A Proposal of a Demand Shifting Function to Address the Demand-Capacity Imbalance of Capacity-Constrained Systems that Shift Demand by Discount Setting

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Abstract – *This paper addresses the problem of low profit of service establishments because of the imbalance between capacity and demand. The hypothesis of this paper is that profit can be maximized by shifting demand from high demand periods to low demand periods. The proposal of this research is to shift the demand through discount setting that is structured and optimized, which is the demand shifting function to be applied to maximize profit. This paper states general framework and the specific method to arrive at the recommended demand shifting function. Furthermore, it is applied to a real test case of a spa service facility. Results show that for the test case, profit is increased and the variability of demand among time periods is decreased.*

Keywords—demand-capacity imbalance, demand shifting function, service facility

I. INTRODUCTION

In many organizations, a usual problem encountered by the operations manager is how to address demand given fixed capacity. In production systems, during periods of low demand, the product units produced by the fixed capacity in excess of the demand, can be stored as “anticipated inventory” and be used for periods when demand is higher than the capacity. This cannot be done in service systems because the service, which is the product of the system, cannot be stored or stocked as inventory for future use. Thus, establishments such as restaurants or personal service facilities like wellness centers, must be able to think of ways to keep the difference between demand and capacity as low as possible. For these service establishments, when demand is higher than their fixed capacities, the unserved demand translates to opportunities for profit. On the other hand, when demand is lower than the capacity, costs are incurred because of running the establishment at the fixed capacity while revenues are not generated because of the low demand for the service. A commonly-used method for keeping the balance between demand and capacity for service organizations is the use of promotional discounts, however no proper scientific approach yet [1,2]. Offering promotional discounts during times when demand is lower than the capacity, results to the stimulation of demand on those low demand periods. At times, the “stimulated demand” comes from completely new customers. In other cases, the stimulated demand comes from customers in high demand periods, who shift their demand from a high demand period to a low demand period, because they have higher probability of being served if they shift their demand. This demand shifting occurrence is the subject of this research. Specifically, the interest of this research is on the mathematical approach on demand shifting via the use of promotional discounts.

II. THE PROBLEM AND OBJECTIVE OF THIS RESEARCH

This paper aims to solve the problem of how to maximize profit by minimizing the gap between demand and capacity.

High demand compared to capacity results in lost sales because of customer balking. On the other hand, low demand compared to capacity results to underutilized resources. These underutilized resources translate to operational overhead or depreciation costs, while not correspondingly generating revenues

Thus, this study aims to solve the problem of profit maximization in a service facility where inventory (of service) is not possible. Profit of a service facility for a given time interval with multiple time periods i inside is:

$$Total\ Profit = \sum_{i=1}^{n} (a_i d_i + (a_i + b)[MIN(c - d_i, 0)]) \qquad (Eqn. 1)$$

where i = *time period*
a_i = *revenue per demand served for time period i (Php)*
b = *shortage cost penalty (Php)*
c = *capacity (units)*
d_i = *demand for time period i (units)*

The Equation above was derived from three papers [3, 4, 5] but shall be discussed in detail in another paper as the construction and validation of the resulting nonlinear program to be discussed further is a scope of another paper.

$$Profit = a_i c - b(d_i - c)\ if\ c < d_i \qquad (Eqn. 2)$$
$$Profit = a_i d_i\ if\ c > d_i \qquad (Eqn. 3)$$

The Equation inside the summation of Equation 1 when evaluated per time period i becomes:

Equation 2 gives the profit when there is shortage of capacity or when demand is higher than capacity. Shortage cost penalty per unserved demand is subtracted from the revenues from served demand. Equation 3 gives profit when there is underutilization of capacity or when demand is lower than capacity. Profit is simply revenues multiplied to the demand served less the relevant costs.

Relevant components that have to be balanced are: (a) revenue and (b) shortage cost. Profit will increase if revenue increases. The system's total revenue for a period such as a day, can increase if the revenue not generated due to missed or unserved demand of balking customers at a high demand period, e.g. peak hour of the day, will be realized when shifted to a period of low demand, e.g. lean hour of the day. Note, however, that revenue that would have been realized from balked customers, may not be completely generated with the shifting of demand because discounts will be given, i.e., the discount may result to reduced revenue because of the lower (discounted) price of the service, which is offered with the intention of shifting demand. Another way to look at this is to increase profit by decreasing the shortage (of capacity) cost in originally high demand periods by shifting them to low demand periods.

