A SIMULATION STUDY ON SPECIFIC CASES OF REENTRANT-LINE SYSTEMS

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ABSTRACT

A reentrant line is a queueing network wherein work-in-process materials (WIP) pass through one or more stations several times before eventually leaving the system. Determining the scheduling mechanism that optimizes system performance is a common concern in reentrant-line configurations. A study by Daway and Matias [1] proposed an inventory control policy, called the Bounded Inventory Level Policy (BILP), which attempts to strike a balance between the frequency of setups and the variability in the internal flow processes for the purpose of reducing the long-run average total holding cost per unit output. The mathematical analysis exhibited cases wherein BILP is superior to existing inventory control policies. A simulation study was conducted on specific cases of reentrant-line systems with the same objectives as those of the analytical study.

Keywords: Bounded Inventory Level Policy, Reentrant Lines, Scheduling Policies

1. INTRODUCTION TO THE SPECIFIC CASE

The specific case examined by Daway and Matias [1] is described herein. The queueing system is composed of one service facility and a single-capacity server. All raw materials entering the system are required to visit the service facility twice before eventually leaving as finished goods. Such is the characteristics of the "two-stage, single-server tandem queuing system with setups", which is illustrated in the following figure. This system was called the "SRL" for brevity.

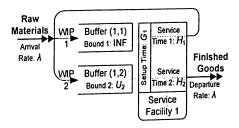


Figure 1. A Two-Stage, Single-Server Tandem Queuing System with Setups (Source: Daway & Matias[1])

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1.1 Design of Experiments

The simulation model formulated for the SRL was programmed to run under BILP with adjustable parameter levels. This suggests that such could also be run under the exhaustive policy by setting each BILP parameter as high as possible. As it is the purpose of this study to demonstrate the superiority of BILP over the exhaustive policy in various reentrant-line settings, it must then be shown via experimentation that there exist scenarios of the SRL wherein the optimal decision vector consists of components which are not all finite.

Since the alternative policies would be compared on the basis of the long-run average total holding cost per unit output coming out of the reentrant-line system of interest (E[C]), every simulation made in this study should be run for a period of time that is long enough for the process to achieve steady-state. Otherwise, the simulation results may not be reflective of the actual performance of the system even if the necessary condition for stability (i.e., $\rho_m < 1$, for m = 1 to M) is met. For this reason, each scenario was replicated ten (10) times under the terminating condition of 100,000 entries and exits.

So as to determine the optimal performance of the SRL under BILP $(E[C^{U^*}])$, simulation optimization software was used. Recall that letting U approach infinity would cause BILP to revert to the exhaustive policy. Thus, having $U \to \infty$ implies that the exhaustive policy should be the choice for inventory control, at least in the applicable SRL scenarios. Of course, it shall be shown in this study that not all situations would lead to such.

Now the experiment conducted on the SRL was so designed in such a way that the functional behavior of $E[C^{l}]$ with respect to the input parameters would manifest. Thus, several scenarios involving different combinations of ρ_1 , ρ_2 , and ρ_G , were considered. To summarize the factorial design of the simulation experiment on the SRL, the following table was constructed. Note that the random variables H_1 , H_2 , and G_1 were each assumed to have an independent exponential distribution whereas $\lambda = 0.25$ and $c_1 = c_2 = 1$.

Table 1. The Factorial Design of the Simulation Experiment on the Ski											
E[H₁]	$E[H_2]$	$E[G_1]$ $E[H_1]$		$E[H_2]$ $E[G_1]$		$E[H_1]$	$E[H_2]$	$E[G_1]$			
0.50	3.00	1.00	0.50	3.00	2.00	0.50	3.00	3.00			
1.00	2.50	1.00	1.00	2.50	2.00	1.00	2.50	3.00			
1.50	2.00	1.00	1.50	2.00	2.00	1.50	2.00	3.00			
2.00	1.50	1.00	2.00	1.50	2.00	2.00	1.50	3.00			
2.50	1.00	1.00	2.50	1.00	2.00	2.50	1.00	3.00			
3.00	0.50	1.00	3.00	0.50	2.00	3.00	0.50	3.00			

