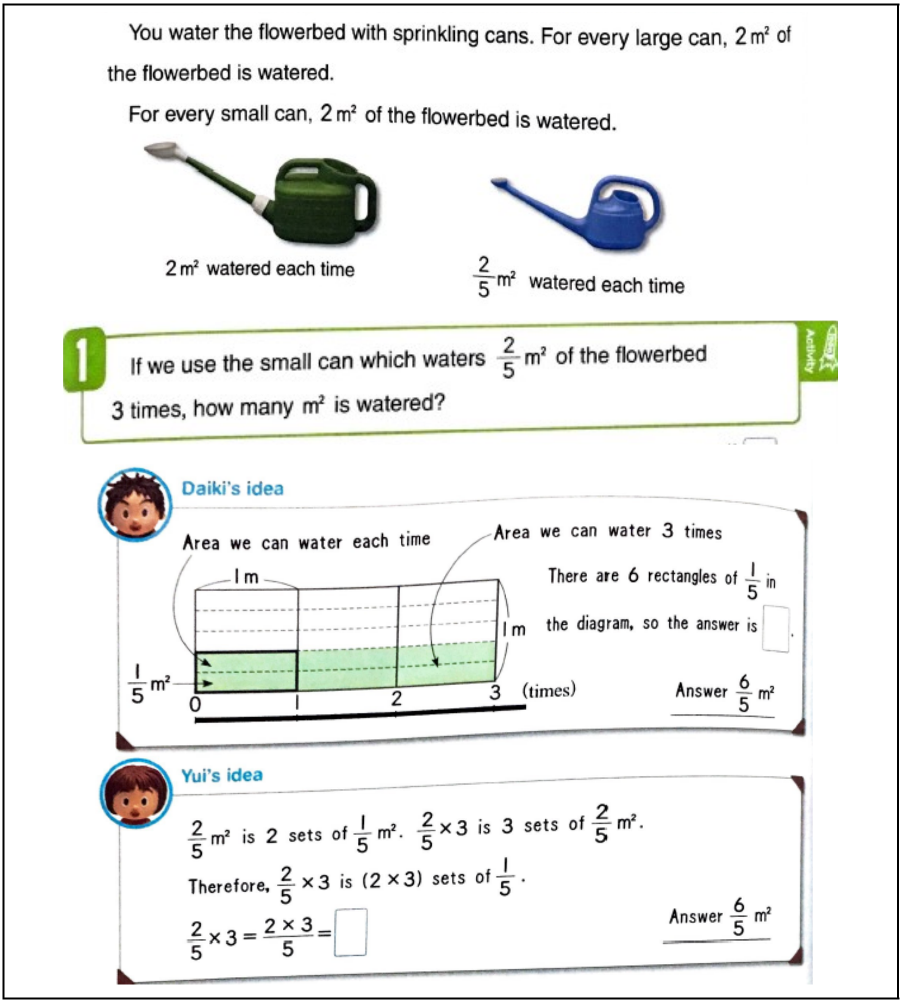


Figure 10. Example of decomposing numbers to answer MTST Item III-4

The solutions presented in figures 9 and 10 demonstrate a good grasp of numbers, the meaning of difference, and practical applications of properties of operations. According to Jo Boaler (2015), students who can work flexibly with numbers demonstrate a strong number sense – they understand numbers and can use them to solve problems and make judgments.

The ability of the Japanese students to come up with different and unique solutions can be traced from how mathematics classes are conducted in Japan. There were three phases in the Japanese mathematics classes: problem-posing, generating solutions, and practice. In the problem-posing phase, the teacher gave an unfamiliar problem to the students. Through a story-telling approach, the teacher elaborated on what the problem meant and why it was important that the problem be solved. The problem given initiated the critical thinking of the students. In the next phase, pair or group discussions alternately happened with whole-class discussions. The students discussed with their seatmates their ideas on how to solve the problem. During this phase, the students were asked to be conscious about their thinking processes. While the students were working, the teacher moved around the room to observe and occasionally help or give suggestions to some students. After some minutes, the teacher asked some students to share different solutions. Then the different solutions were summarized, and important concepts were emphasized. The students naturally developed creative thinking in this phase as they suggested different solutions to solve the problem. The third phase was when the students practiced what they learned. The teacher gave only 2 problems which the students answered individually.