Thus, the scope of this study is demand shifting. The ultimate objective is to make demand level or almost equal among different periods. With demand almost equal among periods, a facility's capacity will be better able to meet demand, given the assumption that capacity is fixed for most periods. This study would be most applicable to facilities with highly time-varying demand that significantly experience shortage costs resulting from missed sales opportunities. The decision variables are (a) when to impose demand shifting and (b) how much demand shifting should be applied to maximize profit.

III. REVIEW OF RELATED LITERATURE

Existing literature within the area of price manipulation include the following:

Table 1. Summary of Related Literature in Price Manipulation

Papers	Description
Whitin [6], Zabel [7], Arcelus and Srinivasan [8], Chen and Simchi-Levi [9], Chen and Simchi-Levi [10], Chen and Simchi-Levi [11]	These papers are on the topic of inventory management. They tackle price manipulation wherein price is treated as inversely proportional to demand. Therefore, price in each time period is determined so as to maximize profit. These research works recognize the correlation between price and demand but do not aim for the active manipulation or shifting of demand to and from different time periods.
Littlewood [12], Gallego and van Ryzin [13], Feng and Gallego [14], Belobaba and Wilson [15], Chatwin [16], Zhao and Zheng [17], Wang, Wang, and Lynch [18], McAfee and Te Velde [19]	These papers are on the topic of yield management. They tackle price manipulation but only for one particular period at a time. Similarly, price is treated as inversely proportional to demand so price is adjusted to maximize profit by determining the optimal point. Since price manipulation is only for a single period, there is no demand shifting.
Taylor et al [20], Spees and Lave [21], Samadi et al [22], Faria and Vale [23], Xu and Li [24], Temple and Ma [25], Kobayashi et al [26], Joe-Wong et al [5]	These papers are on the topic of demand response systems particularly used by the electric grid industry. They tackle price manipulation wherein price is treated as inversely proportional to demand so price in each time period is determined so as to minimize total cost or to reduce variability of demand. However, none introduced actual demand shifting except the last paper mentioned, but which has limited applications to the electric grid industry.

So far, none of the existing literature addresses profit maximization of systems where shortage cost due to unmet demand is significant and where the system is characterized by fixed capacity and highly time-varying demand.

Also, there is only one paper in literature [5], that provides a demand shifting function which involves the actual shifting of demand from one time period to another when price is changed. This paper is a significant basis for the proposal in this paper.

IV. HYPOTHESIS AND PROPOSAL OF THIS RESEARCH

The hypothesis of this research is that profit can be maximized by minimizing the gap between demand and capacity, which can be realized by shifting the demand. Demand shift can be done by the offering of promotional discounts.

Furthermore, the structure of the promotional discount and the time period of the shift can be modeled mathematically and optimized. This optimized promotional discount is proposed by this research to be applied with the demand shifting function to maximize profit.

4.1 General Proposal

The framework for profit maximization using demand shifting is as follows:

1. The time interval of the operations of the facility is divided into multiple smaller time periods, e.g. operations of a day is divided into hours. This is to see the variability of the demand among the smaller time periods.
2. The average demand per time period is computed using historical arrival rate of demand and the duration of the stay. For example, the average demand for time period 1, is computed using the historical arrival rate of demand during time period 1 and the duration of stay of historical demand within this time period 1.
3. A demand shifting function is used by this research to maximize profit by balancing demand and capacity through the different time periods across the whole time interval.