Table 1. The Factorial Design of the Simulation Experiment on the SRL

(Notice that although the values of both $E[H_1]$ and $E[H_2]$ range from 0.50 to 3.00, their sum was maintained at 3.50 so that ρ would always be equal to 0.875 and less than 1.00.) By varying the levels of $E[H_1]$, $E[H_2]$, and $E[G_1]$ across the SRL scenarios, the quantities ρ_1 , ρ_2 , and ρ_G would also change accordingly. And so with this experimental design, clues on how the optimal performance of the SRL responds to the input parameters may be revealed to such extent that the situations wherein BILP outperforms the exhaustive policy would become determinate.

1.2 Evaluation of Alternative Policies

Before proceeding to the experimentation proper, test runs were performed to verify whether or not the simulation model formulated for the SRL agrees with its analytical counterpart. The initial results of the simulation are thus presented in the following table:

Table 2. SRL Model Verification

					$E[W_1]$		$E[W_2]$			
$E[H_1]$	$E[H_2]$	<i>E</i> [<i>G</i> ₁]	<i>E</i> [<i>K</i> "]	Sim	Com	% Diff	Sim	Com	% Diff	
0.50	3.00	3.00	12.06	32.38	33.14	2.31	35.79	36.64	2.33	
1.00	2.50	3.00	12.14	28.60	28.97	1.29	37.51	37.97	1.19	
1.50	2.00	3.00	12.39	25.10	25.37	1.03	40.14	40.61	1.17	
2.00	1.50	3.00	12.66	22.54	22.44	0.45	45.83°	45.77	0.15	
2.50	1.00	3.00	12.79	20.31	20.43	0.60	56.23	56.51	0.49	
3.00	0.50	3.00	13.34	20.00	20.03	0.18	82.86	83.61	0.91	

The computed values of $E[W_1^{\omega}]$ and $E[W_2^{\omega}]$ were obtained using the formulas found in Eqs. (5) and (7), respectively. As can be perceived in Table 2, the simulated values are very close to what can be derived using the analytical model for performance. And so, it appears that the simulation model could be deemed as a good representation of the SRL.

Using simulation software, both $E[C^{U^*}]$ and $E[C^{\infty}]$ were determined in each scenario. The results of the simulation experiment on the SRL were then summarized as follows:

Table 3. Results of the Simulation Experiment on the SRL: Evaluation of Alternative Policies

E[H ₁]	E[H ₂]	E[G ₁]	U	$E[W_1^{\upsilon}]$	$E[W_2^U]$	E[C"]	% Diff	
0.50	3.00	1.00	41	18.18	19.43	37.61	2.06	
0.50		1.00	€0	18.42	19.98	38.40	2.00	
1.00	2.50	1.00	26	17.27	19.25	36.52	2.59	
1.00	2.50	1.00	∞	16.29	21.19	37.49	2.00	
1.50	2.00	1.00	19	18.01	18.69	36.70	4.14	
1.50	2.00	1.00	∞	14.68	23.61	38.29	7.14	
2.00	1.50	1.00	26	15.88	23.16	39.04	7.81	
2.00	1.50	1.00	∞	13.78	28.57	42.34	1.01	
2.50	1.00	1.00	19	20.80	22.16	42.96	15.88	
2.50	1.00	1.00	- 00	13.36	37.71	51.07	13.00	
3.00	0.50	1.00	15	27.52	20.15	47.67	39.63	
5.00	0.50	1.00	60	15.08	63.88	78.96	00.00	
0.50	3.00	2.00	38	25.84	26.81	52.65	2.88	
0.50	0.00	2.00	80	25.86	28.35	<u>54.21</u>	2.00	
1.00	2.50	2.00	65	22.39	29.11	51.50	1.07	
1.00	2.50		∞	22.56	29.50	52.06		
1.50	2.00	2.00	33	21.76	29.29	51.05	3.85	
1.50			∞	20.39	32.71	53.10	3.00	
2.00	1.50	2.00	39	19.95	33.40	53.35	5.54	
2.00	1.30	2.00	∞	18.45	38.02	56.47	3.34	
2.50	1.00	2.00	25	26.02	31.02	57.04	11.29	
2.50	1.00	2.00	8	16.95	47.35	64.30	11.25	
0.00	0.50	2.00	25	30.65	33.52	64.17	29.65	
3.00	3.00 0.50		®	17.54	73.67	91.21		
	3.00	2.00	86	32.21	35.60	67.81	0.53	
0.50		3.00	3.00	3.00	80	32.38	35.79	68.17
	2.50	2.00	61	28.40	36.83	65.24	1.32	
1.00		3.00	∞	28.60	37.51	66.11	1.02	
	2.00		54	25.49	39.20	64.69	0.84	
1.50		3.00	∞	25.10	40.14	65.24	0.04	
		3.00	57	23.01	43.68	66.69	2.46	
2.00	1.50			22.54	45.83	68.37	2.40	
			40	26.13	44.49	70.62	7.74	
2.50	1.00	3.00	₩ ₩	20.31	56.23	76.54		
			35	32.32	46.23	78.54	23.64	
3.00	0.50	3.00	- 60	20.00	82.86	102.85	23.04	

