A Simple Hinge Model for Displacement-based Nonlinear Analysis of Reinforced Concrete Columns

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Abstract – *During strong earthquakes, reinforced concrete (RC) structures experience cyclic lateral loads that result to degradation in load-carrying capacity, and failure of columns in shear and/or flexure. This study presents a simple hysteretic hinge model that may be used in displacement-based analysis of RC columns, classified as flexure critical, shear critical, and shear-flexure critical, subjected to cyclic loads. The proposed hinge model made up of zero-length nonlinear springs can simulate the hysteretic behavior of reinforced concrete material in axial, shear, and flexure. The nonlinear parameters of the springs were derived from geometric and material properties of the column and estimated using Response-2000 software. Pushover analysis and response to cyclic loading were performed using the Open System for Earthquake Engineering Simulation (OpenSees) program and validated by comparing the force-displacement response of select forty-three RC columns available in the PEER Structural Performance Database. Results show that for the six rectangular columns, the numerical experiments using the proposed hinge model and the actual force-displacement curves gave R-squared values greater than 0.80 signifying good agreement of results. Therefore, it was concluded that the model can reasonably replicate nonlinear behavior of shear-, shear-flexure, and flexure-critical columns subjected to cyclic loading and, therefore, may be used to assess performance of actual RC columns.*

Keywords—hinge model, reinforced concrete, performance assessment, OpenSees, column

I. INTRODUCTION

The seismic behavior of reinforced-concrete (RC) structures is strongly affected by the earthquake intensity as well as the combined effect of bending, shear, and axial force interaction [1]. Under this circumstance, columns and beams experience cyclic lateral load in which these elements certainly suffer strength degradation of shear and axial load carrying capacity in local areas such as hinging regions.

In order to design earthquake-resistant RC members or to improve the performance of columns, there is a need to understand and investigate their nonlinear behavior during cyclic loading. This is usually done by performing experiments on actual columns and/or studying the response of numerical models. Recognizing the cost of performing experiments, numerical models are becoming popular and have evolved in the past decades. The seismic response of numerical models of columns subjected to cyclic loads depends greatly on the assumed hinge models and the corresponding nonlinear constitutive relations. Although several models have been proposed [2, 3, 4, 5, 6, 7], their implementations are generally complicated and costly.

1.1. Objective and Scope

This study presents a simple hinge model that can be used in nonlinear analysis and seismic performance evaluations of RC frames. Specifically, it aims to:

i. Propose a column hinge model that simulates axial, shear, and flexural capacities of actual columns; and
ii. Compare the nonlinear behavior and cyclic response with experimental columns.

In this paper, the authors present hinge model for rectangular RC columns and validation procedure using experimental results from literature and data obtained from the Pacific Earthquake Engineering Research Center (PEER) Structural Performance Database. Nonlinear capacities in axial, shear, and flexure are estimated using Response-2000 software and displacement-based analyses are performed using Open System for Earthquake Engineering Simulation (OpenSees).

1.2. Related Studies

Seismic assessment of RC structures requires nonlinear analysis to obtain their response during earthquake and provide better understanding of how well the members are designed [8]. Specifically, seismic response of structures depends on the hysteretic behavior of the plastic hinge regions [9]. Analyses of frames commonly make use of hinge models proposed by FEMA 356 [10] and ASCE 41 [11] although some reported that these guidelines tend to overestimate the column initial stiffness and underestimate the ultimate displacements [12].

Several hinge models with corresponding hysteresis rules such as segment-multi-spring model [2], shear-flexure interaction model [3], rigid body-spring discrete element models [4,6], and fiber hinge model [5,7] were also proposed in the past and gave satisfactory results in assessment of RC members. For hinge models that require evaluation of shear primary curve of concrete sections [3, 6, 7, 9, 13, 14,], the modified compression field theory (MCFT) [15] was used. The MCFT is an analytical model that is capable of predicting load-deformation response of RC elements subjected to in-plane shear and normal stresses. This model is implemented in a sectional analysis program, Response-2000, to calculate the strength and ductility of a RC cross-section subjected to simultaneous shear, moment, and axial load using MCFT [16, 17]. The theory, however, has the following limitations: it tends to underestimate the ultimate capacity and overestimate the stiffness for short columns with aspect ratio equal to 2.5; it does not include bar slip deformation; and it is a force-based approach which will stop once the peak strength of the section is reached [3].