Japanese textbooks are designed to mimic the classroom experiences of the students. A problem is presented and the whole discussion is anchored on the problem. In most lessons, multiple solutions are presented after the problem. An example is presented in Figure 11.



You water the flowerbed with sprinkling cans. For every large can, 2 m^2 of the flowerbed is watered.

For every small can, 2 m^2 of the flowerbed is watered.

2 m^2 watered each time

$\frac{2}{5}$ m^2 watered each time

1 If we use the small can which waters $\frac{2}{5}$ m^2 of the flowerbed 3 times, how many m^2 is watered?

Daiki's idea

Area we can water each time

Area we can water 3 times

1 m

1 m

$\frac{1}{5}$ m^2

0 1 2 3 (times)

There are 6 rectangles of $\frac{1}{5}$ in the diagram, so the answer is ☐.

Answer $\frac{6}{5}$ m^2

Yui's idea

$\frac{2}{5}$ m^2 is 2 sets of $\frac{1}{5}$ m^2. $\frac{2}{5} \times 3$ is 3 sets of $\frac{2}{5}$ m^2.

Therefore, $\frac{2}{5} \times 3$ is (2×3) sets of $\frac{1}{5}$.

$\frac{2}{5} \times 3 = \frac{2 \times 3}{5} =$ ☐

Answer $\frac{6}{5}$ m^2

Figure 11. Multiple solutions in Grade 5 Mathematics textbook (p. 168) by Study with Your Friends Mathematics Textbook for Elementary School, 2015, Japan: Gakko Tosho

Some lessons in the Filipino modules also contain a word problem at the beginning but the discussion is not sustained throughout the lesson. The word problem only serves as an introduction. Moreover, only one solution is presented, usually the method that is easiest computationally. An example is presented in Figure 12.

Leksyon 10.10 Addition ng Unlike Fractions
Lesson 10.10 Addition of Unlike Fractions

Bumili si Alma ng $\frac{3}{4}$ kg. ng manok at $\frac{2}{5}$ kg. ng baka.

Ilang kilogram ng manok at baka ang kanyang nabili?

Alma bought kg of chicken meat and kg of cow meat. How many kilograms of meat did she buy in all?

I-add ang $\frac{3}{4}$ at $\frac{2}{5}$. Pansinin na magkaiba ang mga denominator. Hindi sila maaaring i-add kaagad. Kailangang gawing pareho ang mga denominator. Paano?

Add and . Notice that the denominators are different. The two fractions cannot be added right away. The denominators need to be the same. How?

Hahanapin natin ang kanilang mga equivalent fraction para maging pareho ang kanilang denominator. Kukunin natin ang LCD ng dalawa.

Let us find the fractions equivalent to the two fractions which have the same denominator. Get the LCD of the two denominators.

The LCD of 4 and 5 is 20.

Ang LCD ng 4 at 5 ay 20.

Write the equivalent fractions.

Isulat ang mga equivalent fraction.

$$\frac{3}{4} = \frac{15}{20}$$

$$+\ \frac{2}{5} = \frac{8}{20}$$

Add.

I-add.

$$\frac{15}{20} + \frac{8}{20}$$

Write the answer as a mixed number in simplest form.

$$\frac{23}{20} = 1\frac{3}{20}$$

Isulat ang sagot bilang mixed number sa simplest form.

Samakatuwid, bumili si Alma ng $1\frac{3}{20}$ kg. ng manok at baka.

Therefore, Alma bought kg of chicken and cow meat.

Figure 12. Problem and solution in Grade 4 Mathematics module (p. 178) by Bennagen & Sibbaluca, 2004, Quezon City: UP Integrated School

The observed classes in the Philippines also did not provide evidence of promoting flexible thinking. The observed classed involved three phases: review of the previous lesson, whole-class discussion, and practice. During the review, the teacher asked the students about what they did and learned in their previous meeting. In the whole -class discussion, the teacher guided the students through the development of mathematical concepts by asking a series of questions. The questions were directed to the whole class, and the teacher called out some students to answer. Incorrect answers were processed while correct answers were emphasized. This was the phase where most of the critical and creative thinking questions were asked. During the practice phase, the teacher gave 10 problems and the students answered by pair. The teacher monitored the students as they work on the problems. The solutions and answers to the problems were discussed afterward.