It can be observed in the above table that there exist a finite U (i.e., U^*) in each of the scenarios involved in the experiment such that $E[C^U] < E[C^\infty]$. In addition, simulation results show that the percentage decrease in E[C] that would follow from subjecting the SRL to BILP with U^* could reach up to about 39.63%. Hence, it has just been established via simulation that bounding the inventory level at $B_{1,2}^*$ could lead to significant performance improvements.

Table 3 also reveals that much of the reduction in E[C] that could be achieved by limiting U to some finite value would come from the ensuing decrease in $E[W_2]$. Since $E[W_1]$ generally increases as the assigned value of U decreases, it can be inferred that $E[C^{\infty}] - E[C^{U^*}]$ increases with c_2 ic₁. Furthermore, it seems that the relative difference between $E[C^{\infty}]$ and $E[C^{U^*}]$ increases with $E[H_1]$ but decreases with both $E[H_2]$ and $E[G_1]$. This observation would even become more apparent in the U versus $E[C^U]$ graphs presented below.

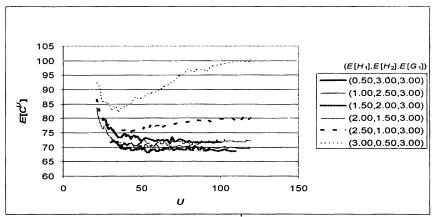


Figure 2. Sample U Versus $E[C^U]$ Graphs Where $E[G_1] = 3.00$

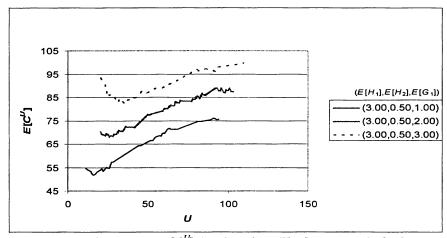


Figure 3. Sample U Versus $E[C^U]$ Graphs Where $E[H_1] = 3.00$ and $E[H_2] = 0.50$

(Please refer to the Appendix for the remaining U versus $E[C^U]$ graphs.) Notice in Fig. 2 that the scenarios with larger positive differences between $E[H_1]$ and $E[H_2]$ tend to have graphs that are more U-shaped rather than L-shaped. Thus, it can be said that $E[C^{\infty}]$ $E[C^{U^{\bullet}}]$ does increase with $E[H_1]$ $E[H_2]$. On the other hand, the gap between the lowest and the highest points of each graph in Fig. 3 seems to narrow slightly as $E[G_1]$ is increased from 1.00 to 3.00. And so, this actually illustrates that $E[C^{\infty}]$ $E[C^{U^{\bullet}}]$ decreases with $E[G_1]$.