The Finite Element Method (FEM) is also extensively used to obtain the approximate numerical response of structures with very good agreement with results of physical experiments [18]. The open source FEM software framework, OpenSees, is commonly used in simulating seismic response of structural systems. Due to its modular framework, it is capable of using a wide range of material models, elements, and solution algorithm for nonlinear analysis [8].

II. METHODOLOGY

The proposed plastic hinge model is composed of three nonlinear springs for axial, shear, and flexure displacements with corresponding hysteretic rules. Experimental test setup of RC columns and the equivalent numerical model of the plastic hinge are shown in Figure 1. The model connects two elastic elements represented by a zero-length element. For simulation of cyclic tests on columns, OpenSees program was used since it is capable of conducting displacement-based analysis. Moreover, its object-oriented framework enables users to define element and analysis procedure to optimize calculation.

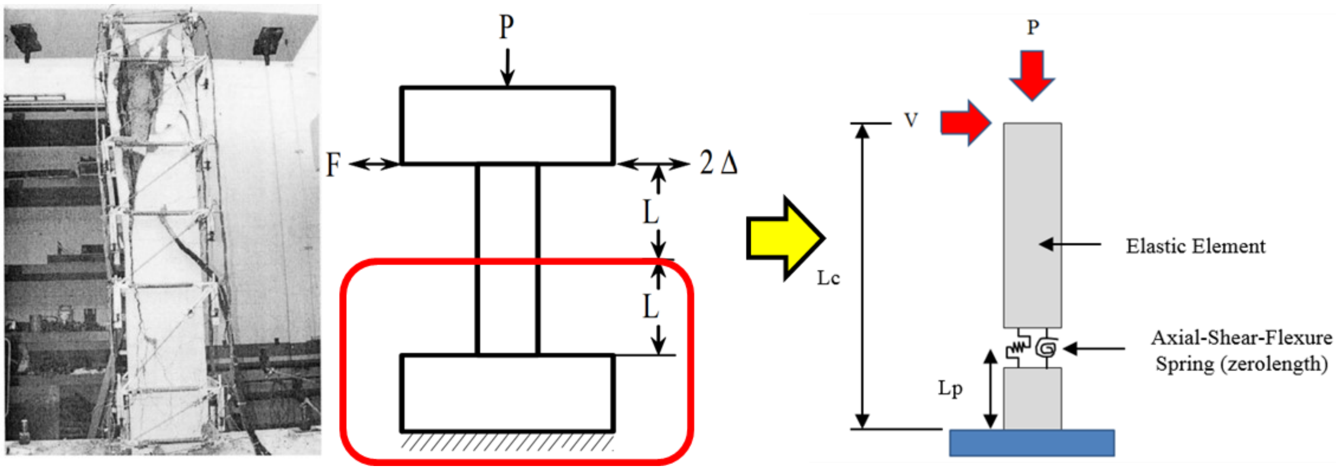

Figure 1. Cyclic loading of RC columns: (a) Actual column [19] (b) experimental setup (double cantilever) [20] (c) numerical model of proposed hinge.

Since cyclic degradation of shear is expected to be large within flexural plastic hinge [21], the location of the proposed hinge model was assumed to occur at L_p distance from the face of the support which is equivalent to the plastic hinge length. The length of plastic hinge L_p is approximated by Paulay and Priestly (1992) [22] as:

$$L_p = 0.08L_c + 0.022d_b f_y \tag{1}$$

where L_c is the distance from the face of support to the point of contraflexure (in mm); f_y and d_b are the yield stress (in MPa) and diameter (in mm) of longitudinal reinforcement.

2.1. Proposed Hinge Model Configuration

The authors proposed a hinge model composed of three independent springs that captures nonlinear capacities of the column section in axial, shear, and flexure directions. The assumed backbone curve are estimated using empirical formulas and/or computed using Response-2000 and the appropriate hysteretic rules determined using numerical investigation.