The process is as illustrated in the example below:

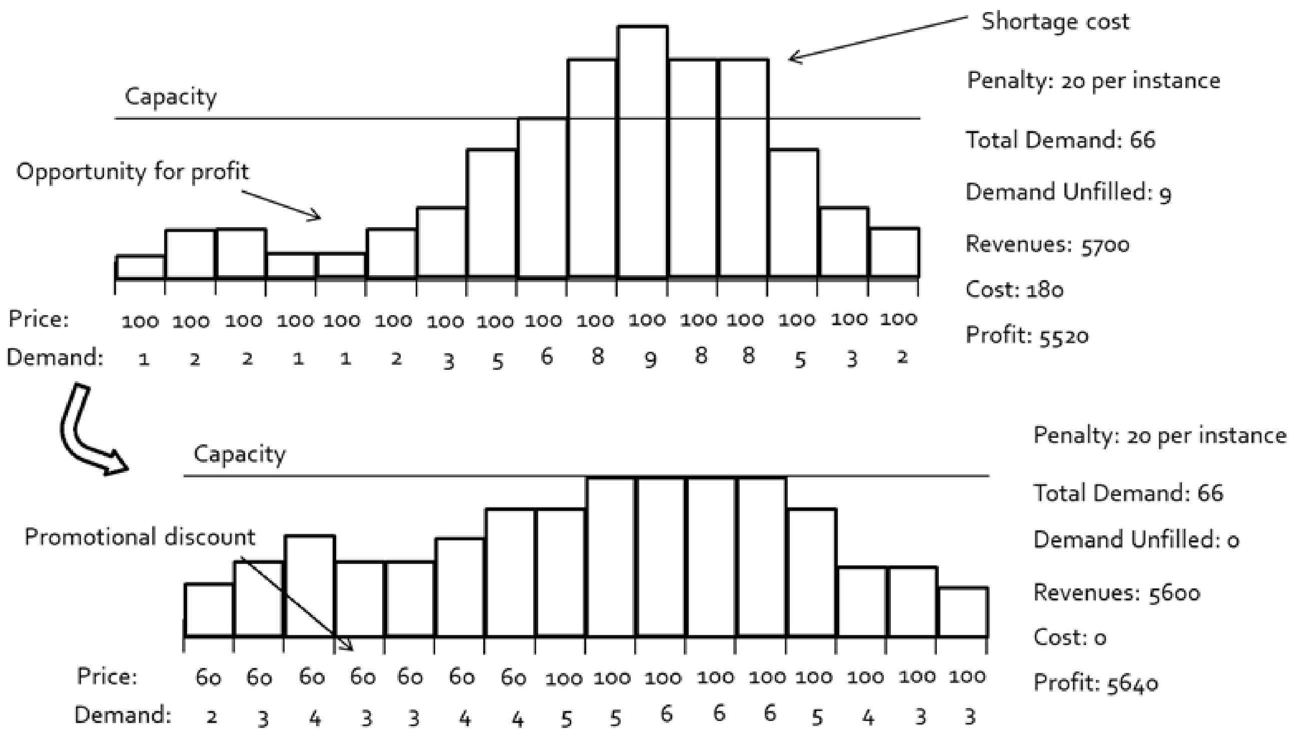


Figure 1. Illustration of possible effect to demand and profit of discount setting

Given the framework above, the following are then needed as input data:

1. Arrival rate of customers per time period
2. Stay duration of customers per time period
3. Capacity of service facility
4. Revenue per demand served (i.e. product price)
5. Penalty per demand unserved (i.e. shortage cost)

This demand shifting function is incorporated in a nonlinear program that is used to maximize overall profit. While within the scope of the overall research, the nonlinear program is not within the scope of this paper and will be discussed in another paper.

4.2 Shifting Function

The demand shifting function used is:

$$s_{ik}(r_i) = \gamma r_i (D_k - D_i)^{+}$$

(Eqn. 4)

where $s_{ik}(r_i)$ = *proportion of demand shifting from time period k to time period i*
r_i = *amount of discount (price reduced) at time period i (Php)*
D_i = *demand at time period I (units)*
$(D_k - D_i)^+$ = *positive part of difference in demand between time period k and time period I (i.e. how much is demand in time period k greater than demand in time period i, else zero)*
γ = *normalizing constant (which can also be adjusted depending on behavior and type of service facility to simulate strength of shifting)*

The function, Equation 4, is formulated such that proportion of demand that moves from time period k to time period i, is proportional to:

a) the amount of discount at time period i, i.e. time period shifted to (higher discount, higher shift),
b) the positive part of the difference in demand between time period k and time period i, i.e. difference of demand in time period shifted to from time period shifted from (higher difference in demand, higher shift).

