A Passage To Schroeder-Bernstein

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Dedicated to fellow parents of the graduates of 2008

McDonald's, Commonwealth Avenue, Quezon City, southwest corner, 9:30 pm. By now, Vigile[2] has struggled for the better part of two hours on the problem: "The set A union B is countably infinite if and only if A and B are each countably infinite." How teasingly elementary until one tries to actually show that it concerns a map of a certain variety into the set of all integers. Bouncing between excitement and despondence, she had tried different approaches but none had bagged the prize. The proof by contradiction, I suggested, looked promising for a while but died upon closer scrutiny. The exasperation, growing with each dead end, was palpable.

"I have spent hours on this one problem alone and I still have four to go," she moaned flashing in my direction the problem set for Math 109. "I'm so dumb," she lamented.

In creeping desperation myself, I proposed another tack: this set is (a) not finite, and (b) not uncountable like a continuum, ergo. But again the trail grows cold. That's the curse and the glory of Mathematics: absolute truth is never just a hand wave away. You have to slog through a series of steps, each as self-evident as one plus one equals two. "Self-evident," that is, only to the practiced eye, for there is a high price to pay on the road

to such peaks of clarity. Like hours of unrequited struggle! "Absolute proof" shrieks *The Economist,* in one charming rumination over algorithmic proofs, is not negotiable, not even by super zippy Boolean off and ons.

Why don't we try another problem: "The set of all polynomials is countably infinite." One has to construct the set and find a way of counting the elements. Counting the infinite? One can almost hear the great Gordan exclaim: *"Das ist nicht Matematik; das ist theologie!"* It doesn't get any easier, it seems. A double bagel beckons. No wonder the despondence.

McDonald's Commonwealth closes at half past twelve; time to clear out. Nothing it seems to show for all that mental contortion. "It's never totally lost," I observed, drawing on the wisdom of the ages but it never seemed so lame. Energy, it is true, is always conserved but that is a global and not a local imperative. Locally, one can really feel deprived. Till lately, I have always seemed to manage to pull the rabbit out of the hat; to suggest a path to catharsis. Not this time. It was distressing, clearly but other emotions seem to be at work as well. Like guilt?

And why not? She had read *Fermat's Dilemma* on my suggestion having thoroughly enjoyed it myself. She had then been pleasantly intrigued by Euclid's proof of the irrationality of the square root of two. There is a pulse of power in knowing something so compellingly true beyond space and time. I subsequently approved of Math 109 as an elective, which she unearthed from the UP CRS. "No prerequisites," she beamed.

Inquiries from friends about the offering and Dr. Agnes Paras, the teacher, returned glowing reports.

"Very competent researcher and a very good teacher," texted Dr Fidel Nemenzo.
"The real deal. I worship at her altar," raved Academician Pol Nazarea.

But was Vigile cut out for it? Dr. Paras herself had doubts. *"Iha, Econ ka. Bakit ka kukuha nito? Baka magsisi ka,"* she admonished. Vigile would have to sink or swim with Math majors. Obviously Dr. Paras had seen stragglers beat a hasty retreat after a few weeks, wasted most likely. Designs of graduating with honors had expired at the crest of this youthful bravado.

"Kakayanin ko ma'am," Vigile had insisted, more it seemed to me out of *pasubo* than conviction. In keeping with conventional wisdom, she, however, saw to it that if Math 109 proved a mistake, the resulting retreat wouldn't come in the way of graduating with honors.

Math 109 is of course required for Math majors. It is the gateway to the universe of Abstract Mathematics – the universe, may I add, where one gets a glimpse of *how God thinks*. I myself have never stepped into this universe. Instead, like Thomas Hardy's tragic "Jude", I have often paused at its gate, craning to catch faint echoes from the high priesthood's discourses. Popular accounts (e.g., Edna Kramer's "Nature and Growth of Modern Mathematics," S. Nasar's "A Beautiful Mind" and BBC's "Fermat's Last Theorem") allowed the laity no more than a sociological, if still absorbing, eye view of the agonies rewarded or unrewarded by the ecstacies. Confident intimacy born out of true understanding is never in the bargain.

Euclid's proof of the "irrationality of the square root of two" is an exception to the rule that the profound is impenetrable; an absolute proof of both charming accessibility and devastating finality. Akin to this was what savants through the centuries believed was the Holy Grail that could not be contained in Pierre de Fermat's "narrow margin." Vigile had, without warning, declared her own disdain for so-called "computer-aided proof." Was I reading too much into those signs?

McDonald's is my usual hangout whenever a need arises for some heavy mental calisthenics. A cup of coffee makes good company when one ponders a conundrum. The hustle and bustle of *"kita-kits"* are easily blocked out. More enabling, no one has a