The ability to provide multiple solutions coming from different perspectives is an evidence of creativity (Herlina, 2015), a higher-order mathematical thinking skill that is targeted in the modern mathematics teaching and learning. Flexibility is also considered a life and career skill according to the Philippines' K to 12 curriculum. Clearly, the Japanese students' exposure to multiple solutions not only in classroom discussions but also through their textbooks make them more flexible thinkers. This advantage over the Filipino students emerged in the MTST results.

Teacher-made assessments provide further insights about the mathematical thinking skills of the Filipino and Japanese students. Sample tests created by teachers and given to students as graded assessments were collected from both countries and were scrutinized in terms of the level of mathematical thinking (recall, basic thinking, critical thinking, and creative thinking) assessed by each item. The tests are about integers, the topic covered by MTST that is discussed in Grade 7.

The sample test from the Philippines has 25 questions with a total of 50 points. Table 4 shows the distribution of items according to the level of mathematical thinking.

Table 4
Level of Thinking Assessed in Sample Test from Philippines

	Recall	Basic Thinking	Critical Thinking	Creative Thinking
No. of items	1	22	2	0
%	4%	88%	8%	0%
No. of points	1	44	5	0
%	2%	88%	10%	0%

In the sample test from the Philippines, almost all questions (22 out of 25) assess basic thinking (series of operations, application problems). There is one recall question (meaning of a negative exponent) and only two critical thinking items. Examples are presented in Figure 13. There are no creative thinking items found in the sample test from the Philippines.

Recall

True or False: The expression $(-4)^{-3}$ is $-\frac{1}{4^3}$.

Basic thinking

Solve for N: $-2(12 - 15)^2 \div (-3)(2) = \text{N}$

Solve for N given that a = -2, b = -1, c = 2, and d = 3: $c^a - a^{-d} + c^b = N$

Critical thinking

The sum of two integers is (-4). The smaller integer is 4 more than (-2) times the larger integer. What are the two integers?

Figure 13. Sample items from a mathematics quiz about integers from the Philippines

On the other hand, the sample test from Japan contains 51 questions with a total of 96 points. Table 5 shows the distribution of items according to the level of mathematical thinking.

Table 5
Level of Thinking Assessed in Sample Test from Japan

	Recall	Basic Thinking	Critical Thinking	Creative Thinking
No. of items	9	28	13	1
%	18%	55%	25%	2%
No. of points	10	54	30	2
%	10%	56%	31%	2%

The majority (55% of items, 56% of points) of the questions from the Japanese sample test assess basic thinking (mostly operations and series of operations). A quarter of the items are on critical thinking (reasoning, critical thinking problems, etc.). There are also some (18% of items, 10% of points) recall items (reading negative integers, definition of natural numbers, etc.) and one (2% of items and points) creative thinking item (multiple solutions). Examples are presented in Figure 14.

Recall

1 次の［　　　］にあてはまる言葉や数をかきなさい。＜知各１点＞
（１）０より大きい数を正の数、０より小さい数を［　①　］の数といい、－４を［　②　］４と読む。　①

1) Fill in the box with the appropriate wording. <1 point each for knowledge math>
(1) A number greater than 0 is called a positive number, while a number less than 0 is called __(1)__ , which makes -4 to be read as __(2)__4.

Basic thinking

Do the following calculations. <2 points each for technical math>

5 次の計算をしなさい。＜技各２点＞

（１）　－１０＋（－８）	（２）　－ 0.4 ＋ 0.7	（３）　$-\frac{3}{4}-\frac{2}{3}$

Critical thinking

Answer the following using reason. (Explain your reasoning.) <3 pts each for concept math>

7 次の計算を工夫して行いなさい。（どのような工夫をしたかも説明せよ。）＜考各３点＞

（１）　１２＋（－３１）－４５－（－３１）	（２）　23 ×（－ 12）＋ 23 × 112

Creative thinking

3 （－４）－１１の計算について、次の問いに答えなさい。＜知各２点＞
（１）この計算を加法に直すとどのような式になるか。
（２）この計算を【同符号】と【絶対値】と【和】という用語をすべて使って、計算方法を説明せよ。○の中に符号、（　）の中に計算式、□の中に答えを書け。

3) With regards to the calculation of (-4) - 11, answer the following questions. <2 points each for knowledge math>
(1) When adding these two values how do you show this in an equation?
(2) When using the concepts of same signs, absolute values, and sums, how do you show this in an equation?