These two are built in the function so that demand will shift more strongly to time periods with higher discounts and to time periods with lower demand.

The normalizing constant, γ, has two roles:

a) to ensure the value of function is from 0 to 1 (i.e. proportion can only take these values)
b) to simulate strength of shifting depending on behavior and type of service facility

Thus, the highest possible value of γ to ensure the highest value of function is 1 as derived from the Equation of demand shifting function is:

$$\gamma_{max} = \frac{1}{r_{max}(D_{max} - D_{min})} \qquad \text{(Eqn. 5)}$$

where r_{max} = *largest possible value of all* r_i *(which is equal to the revenue per demand served i.e. product price) (Php)*
D_{max} = *largest demand in all time periods (units)*
D_{min} = *smallest demand in all time periods (units)*
$(D_{max} - D_{min})$ = *difference between largest and smallest demand in all time periods*

This is then the value for the normalizing constant which can be adjusted based on the sensitivity of demand shifting to price reduction and the difference in demand.

The construction of the demand shifting function assumes that demand shift always happens when price is changed although at varying rates proportional to price reduction and demand difference. The demand shift is also assumed to be linear and is regardless of any other factors like proximity between time periods, etc. Also, this demand shifting function does not involve queuing, i.e. customers immediately leave when the system is full. Lastly, it is assumed that there will be no additional demand from outside when price is reduced (i.e. demand only shifts from one time period to another and the total does not change throughout the whole time interval).

The normalizing constant, γ, is same along the whole index i (i.e. along all time periods i). This is because the demand shifting function is based on a previous paper [5] which also assumed this and also to simplify the model since we assume that behavior of customers reacting to change in price is the same all throughout the time interval under study.

The paper in which the demand shifting function was based from [5] uses:

$$s_\beta(r,t) = C_\beta \frac{r}{(t+1)^\beta} \qquad \text{(Eqn. 6)}$$

where $s_\beta(r,t)$ = *proportion of demand shifting within time periods t given r*
C_β = *shifting constant to match with real data*
r = *amount of discount*
t = *number of time periods between time periods with shifting*
β = *constant to match customer behavior in real data*

The difference is that while this function is also proportional to amount of discount at time period shifted to, it is inversely proportional to distance between time periods in consideration, with a constant β to match real data. This factor, however (proximity between time periods), is not included in this paper and is replaced by relative demand since real-life applications in service facilities (e.g. hotels, restaurants, contact centers) may include multimodal distributions of demand or special hours that cannot be influenced by demand shifting.

V. APPLICATION OF THE DEMAND SHIFTING FUNCTION TO AN ACTUAL TEST CASE OF A SPA SERVICE FACILITY

The demand shifting function is incorporated in a nonlinear program which maximizes total profit as presented below. Details of the nonlinear program are to be discussed in another paper.

$$\text{Max Profit} = \sum_{i=1}^{n}\left[(P-r_i)\left(D_i + \sum_{k\neq i} D_k\gamma(D_k-D_i)^+ r_i - \sum_{k\neq i} D_i\gamma(D_i-D_k)^+ r_k\right) + (P-r_i+B)(t_i)\right] \qquad (Eqn. 7)$$

$$\text{s.t. } 0 \le r_i \le P \ \forall\, i = 1,2,\dots,n$$

$$t_i \le C - \left(D_i + \sum_{k\neq i} D_k\gamma(D_k-D_i)^+ r_i - \sum_{k\neq i} D_i\gamma(D_i-D_k)^+ r_k\right) \ \forall\, i = 1,2,3,\dots n$$

$$t_i \le 0 \ \forall\, i = 1,2,3,\dots n$$

Decision variables:

r_i = discount at time period I (Php)
t_i = auxiliary variable for positive difference in capacity and demand (t_i = MIN[c – d, 0]) (units)

Parameters:

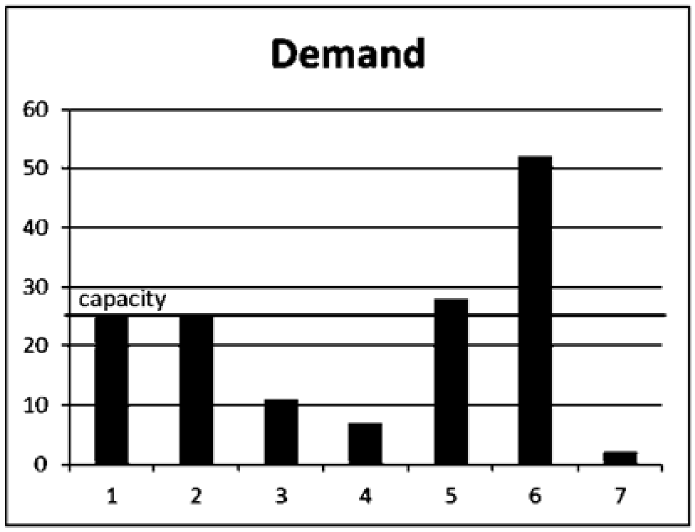
P = selling price of product (Php)
D_i = original demand at time period I (units)
γ = normalizing constant (which can be adjusted to simulate strength of shifting)
B = shortage cost penalty (Php)
C = capacity of service center (fixed) (units)

The nonlinear program can be used for any number of time periods inside the time interval under study. The nonlinear program can be proven, again to be detailed on another paper, to be a quadratic program that is concave on its entire search space, therefore a global optimal solution can be found for any set of inputs in a fast solving time.

The nonlinear program with the demand shifting function is applied on actual data from a spa service facility based on a previous study [27]. Following the framework, the time interval to study is set for one week since variability happens most at the weekly interval. Average demand per day is derived from historical data together with all other inputs. Since there is no information regarding customer behavior and the relationship of demand shift to price change, the normalization constant for demand shifting function is analyzed at different levels. The nonlinear program is ran using Lingo and for each instance, a global optimum is found at < 1 second solving time.

Below is the result when the nonlinear program is ran at the highest possible level for the demand shifting function.

Original (w/o discount):



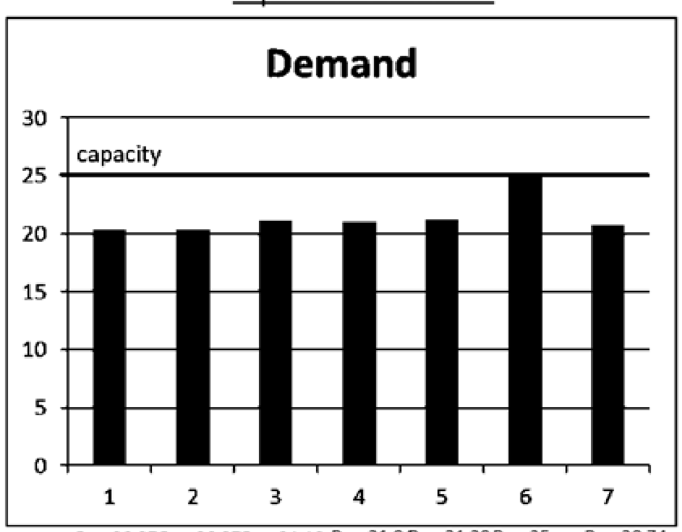
$D_1 = 25$ $D_2 = 25$ $D_3 = 11$ $D_4 = 7$ $D_5 = 28$ $D_6 = 52$ $D_7 = 2$

$r_1 = 0$ $r_2 = 0$ $r_3 = 0$ $r_4 = 0$ $r_5 = 0$ $r_6 = 0$ $r_7 = 0$

$P_1 = 200$ $P_2 = 200$ $P_3 = 200$ $P_4 = 200$ $P_5 = 200$ $P_6 = 200$ $P_7 = 200$

$Z = 23400$ Revenues = 24000 Cost = 600

Optimal solution:



$D_1 = 20.37$ $D_2 = 20.37$ $D_3 = 21.18$ $D_4 = 21.04$ $D_5 = 21.29$ $D_6 = 25$ $D_7 = 20.74$

$r_1 = 3.34$ $r_2 = 3.34$ $r_3 = 32.48$ $r_4 = 36.64$ $r_5 = 0$ $r_6 = 0$ $r_7 = 40.64$

$P_1 = 196.7$ $P_2 = 196.7$ $P_3 = 167.5$ $P_4 = 163.4$ $P_5 = 200$ $P_6 = 200$ $P_7 = 159.36$

$Z = 27562.27$ Revenues = 27562.27 Cost = 0

Figure 2. Comparison of current configuration to effect when discount setting is applied

The profit increases from 23,400 to 27,562.27 which is a 17.79% increase. Also, the variance in daily demand decreases from 282.95 to 2.62 with the range decreasing from 50 to 4.63. This shows that with the demand shifting via discount setting, profit can be maximized and variability in demand decreased.