The key insights of the simulation experiment on the SRL could therefore be summed up by stating that the percentage difference between $E[C^{o}]$ and $E[C^{l}]$ would grow larger if: c_2 wc₁ is higher; ρ_1 is higher; ρ_2 is lower; and ρ_G is lower.

Such findings should not be surprising as they are only consistent with those of the analytical study. As discussed in earlier sections, having a higher ρ_1 while at the same time, lower ρ_2 and ρ_G would effectively reduce the negative impact of having more frequent switchovers as a direct consequence of implementing BILP in the SRL. As such, the benefits gained by bounding the inventory level at $B_{1,2}$ would become more pronounced under the aforementioned circumstances.

1.3 The Frequency of Setups Versus the Variability in the Internal Flow of Processes

The effects of bounding the inventory level at $B_{1,2}^{\bullet}$ on both the frequency of setups and the variability in the internal flow processes could also be made evident in the conduct of the simulation experiment on the SRL. But prior to the actual experimentation, the quantities that would be relevant to the subsequent analysis must first be identified.

Now define the cycle rate as the average number of cycles completed by the system per unit time. Note that during each cycle of the SRL, there would always be two switchovers taking place one from $B_{1,1}$ to $B_{1,2}$ and the other from $B_{1,2}$ to $B_{1,1}$. As such, the cycle rate can be used to directly quantify the frequency of the setups performed by the service facility while the SRL is under operation. On the other hand, the connection between the variability in the internal flow processes and $E[K^2]$ was already established in earlier sections. In addition, let $IDT_{1,1}$ and $IDT_{1,2}$ be random variables representing the interdeparture times of the materials coming out of $B_{1,1}$ and $B_{1,2}$, respectively. (Of course, $IDT_{1,1}$ and $IAT_{1,2}$ should have identical distributions since the materials coming out of $B_{1,1}$ are just the same ones going into $B_{1,2}$.) And so, the variability in the internal flow processes of the SRL should primarily be measured in terms of the quantities $E[IDT_{1,1}^2]$ and $E[IDT_{1,2}^2]$.

Upon simulation, the following results were obtained:

Table 4. Results of the Simulation Experiment on the SRL: The Frequency of Setups and the Variability in the Internal Flow Processes