Axial Spring

Axial capacity of reinforced concrete columns is designed in such a way that the concrete strength approaches its strength first before the longitudinal steel. The backbone curve for the axial strength of columns was considered to be elasto-plastic as shown in Figure 2.a. Assuming the concrete cracking strain ε_y of 0.002 as the strain which the material reaches plastic state in compression and the ultimate force for the RC member can carry P_u defined in Eq. (2), the axial capacity curve can be defined.

$$P_u = 0.85f'_c(A_g - A_{st}) + A_{st}f_y \tag{2}$$

From Eq. (2), P_u is the ultimate axial load (in Newtons); f'_c is the specified concrete strength (in MPa); A_g is the gross area of the RC section (in mm^2); and A_s is the total area of longitudinal reinforcement (mm^2). Furthermore, the ultimate strain ε_y is limited to 0.003, setting this value as the failure point for axial load.

Shear Spring

The "Hysteretic" material default in OpenSees was used to define the hysteretic rule for shear spring. The material is capable of defining a trilinear backbone curve with additional parameters incorporate pinching, damage due to ductility and energy, and unloading stiffness degradation based on ductility. In this study, only the unloading stiffness degradation based on ductility will be considered as shown in Figure 2.b.

The backbone curve was defined using Response-2000 section analysis. From the shear-shear strain plot generated, the values of shear for cracking, yielding of reinforcement, and ultimate shear capacity and their corresponding shear strain were taken to plot the shear capacity curve.

Flexure Spring

The pinched hysteretic model proposed by Ibarra et al. (2005) [23] was used to model the flexure spring in OpenSees. The material incorporates several sources of deterioration i.e. cyclic deterioration, softening of post-yielding stiffness, and also considers residual strength after deterioration. The model has the capability to control the basic and post-capping strength, and unloading and reloading stiffness deterioration from an energy-based deterioration parameter. In this study, these parameters were set to 1.0.

Fischinger et. al (2008) [24] compared several approaches to define the backbone curve of the hysteretic model in terms of effective yield drift, ultimate drift, capping drift, post-capping stiffness, hysteretic rules, and energy dissipation. Based on their study, they adopted Eq. (3) from Fardis and Biskinis (2003) [25] and Eqs. (4) – (6) from Haselton (2006) [26] to define the backbone curve. Response2000 was used to obtain the yielding moment and corresponding yield curvature of the column sections.

$$\theta_y = \phi_y \frac{L_c}{3} + 0.00275 + a_{sl}\frac{\varepsilon_y}{(d-d')}\frac{0.2 d_b f_y}{\sqrt{f'_c}} \tag{3}$$

$$\theta_{cap} = 0.12 \cdot (1 + 0.4a_{sl}) \cdot 0.2^{v} \cdot (0.02 + 40\rho_t)^{0.52} \cdot 0.56^{0.01 f'_c} \cdot 2.37^{10.0\rho} \tag{4}$$

$$\theta_{pc} = 0.76 \cdot 0.031^{v} \cdot (0.02 + 40\rho_t)^{1.02} \leq 0.10 \tag{5}$$

$$M_c/M_y = 1.25 \cdot 0.89^{v} \cdot 0.91^{0.01 f'_c} \tag{6}$$

Where: θ_y is the yield drift [25]; θ_{cap} is the capping drift [26]; θ_{pc} is the post-capping rotation capacity [26]; M_c is the post-capping moment capacity [26]; M_y is the yielding moment capacity from Response2000 section analysis; ϕ_y is the yield curvature from Response2000 section analysis; a_{sl} is zero-one variable indicating slip of the longitudinal bars from their anchorage (1 for with slip, 0 for without slip); *(d-d')* is the distance between tension and compression reinforcement; v is the axial load ratio; ρ_t is the transverse reinforcement ratio (in critical region); and ρ is longitudinal reinforcement ratio. Pinching parameter is integrated in the model by defining the ratio of the force at which reloading begins to the corresponding force at maximum historic deformation demand.

(a)	(b)	(c)