Figure 14. Sample items from a mathematics quiz about integers from Japan

The distribution of items in the sample test demonstrates that the Japanese students are more used to being assessed critically and creatively than their Filipino counterparts. Since instruction and assessment are always interconnected, the structure of the sample tests also gives a preview on the focus of instruction in class. Based on the sample tests, it can be predicted that in the classroom, Japanese students are given more opportunity to think critically and creatively than the Filipino students. This is true in the classes observed. In the generating solutions phase of the Japanese classes, the students were given equal time to struggle with the questions which gave all of them opportunity to think critically and creatively. All students in the Japanese mathematics class had the chance to form and share their thoughts before the mathematical concept was revealed. In contrast, in the discussion phase of the Philippine mathematics class, it was observed that only the students who were quick in giving answers were able to generate their own ideas; others just followed the flow of the discussion and accepted the concepts being formed as their classmates answered the questions posed by the teacher. Some students might have attempted to answer the questions on their own, but their thinking was halted by the reveal of the answer by a quicker classmate. This means that the mathematical concepts may be very meaningful to some while merely rote knowledge to others since they are forced to accept the concepts as the class discussions move forward. This may be a factor to the larger variation of MTST scores of the Filipino students (*SD* = 10 points) compared to the Japanese students (*SD* = 7). Furthermore, Boshen (2016) stated that students become more skilled mathematical thinkers when they are taught how to solve problems using multiple strategies. Japanese students are engaged to generating multiple solutions to a single problem during their classroom discussions. This may explain their significantly better scores in MTST.

Mathematical Communication

Table 6 presents the mean and standard deviation of the results of MCAT.

Table 6
Descriptive Statistics for MCAT

	N	Mean	Std. Deviation	Std. Error Mean
Filipino	70	26.2000	5.67884	.67875
Japanese	70	28.2571	5.27693	.63071

The highest possible score is 40. The scores of the Japanese students are higher (*M* = 28.257) and less varied (*SD* = 5.277) than those of the Filipino students (*M* = 26.200, *SD* = 5.679). A dependent samples t-test was run to determine whether this difference is significant. Table 7 shows the result of the t-test.

Table 7
Dependent Samples t-test for MCAT

Paired Differences							
			95% Confidence Interval of the Difference				
Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
-2.03571	7.08710	.84707	-3.72557	-.34586	-2.403	69	.019

At .05 level of significance, the results of t-test show that there is a significant difference in the mathematical communication ability of Filipino and Japanese students, $t(69) = 2.036$, $p = .019$. This means that Japanese students could better express their mathematical thoughts and understanding than their Filipino counterparts.

The qualitative difference between the mathematical communication ability of the two groups is most evident in the first item of MCAT as presented in Figure 15.

In an international school, $\frac{7}{8}$ of the students are from Asia and $\frac{1}{16}$ are from Europe. What fraction of the entire student population are **neither** from Asia nor Europe?

a. Write an equation that can be used to solve the problem.

b. Use your equation in (a) to solve the problem. Present your solution in detail and **explain each part** of your solution.

Figure 15. MCAT Item 1

The correct equation to solve the problem is $1-\left(\frac{7}{8}+\frac{1}{16}\right)=N$ or $1-\frac{7}{8}-\frac{1}{16}=N$. An example of the desired explanation to this equation is shown in Figure 16.