				Cycle Rate		$E[K^2]$		E[IDT _{1,1} ²]		$E[IDT_{1,2}^2]$	
<i>E</i> [<i>H</i> ₁]	E[H 2]	<i>E</i> [G₁]	U	Sim	% Diff	Sim	% Diff	Sim	% Diff	Sim	% Diff
0.50	3.00	1.00	41	0.0478	-1.01	60.22	3.50	131.61	2.36	31.98	0.09
0.50	3.00	1.00	∞	0.0473		62.40		134.80		32.01	
1.00	2.50	1.00	26	0.0467	-1.80	61.14	10.69	99.02	6.39	40.14	2.82
			∞	0.0458		68.46		105.78		41.30	
1.50	2.00	1.00	19	0.0450	-1.68	61.62	21.51	71.87	13.55	54.73	11.70
			®	0.0443		78.51		83.13		61.98	13.94
2.00	1.50	1.00	26	0.0425	-2.20	80.20	20.27	58.57	10.56	89.08	
<u> </u>			∞ 10	0.0416 0.0399		100.58		65.49		103.51	
2.50	1.00	1.00	19 ∞	0.0399	-1.75	81.84 139.86	41.49	41.67 50.85	18.06	120.41 188.30	36.05
			15	0.0392		79.16		32.80		149.55	
3.00	0.50	1.00	∞ ∞	0.0345	-9.27	268.05	70.47	39.48	16.92	419.69	64.37
		2.00	38	0.0300		126.50	7.54	176.80	4.50	35.64	0.26
0.50	3.00			0.0293	-2.23	136.82		185.12		35.73	
4.00	2.50	2.00	65	0.0292	-0.18	141.49	1.42	140.62	0.96	49.84	0.52
1.00			80	0.0291		143.53		141.99		50.10	
1.50	2.00	2.00	33	0.0288	-2.45	143.96	12.80	102.02	7.22	73.87	6.11
1.50			•	0.0281		165.10		109.97		78.68	
2.00	1.50	2.00	39	0.0280	-2.54	168.83	14.30	76.71	7.10	119.11	9.24
2.00			8	0.0274		197.01		82.58		131.24	
2.50	1.00	2.00	25	0.0271	-1.83	162.32	35.73	50.42	15.56	158.53	30.87
				0.0266		252.56		59.71		229.31	
3.00	0.50	2.00	25	0.0259	-3.74	183.43	56.36	36.93	13.43	229.51	51.83
			80	0.0250		420.32		42.66		476.41	
0.50	3.00		86	0.0208	-0.26	236.28	0.84	229.60	0.55	40.03	0.18
			80	0.0207		238.28		230.87		40.10	
1.00	2.50		61 ∞	0.0207 0.0206	-0.50	245.77 251.89	2.43	175.92 178.03	1.18	59.28 59.63	0.58
		 	54	0.0206		263.45		131.44		93.26	
1.50	2.00	3.00	∞	0.0202	-1.93	275.50	4.37	133.62	1.63	94.59	1.40
 		3.00	57	0.0202	-1.21	303.31	6.01	95.65	2.79	150.18	n/
2.00	1.50		8	0.0200		322.70		98.39		156.14	
0.50	1.00	3.00	40	0.0200	-2.25	308.60	23.13	62.64	9.49	218.67	18.66
2.50			∞	0.0195		401.47		69.20		268.82	
3.00	0.50	3.00	35	0.0193	0193 -3.01	332.81	46.56	41.57	11.36	306.74	12.36
3.00			••	0.0187		622.81		46.89	11.30	532.16	42.36

It can be observed in each of the SRL scenarios presented in Table 4 that limiting the value of U from infinity to U would increase the cycle rate, and at the same time, decrease $E[K^2]$, $E[IDT_{1,1}^2]$ and $E[IDT_{1,2}^2]$. Hence, it has just been shown through simulation that bounding the inventory level at $B_{1,2}$ would not only lead to a higher frequency of setups, but a significantly lower variability in the internal flow processes as well. Moreover, the functional behavior of such quantities with respect to U could further be described by constructing graphs out of the simulated data as follows.

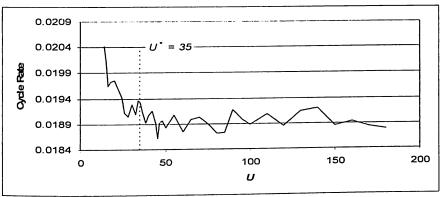


Figure 4. Sample *U* Versus the Cycle Rate Graph Where $E[H_1] = 3.00$, $E[H_2] = 0.50$, and $E[G_1] = 3.00$

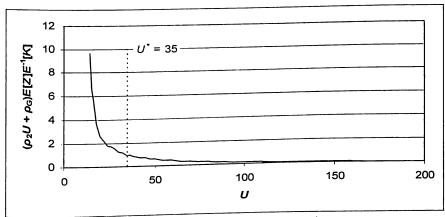


Figure 5. Sample *U* Versus $(\rho_2 U + \rho_G) E[Z] E^{-+}[K]$ Graph Where $E[H_1] = 3.00$, $E[H_2] = 0.50$, and $E[G_1] = 3.00$

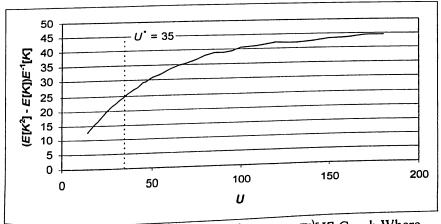


Figure 6. Sample *U* Versus $(E[K^2] \text{ er} E[K])E^{-}[K]$ Graph Where $E[H_1] = 3.00$, $E[H_2] = 0.50$, and $E[G_1] = 3.00$