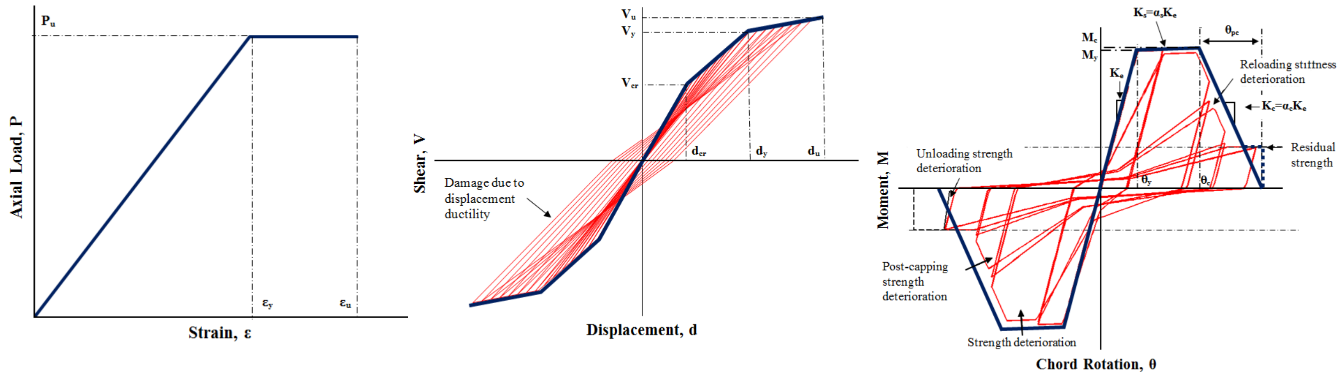

Figure 2. Hinge Parameters and Nonlinear Hysteresis model for (a) Axial (b) Shear and (c) Flexure.

2.2. Nonlinear Analysis

Nonlinear displacement-controlled analysis is conducted, using OpenSees, for the column models subjected to displacement-time histories that were derived from load-displacement data available in the PEER database. The values of lateral force and the displacement at the top of the column models are computed using a Tcl script that optimizes that use of the different implementations of Newton's algorithm available to solve the nonlinear problem.

2.3. Validation using Experimental Studies

Forty-three select columns from PEER Structural Performance Database [27] were modelled to validate the proposed hinge model. These experimental columns were reported to fail in flexure, shear, or shear-flexure. For brevity, six of these columns (two for each mode of failure) are presented in this paper and are listed in Table 1 with their relevant parameters.

To quantify how close the results from numerical model and experiment, coefficient of determination or *R-squared* values were computed for each column using Eq. (7).

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(f_i - y_i)^2}{\sum_{i=1}^{n}(y_i - y_{mean})^2} \quad (7)$$

Where f_i and y_i are the corresponding forces for numerical model and experiment, respectively, at a certain point in displacement-time history and y_{mean} is the mean value of all load values in experiment.

III. RESULTS AND DISCUSSION

3.1. Hinge Parameters

The selected columns were modelled in Response-2000 based on the properties presented. Section analysis of the program generated shear-shear strain and flexure-chord rotation curves that were used to define the spring parameters. Computed hinge parameters for specimens are shown in table 2.

Table 1. Select experimental columns from PEER Structural Performance Database [27].

	Properties	Specimen No. 1 Azizinamini et al. 1988, NC-4	No. 2 Gill et al. 1979, No. 4	No. 3 Lynn et al. 1998, 3CMD12	No. 4 Zhou et al. 1987, No. 104-08	No. 5 Sezen and Moehle No. 2	No. 6 Wight and Sozen, 1973, Spec. WS048E
	f'_c (MPa)	39.8	23.5	27.6	19.8	21.1	26.1
	Axial Load (kN)	2580	4265	1512	406	2669	178
Geometry	Width B (mm)	457	550	457.2	160	457.2	152
	Height H (mm)	457	550	457.2	160	457.2	305
	Length L (mm)	1372	1200	1473.2	160	1473.2	876
	Conc. cover (mm)	41.3	38	38.1	12.5	65.1	35
Longitudinal	Diameter (mm)	25.4	24	31.8	9.5	28.7	19
	No. of bars	8	12	8	8	8	4
	Reinf. ratio ρ	0.0194	0.0179	0.0303	0.0222	0.0247	0.0245
	f_y (MPa)	439	375	331	341	434.4	496
Transverse	No. of legs	3.4	4	3.4	2	3.4	2
	f_{yt} (MPa)	616	294	399.9	559	476	345
	Diameter d_s (mm)	9.5	12	9.5	5	9.5	6.3
	s (mm)	102	62	304.8	40	304.8	89
	Failure	Flexure	Flexure	Shear	Shear	Shear-Flexure	Shear-Flexure

Table 2. Computed hinge parameters.