Student F-25

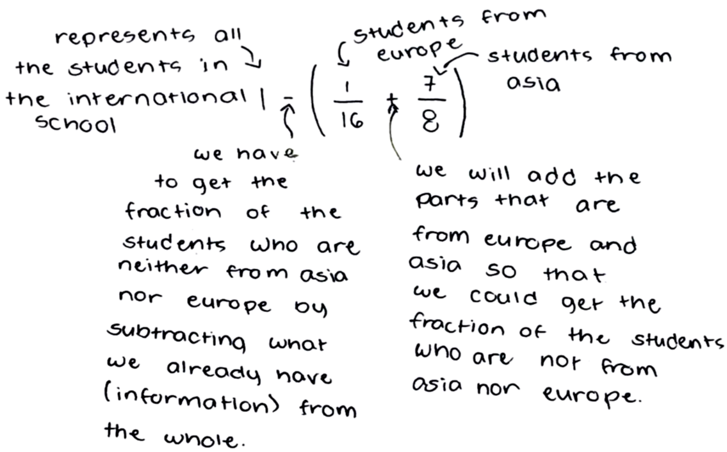


Figure 16. Example of desired explanation in MCAT Item 1b

Note that in the desired explanation, the relevance of each operation is discussed. For example, student F-25 explains that addition is applied to get the total fraction of students who are from Asia and Europe, and subtraction is applied to get the fraction of students who are neither from Asia nor Europe. However, many Filipino students did not explain the relevance of each operation but merely enumerated the steps in adding and subtracting fractions instead of discussing why performing such operations was necessary to solve the problem. Examples of such explanations are listed below:

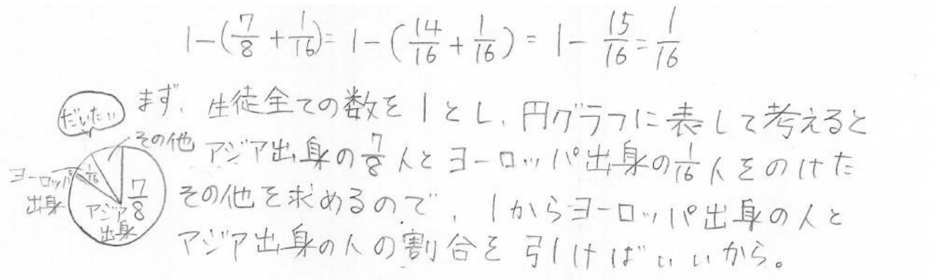
F-31 *Convert $\frac{7}{8}$ to $\frac{14}{16}$ so it can be like fractions with $\frac{1}{16}$.*

F-10 *Make the fractions like fractions by getting the least common denominator.*

F-41 *You may ask where I got $\frac{16}{16}$. $\frac{16}{16}$ is equal to 1. Why $\frac{16}{16}$? Because the denominator of the other fractions is 16.*

It is notable that these detailed explanations of the process of adding or subtracting fractions are not present in the Japanese group. Most of the Japanese students anchored their explanations to the relevance of such process on the problem. An example is shown in Figure 17. Student J-13 did not explain how to subtract fractions; instead, he/she explained why the two fractions need to be subtracted from 1. Furthermore, Student J-13 supplemented his/her explanation with an illustration of a pie chart.

Student J-13



First suppose that the number of all students is 1. Showing it in a pie chart, I need to obtain the value of others. So I except people from Asia which is 7/8 and people from Europe which is 1/16. I subtracted those fractions from 1.

Figure 17. Example of a response of a Japanese student in MCAT Item 1b

Sample classroom tests and textbooks provide a good explanation for the difference in the mathematical communication ability of the two groups. The sample test from the Philippines has 3 items (worth 3 points) that are related to mathematical communication (Figure 18). These are items that ask for a number sentence before solving the given problem. Such items allow students to express mathematical processes through mathematical symbols.

V. Read and solve. Write the number sentence as part of your solution. Write the correct label in your final answer. **(10 points)**

1. Bart's investment in a certain company gained P700 during the first month, lost P600 during the second month, lost another P500 during the third month but gained P100 during the fourth month.
   A. How much money did Bart have at the end of 4 months? **(2 pts)**
   How much is the **AVERAGE** investment gain or loss in the four months? **(1 pt)**

Figure 18. Mathematical communication ability item in the sample test from the Philippines

Items that require translating a word problem to a workable number sentence are also present in the Filipino modules (Figure 19).

Answer completely. Write the number sentence and write the correct unit and label. Encircle your final answer in simplest form.

B. Sagutin nang kumpleto. Isulat ang number sentence at lagyan ng tamang unit at label.

Bilugan ang sagot na nasa simplest form.

6) Inabot ng $\frac{5}{6}$ ng isang oras si Jon sa pagtugtog ng piano. Si Jovy naman ay tumagal ng $\frac{3}{4}$ ng isang oras sa pagtugtog ng violin. Sino ang mas matagal nag-ensayo sa pagtugtog ng instrumento? Ipaliwanag ang sagot.