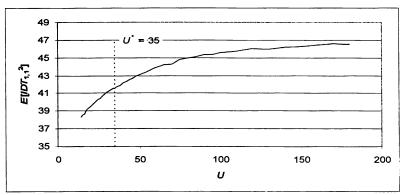


Figure 7. Sample *U* Versus $E[IDT_{1,1}^2]$ Graph Where $E[H_1] = 3.00$, $E[H_2] = 0.50$, and $E[G_1] = 3.00$

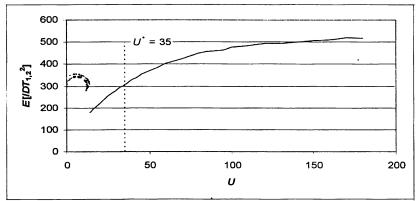


Figure 8. Sample *U* Versus $E[IDT_{1,2}^{-2}]$ Graph Where $E[H_1] = 3.00$, $E[H_2] = 0.50$, and $E[G_1] = 3.00$

Aside from that in Fig. 4, a sample U versus $(\rho_2 U + \rho_G) E[Z] E^{-1}[K]$ graph is also displayed in Fig. 5. Seeing that both the cycle rate and $(\rho_2 U + \rho_G) E[Z] E^{-1}[K]$ generally decrease with U, it can now be said that tightening U would result in a higher setup frequency. In contrast, $(E[K^2] - E[K])E^{-1}[K]$, $E[IDT_{1,1}^{-2}]$, and $E[IDT_{1,2}^{-2}]$ seem to monotonically decrease as U decreases. (Notice how similar the shapes of the graphs in Figs. 6 7, and 8 appear.) Therefore, it must be the case that assigning a lower value to U would lessen the variability in the internal flow process of the SRL.

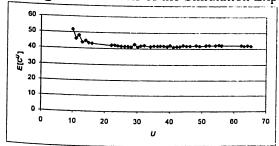
As for the performance of the SRL under BILP, $E[C^U]$ is an increasing function of both $(\rho_2 U + \rho_G)E[Z]E^{-+}[K]$ and $(E[K^2] - E[K])E^{-}[K]$. This implies that $E[C^U]$ can be regulated by keeping the said quantities under control. Now it was revealed in Fig. 5 that $(\rho_2 U + \rho_G)E[Z]E^{-+}[K]$ could actually be minimized by setting U as high as possible. However, doing so would in turn maximize $(E[K^2] - E[K])E^{-}[K]$, according to the graph in Fig. 6. And so in order to get the most out of BILP (i.e., maximize $E[C^{\infty}] - E[C^U]$), U must be chosen in such a way that a balance would be achieved between the competing effects of the frequency of setups and the variability in the internal flow processes. Fortunately, such could be accomplished through simulation optimization.

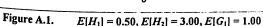
2. CONCLUSION

It was established through simulation conducted on a specific reentrant-line setting that BILP, which was found to work by striking a balance between the frequency of setups and the variability in the internal flow processes, could be superior to the exhaustive policy when it comes to minimizing the long-run average total holding cost per unit output. The optimal level of the decision vector, **U**, must be obtained using simulation optimization software in order to maximize the potential benefits of having BILP for inventory control.

APPENDIX

Figure A. Results of the Simulation Experiment on the SRL: U Versus $E[C^U]$ Graphs