Calculated Parameters		Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6
	Axial load ratio v	0.310	0.600	0.262	0.801	0.605	0.147
	Transverse reinf. ratio ρ_t	0.0052	0.0133	0.0017	0.0061	0.0017	0.0046
Axial Spring	P_u (kN)	6932.186	7964.81	4759.885	422.494	3661.292	1005.146
	ε_y	0.002	0.002	0.002	0.002	0.002	0.002
Shear Spring	V_{cr} (kN)	184.493	385.068	112.975	61.158	158.225	18.67
	d_{cr} (mm/m)	0.089	0.172	0.062	0.368	0.106	0.049
	V_y (kN)	407.886	527.652	287.189	72.151	245.29	76.151
	d_y (mm/m)	1.494	0.512	1.541	0.678	0.48	2.076
	V_u (kN)	438.472	538.605	291.274	74.892	267.794	78.742
	d_u (mm/m)	2.157	0.646	1.878	1.751	0.78	2.429
Flexure Spring	M_y (kN-m)	560.862	635.597	424.51	11.586	352.794	67.14
	ϕ_y (rad per km)	11.383	6.227	7.775	13.938	7.954	15.63
	θ_y (rad per km)	0.010307	0.006875	0.008643	0.005810	0.011663	0.012034
	θ_{cap} (rad per km)	0.044264	0.047763	0.034702	0.025074	0.019769	0.061665
	θ_{pc} (rad per km)	0.057003	0.051444	0.025990	0.012157	0.007895	0.090284
	M_c (kN-m)	652.384	727.249	512.272	13.495	423.016	82.443
Pinching Factor	K_d *(no pinching if equal to 1.0)*	1.0	1.0	0.5	1.0	1.0	0.5

3.2. Pushover and cyclic loading of columns

The load-displacement curves for pushover and cyclic load analysis on the selected column were computed and plotted in Figures 3 and 4 along with the experimental results (shown in dashed lines). For these columns, the pushover curves enveloped the experimental data except for column specimens 2 and 6 where the initial stiffness was underestimated. The pushover curves also replicated nonlinear behaviour of columns such as capping and strength degradation.

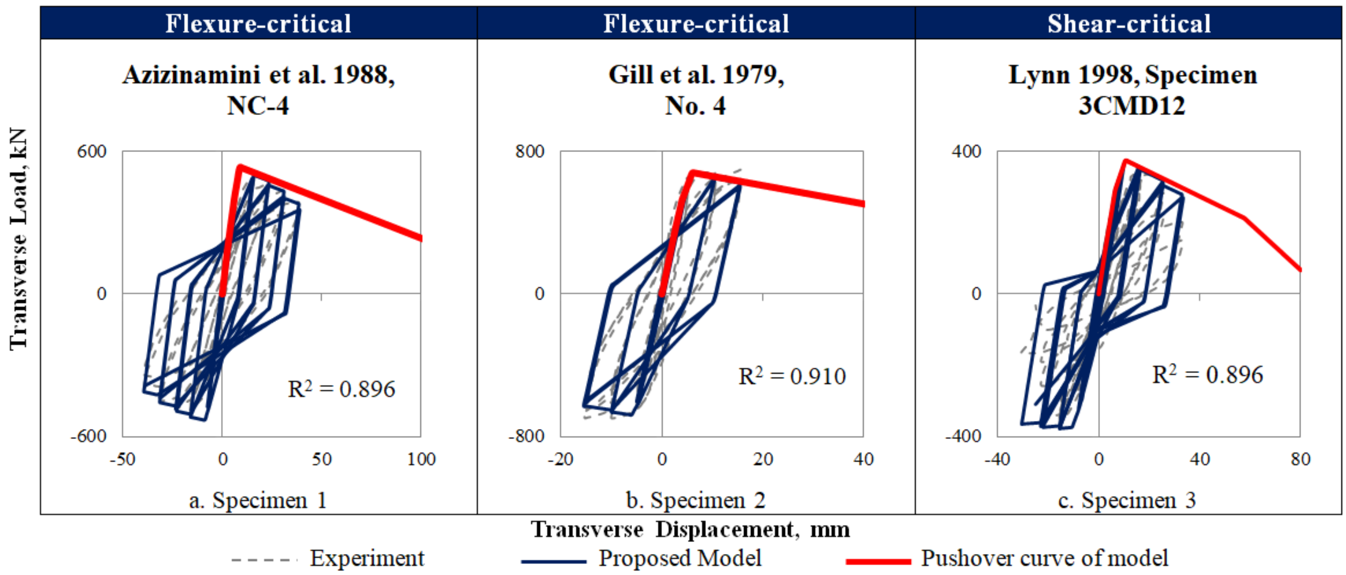

Figure 3. Cyclic loading of specimens 1 to 3.