V. DISCUSSIONS

Sensitivity analysis is performed on the value of the demand shifting function normalization constant which affects how much demand shifts in relation to price change due to discount setting.

Below are the results of running the same data using different values of γ.

Table 2. Sensitivity Analysis of NLP Solution for Different Levels of γ

Demand shifting function γ	Remarks on γ	Optimal solution discount setting	Demand per time period after shifting	Abs. increase in Profit (compared to original: 23400)	% increase in Profit (compared to original: 23400)	Actual proportion of demand that shifted	Variance in demand (original: 282.95)	Range of demand (original: 50)
$\frac{1}{10000}$	Highest possible value	$r_1 = 3.34$, $r_2 = 3.34$, $r_3 = 32.48$, $r_4 = 36.64$, $r_5 = 0$, $r_6 = 0$, $r_7 = 40.64$	$D_1 = 20.37$, $D_2 = 20.37$, $D_3 = 21.18$, $D_4 = 21.04$, $D_5 = 21.29$, $D_6 = 25$, $D_7 = 20.74$	4162.27	17.79%	58.62%	2.62	4.63
$\frac{1}{20000}$	50% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 38.84$, $r_4 = 48.88$, $r_5 = 0$, $r_6 = 0$, $r_7 = 58.28$	$D_1 = 21.54$, $D_2 = 21.54$, $D_3 = 17.03$, $D_4 = 16.36$, $D_5 = 23.52$, $D_6 = 34.56$, $D_7 = 15.44$	2134.94	9.12%	38.47%	42.96	19.12
$\frac{1}{30000}$	33% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 30.32$, $r_4 = 48.28$, $r_5 = 0$, $r_6 = 0$, $r_7 = 64.81$	$D_1 = 22.68$, $D_2 = 22.68$, $D_3 = 14.06$, $D_4 = 13.16$, $D_5 = 25$, $D_6 = 40.47$, $D_7 = 11.96$	1361.48	5.82%	25.57%	98.46	28.51
$\frac{1}{40000}$	25% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 21.60$, $r_4 = 47.47$, $r_5 = 0$, $r_6 = 0$, $r_7 = 71.11$	$D_1 = 23.25$, $D_2 = 23.25$, $D_3 = 12.56$, $D_4 = 11.53$, $D_5 = 25.75$, $D_6 = 43.45$, $D_7 = 10.20$	831.91	3.56%	19.06%	135.49	33.25
$\frac{1}{50000}$	20% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 4.87$, $r_4 = 38.32$, $r_5 = 0$, $r_6 = 0$, $r_7 = 68.73$	$D_1 = 23.83$, $D_2 = 23.83$, $D_3 = 11.15$, $D_4 = 9.92$, $D_5 = 26.50$, $D_6 = 46.42$, $D_7 = 8.34$	548.39	2.34%	12.55%	178.54	38.08
$\frac{1}{100000}$	10% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 0$, $r_4 = 0$, $r_5 = 0$, $r_6 = 0$, $r_7 = 57.69$	$D_1 = 24.63$, $D_2 = 24.63$, $D_3 = 10.94$, $D_4 = 6.98$, $D_5 = 27.53$, $D_6 = 50.33$, $D_7 = 4.96$	183.39	0.78%	3.95%	247.17	45.37
$\frac{1}{200000}$	5% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 0$, $r_4 = 0$, $r_5 = 0$, $r_6 = 0$, $r_7 = 36.01$	$D_1 = 24.90$, $D_2 = 24.90$, $D_3 = 10.98$, $D_4 = 6.99$, $D_5 = 27.87$, $D_6 = 51.53$, $D_7 = 2.83$	29.90	0.13%	1.11%	272.54	48.7
$\frac{1}{300000}$	3.33% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 0$, $r_4 = 0$, $r_5 = 0$, $r_6 = 0$, $r_7 = 14.33$	$D_1 = 24.97$, $D_2 = 24.97$, $D_3 = 10.99$, $D_4 = 6.99$, $D_5 = 27.97$, $D_6 = 51.88$, $D_7 = 2.22$	3.16	0.0135%	0.3%	280.26	49.66
$\frac{1}{400000}$	2.5% of original	$r_1 = 0$, $r_2 = 0$, $r_3 = 0$, $r_4 = 0$, $r_5 = 0$, $r_6 = 0$, $r_7 = 0$	$D_1 = 25$, $D_2 = 25$, $D_3 = 11$, $D_4 = 7$, $D_5 = 28$, $D_6 = 52$, $D_7 = 2$	0	0%	0%	282.95	50