It took Jon 5/6 of an hour playing the piano. While Jovy played the violin for 3/4 of an hour. Who took rehearsing playing a musical instrument longer? Explain your answer.

Figure 19. Item in Mathematics 4 module that promotes mathematical communication (p. 188) by Bennagen & Sibbaluca, 2004, Quezon City: UP Integrated School

On the other hand, the Japanese test has 3 items (worth 10 points) that test mathematical communication. One of these items asks the students to explain how to solve a problem (Figure 19). The other two require the students to solve using reason instead of calculation and then explain their reasoning afterward, e.g., explain your reasoning on how to solve 12 + (-31) - 45 - (-31).

6 青木さんは、計算問題を、1日 20 問ずつ行うことを目標としている。下の表は、月曜日から土曜日までに行った計算問題数の目標との違いである。このとき次の問いに答えなさい。<(1)～(4)技各2点、(5)考4点>

Aoki's goal is to complete 20 math problems a day. The chart below shows the difference between her goal and the actual number of math problems she completed, from Monday through Saturday. Use the chart to answer the following questions.

Day of the week	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
曜　日	月	火	水	木	金	土	日
目標との違い	＋ 10	－ 5	－ 15	ア	0	+13	

日曜日は目標を変える

Difference from goal

（５）青木さんは、月曜日から土曜日までに自分が行った問題数の平均を求めようと考えた。そこで青木さんは「仮平均を２０問」に設定して、求めることにした。どのようにして求めればよいか考え方を書き、月曜日から土曜日までに青木さんが行った問題数の平均を求めよ。

Aoki decided to find the average number of math problems she completed from Monday to Saturday. Aoki set the tentative average to 20. How can one come to this tentative average? **Explain the method**, then calculate the actual average number of math problems Aoki completed from Monday to Saturday.

Figure 20. Mathematical communication ability item in the sample test from Japan

The Japanese textbook also contains some items which require students to explain their answer. An example is shown in Figure 20.

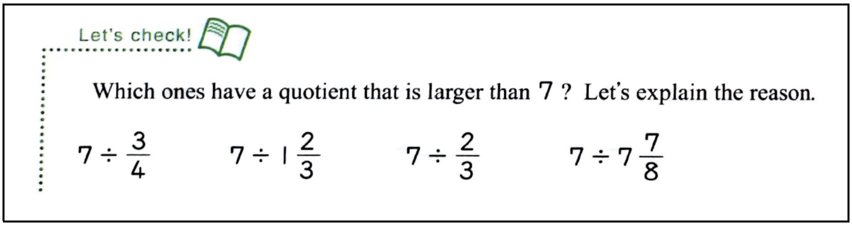


Figure 21. Item in Mathematics 6 textbook that promotes mathematical communication (p. 50) by Study with Your Friends Mathematics Textbook for Elementary School, 2015, Japan: Gakko Tosho

The presence of such items in the Japanese tests and textbooks illustrates why when asked to explain, the Japanese students utilize their reasoning skills instead of giving the step-by-step procedure. This also supports the advantageous results of MCAT in favor of the Japanese group. Meanwhile, the number sentence requirement in the sample tests from the Philippines shows the importance they give on the skill of representing mathematical thoughts through numbers and symbols.

The flow of the classroom discussions also explains the results of MCAT. Both Filipino and Japanese students were given opportunities to express their mathematical thinking in their classes. The difference lies in the kind of thinking the students were communicating. The Filipino students were mostly engaged in mathematical discourse during the practice phase of the lesson wherein they answered a series of questions on with a partner. This gave them the chance to explain their answers, and sometimes justify their solutions in case they had a different answer from their partner. This explains why the object of the Filipino students' explanations in MCAT is algorithms. Meanwhile, the Japanese students were engaged in mathematical talk during the generating solutions phase of the lesson. Since this phase was focused on formulating solutions, the Japanese students were trained to explain the mathematical processes that went through their minds, as well as comprehend the mathematical thoughts of others. Moreover, they were experienced in making meanings of the mathematical processes which explains why their explanations in the MCAT were geared towards reason.