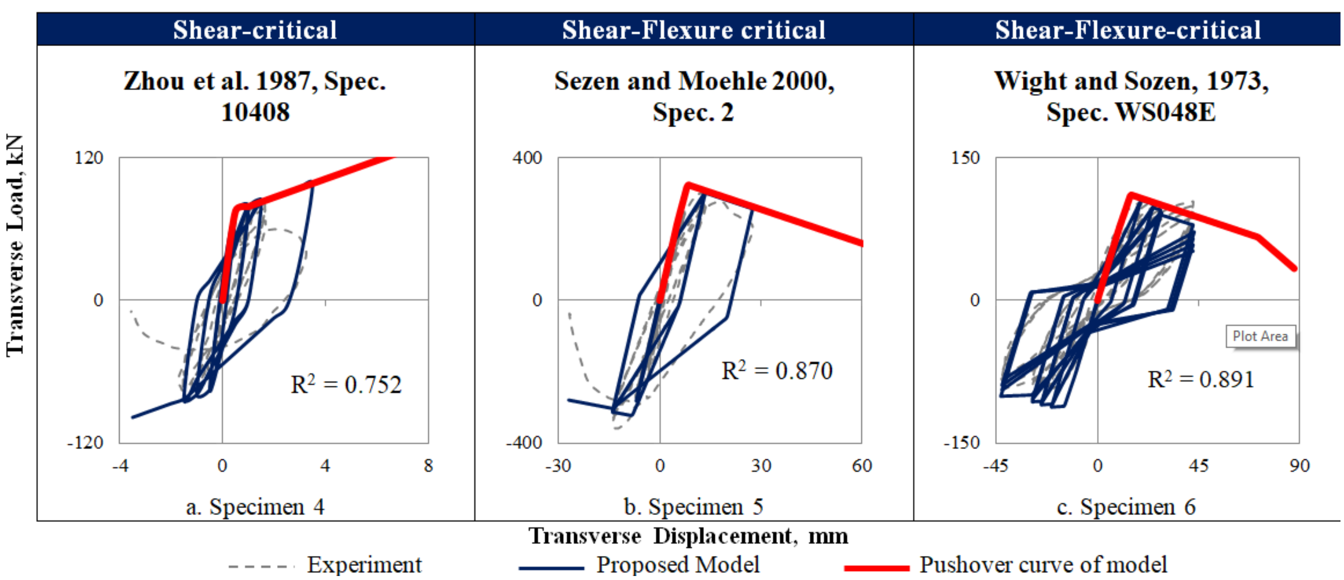

Figure 4. Cyclic loading of specimens 4 to 6.

The column models subjected to cyclic loads (shown as solid blue lines) generally predicted the capacity and ductility of columns. The degradation of both stiffness and strength, however, were not accurately simulated as pronouncedly evident in shear-critical columns specimens 3 and 4. This is because of the assumed material models that are simplified into linear and abrupt functions as shown in Figure 2. The models of shear-critical columns tend to have higher capacity even after several cycles of loading. Pinching factors assumed, specifically to specimen 3 and 6, were enough to replicate the pinching effect in the experiment.

Although MCFT tends to underestimate the ultimate capacity and overestimate the stiffness for short columns, it can be observed that the hinge model was able to envelope the load-deformation curve from experiments for shear-critical columns.

IV. CONCLUSION

A simple hinge model comprised of three nonlinear springs for axial, shear, and flexure was presented for the purpose of conducting performance assessment of RC columns, and consequently, RC frames. The nonlinear parameters of the springs were estimated using empirical formulas and by using Response-2000. Hysteretic behavior of materials was assumed and was shown to influence the nonlinear deterioration of stiffness and strength. For the six columns, presented, displacement-based pushover analysis were performed using OpenSees and responses were compared with experimental results from PEER Structural Performance Database.

Results show that the proposed numerical model of selected columns and the experimental results indicate good agreement in the nonlinear behavior when the columns are subjected to monotonic and cyclic loading. These were shown in superimposed figures and the computed *R-squared* values that are greater than 0.80 and averaging at 0.87 for the six columns presented.

The authors recommend further investigation to come up with an empirical formula in determining pinching factor and deterioration parameters such as post-capping and stiffness degradation. Hysteretic behavior can be

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