The value of demand shifting function normalization constant has an effect on discount setting, resulting profit, and resulting variance in daily demand. This means that actual behavior of customers with respect to price change is important to characterize. However, it can be noted that there is a low value of the demand shifting function constant such that the original configuration (zero discount) is already the optimal solution. At this point, it is not advisable to promote any discount on prices otherwise it will lead to losses in profit. This can be the lower bound for characterized customer behavior up to when discount setting should not be used anymore.

It should be noted that in real-life, the choice of γ will be based on two factors: a) profitability, and b) practicality.

For profitability, the upper bound to use for γ is based on the Equation 5 in section 4.2. This is the highest possible value of γ that allows a total possible demand shift of up to 100%. This is also the γ that will give the highest profit. However, this value of γ may not always be practical or realistic. The lower bound to use for γ is such that it is not any more profitable to use discount setting i.e. the optimal solution that gives the highest profit is the original configuration.

For practicality, the choice of γ corresponds to an actual percentage of demand shift. When the highest possible value of γ is used, it does not directly mean that 100% of demand will shift. Actual demand shift is different to total possible demand shift or the allowed shift provided by the value of γ. But it can be seen from the sensitivity analysis (see Table 2) and if used on initial trials the actual demand that shifts given a certain value of γ. The user can therefore choose the level of γ in which the actual demand that shifts is closest to what may happen in real-life when price is changed or discount is applied.

If the scheme is applied, the choice of demand shifting function normalization constant γ must be chosen carefully and rationally. The choice should be based on these two factors especially that research in this specific topic (e.g. reaction of customers on shifting their time of demand consumption when there is price change or discount set) is little. There are no industry standard values yet for demand shifting function constants.

VII. SUMMARY AND CONCLUSION

This paper addresses the problem of low profits due to the imbalance of capacity and demand particularly for service systems where inventory cannot be used to augment capacity. The objective of this paper is to maximize profit by minimizing the gap between demand and capacity. High demand compared to capacity results to lost sales because of customer balking. On the other hand, low demand compared to capacity results to underutilized resources, that result to continued overhead costs while not generating revenues.

To solve the aforementioned problem, the hypothesis of this paper is that profit can be maximized by minimizing the gap between demand and capacity, which can be realized by shifting the demand. Demand shift can be done by the offering of promotional discounts, which in turn, can be structured and optimized that becomes the optimal demand shifting function that is proposed by this paper. The demand shifting function is constructed and incorporated in a nonlinear program that maximizes profit and reduces demand variability of a service facility to address the imbalance in demand given constrained capacity.

The nonlinear program with the demand shifting function is applied on actual data of a spa service facility. It is seen from the results that using demand shifting can increase profit and reduce variability in demand. However, since the normalization constant in the demand shifting function which also serves as the constant to simulate strength of shifting is not easily quantifiable by the service facility, it is important to do sensitivity analysis and run the nonlinear program with the constant at different levels. It can be seen from the results that this constant affects how much profit increases and how much variability in demand decreases. Also, there is a low value of the constant such that profit does not increase, variability in demand does not decrease, and the original price setting is already the optimal solution. This value of the constant can be set as the lower boundary for customer behavior up to when discount setting should not be used anymore. The choice of γ should also be based on which value provides a demand shift close to what may happen in real-life.

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