Mathematical Thinking and Mathematical Communication Ability

Pearson's product-moment correlation reveals that there is a strong positive correlation ($r = .599$, $p < .001$) between the two variables, mathematical thinking and mathematical communication ability, at .05 level of significance. Simple linear regression was run to determine whether mathematical thinking skills predicts mathematical communication ability. The results are shown in Tables 8.1, 8.2, and 8.3.

Table 8.1
MTST-MCAT Linear Regression – Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.570[a]	.325	.320	4.58433	2.080

a. Predictors: (Constant), MTST

b. Dependent Variable: MCAT

Table 8.2
MTST-MCAT Linear Regression – ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	1394.463	1	1394.463	66.352	.000[b]
	Residual	2900.223	138	21.016		
	Total	4294.686	139			

a. Dependent Variable: MCAT b. Predictors: (Constant), MTST

Table 8.3
MTST-MCAT Linear Regression – Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	13.812	1.692		8.162	.000	9.392	18.231
	MTST	.281	.035	.570	8.146	.000	.191	.372

a. Dependent Variable: MCAT

Table 8.1 gives the R value for the simple linear regression model. The MTST scores accounted for 32.5% of the variation in the MCAT scores with adjusted R^2 = 32.0%. The R value is .570 indicating a large size effect. The ANOVA result in Table 8.2 shows that the regression model significantly ($p < 0.001$) predicts MCAT scores. Table 8.3 shows that the value of the regression coefficient (.281) is significantly ($p < 0.001$) contributing to the prediction of scores in MCAT. The regression equation is

MCAT score = 13.812 + 0.281 (*MTST score*).

This finding suggests that MTST score significantly predicts MCAT score. This means that a student who can think critically and creatively in mathematics is more likely than not to be able to communicate his or her mathematical thoughts effectively.

Conclusions and Recommendations

The present study reveals that Japanese students have better mathematical thinking skills than Filipino students. The students' solutions to the sample mathematical problems show that Filipino students tend to represent knowledge symbolically while Japanese students tend to represent knowledge through images and models. Furthermore, Japanese students are flexible problem solvers while Filipino students are fixed on one approach to solving mathematical problems, usually the computational solution. Textbooks and modules, teacher-made assessments, and classroom observations provide evidence that Japanese students are given more opportunities to practice higher order mathematical thinking skills than their Filipino counterparts.

In addition, Japanese students are the better communicators of mathematical thinking. Filipino students communicate their mathematical thoughts by explaining procedures and algorithms. Japanese students communicate mathematically by explaining the relevance and importance of the mathematical processes in relation to the problem. Textbooks and modules, teacher-made assessments, and classroom observations reveal that Japanese students are exposed to making meanings of the mathematical processes while Filipino students are experienced in expressing their mathematical thoughts in symbols, specifically, number sentences.

Finally, mathematical thinking predicts communication ability. This implies that effective communication of mathematical thoughts is an evidence of higher order mathematical thinking skills.

With this, it is recommended that Filipino students must be given more opportunities to think critically and creatively during mathematics classes. To accomplish this, mathematics lessons should be designed such that all students are given time to dig deeper into mathematical concepts. Likewise, teachers could encourage students to communicate their mathematical thoughts through activities that require them to talk or write about their thinking. Authors of learning materials should also write textbooks that put emphasis on critical and creative thinking, i.e., mathematical concepts and solutions to word problems should be scaffolded not immediately reported, and practice exercises could include items that require multiple solutions or multiple answers. Since mathematics education goals have shifted, there should be a follow-through of the assessment of mathematics learning. Upon including creativity and critical thinking in the delivery of math lessons to the students, test writers could write achievement tests that assess mathematical thinking beyond computations and applications of operations. Open-ended questions could be integrated to assess critical and creative thinking as well. Items that ask students to communicate their thinking by explaining or describing mathematical processes can be integrated in the achievement test since mathematical communication is proven to be evidence of mathematical thinking.

Future action-researches could focus on teaching strategies that give more time for students to struggle with mathematical problems to produce multiple solutions rather than rehearse operations and algorithms. Researchers who wish to study mathematical thinking and mathematical communication ability could improve the analysis of data by conducting student- and teacher-interviews. Researchers interested in cross-national studies can re-do the present study to explore the differences in the thinking processes of students from other nations. It would be interesting to determine if there are differences in the mathematical thinking and communication ability of students from countries which consistently score high in international comparative studies, like Finland and Japan.

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