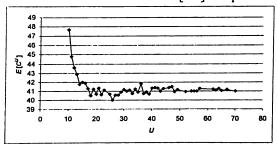


Figure A.2. $E[H_1] = 1.00, E[H_2] = 2.50, E[G_1] = 1.00$

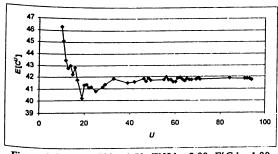


Figure A.3. $E[H_1] = 1.50, E[H_2] = 2.00, E[G_1] = 1.00$

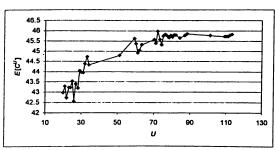


Figure A.4. $E[H_1] = 2.00, E[H_2] = 1.50, E[G_1] = 1.00$

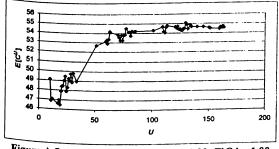


Figure A.5. $E[H_1] = 2.50, E[H_2] = 1.00, E[G_1] = 1.00$

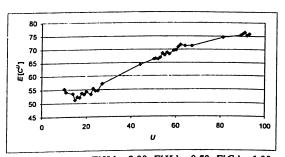


Figure A.6. $E[H_1] = 3.00, E[H_2] = 0.50, E[G_1] = 1.00$

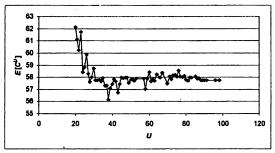


Figure A.7. $E[H_1] = 0.50, E[H_2] = 3.00, E[G_1] = 2.00$

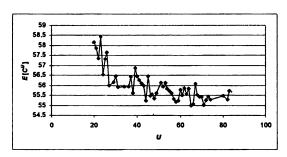


Figure A.8. $E[H_1] = 1.00, E[H_2] = 2.50, E[G_1] = 2.00$

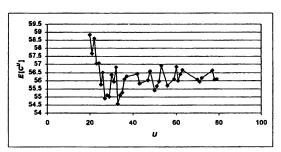


Figure A.9. $E[H_1] = 1.50, E[H_2] = 2.00, E[G_1] = 2.00$

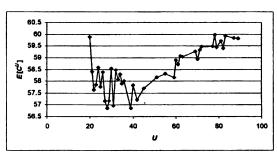


Figure A.10. $E[H_1] = 2.00, E[H_2] = 1.50, E[G_1] = 2.00$

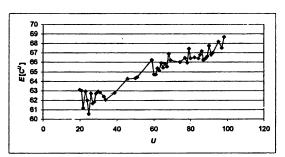


Figure A.11. $E[H_1] = 2.50, E[H_2] = 1.00, E[G_1] = 2.90$

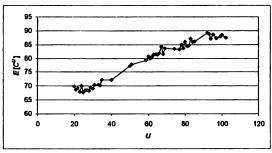


Figure A.12. $E[H_1] = 3.00, E[H_2] = 0.50, E[G_1] = 2.00$

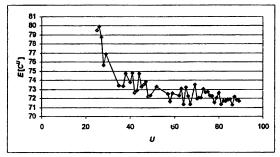


Figure A.13. $E[H_1] = 0.50, E[H_2] = 3.00, E[G_1] = 3.00$

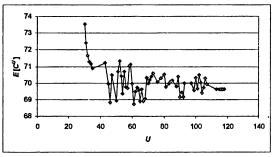


Figure A.14. $E[H_1] = 1.00, E[H_2] = 2.50, E[G_1] = 3.00$

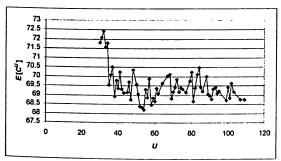


Figure A.15. $E[H_1] = 1.50, E[H_2] = 2.00, E[G_1] = 3.00$

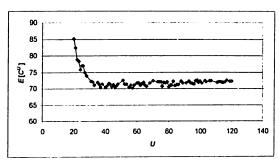


Figure A.16. $E[H_1] = 2.00, E[H_2] = 1.50, E[G_1] = 3.00$

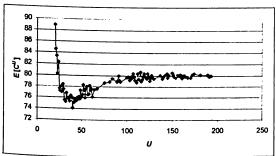


Figure A.17. $E[H_1] = 2.50$, $E[H_2] = 1.00$, $E[G_1] = 3.00$

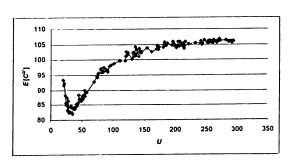


Figure A.18. $E[H_1] = 3.00, E[H_2] = 0.50, E[G_1] = 3.00$

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