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ANGAT RESERVOIR MONTHLY OPERATIONS USING AN OPTIMIZATION-SIMULATION MODEL WITH SEASONAL AUTOREGRESSIVE MODEL TO FORECAST INFLOWS

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ABSTRACT

This paper presents an optimization-simulation model for monthly allocation of the multi-purpose Angat Reservoir for water supply to Metro Manila, irrigation water to Bulacan, hydropower generation and flood control purposes. This reservoir operations scheme employs a dynamic, anticipatory or hedging operation policy over a 6-month planning horizon. This is in contrast to the existing procedure in which the reservoir releases are based on target storage or rule curves defined by upper and lower rule curves (storages) that vary on monthly basis. This paper also presents a monthly forecasting model of reservoir inflows using a seasonal autoregressive model implemented with sequential model parameter estimation scheme to efficiently update the model parameters as new information becomes available. It is demonstrated in this study that the Angat Reservoir monthly allocation procedure using an optimization-simulation model performs better than the currently employed reservoir rule curve-based operations in terms of satisfying water demand and hydropower generation based on 58 years of historical reservoir inflows as well as using forecasted inflows.

1. INTRODUCTION

The Angat Reservoir is a multi-purpose reservoir to supply water for irrigation to over 28,000 hectares of rice and vegetable farms in Bulacan, almost 65% of Metro Manila's domestic water supply needs which is regulated by MWSS and distributed by private concessionaires Manila Water Company (MWC) and Maynilad Water Services (MWS), hydropower generation for NAPOCOR at an annual average of about 200 GW-hr or more for Luzon's power demand, and as flood control storage downstream of the Angat damsite.

Currently, the monthly allocation or release policy of Angat Reservoir for water supply and irrigation (including additional releases to satisfy hydropower generation demands) depends on target end-of-the-month reservoir levels called rule curves. These rule curves are fixed or static since they were developed based on historical hydrology and past experiences with the system and updating of these rule curves is done on a need or periodic basis. This reservoir release policy is not adaptive and could be myopic since it only considers current reservoir levels and that any foresight is assumed to be captured in these target or desirable end-of-month reservoir levels. A more dynamic as well as anticipatory release policy called reservoir hedging rule can be developed for Angat

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Reservoir operations as advocated by Tabios and David (2004). In this hedging rule, an optimization-simulation model is employed to determine the reservoir releases for say the next 6 months based on current reservoir levels and forecasted future inflows. Hedging or foresight in this case is accounted for by judiciously making reservoir releases to optimally satisfy the various water demands over the 6-month window or planning horizon. Although the Angat Reservoir has normally, a less-than-a-year carryover storage (i.e., reservoir storage capacity is about equal to annual water demand), this hedging rule is applied within the year (on monthly basis) to increase the reliability of satisfying the various water demands.

The main objective of this study is to develop a reservoir operation procedure for the monthly releases (allocation) of Angat Reservoir to various water uses using an optimization-simulation model with an anticipatory or hedging rule. The latest study on Angat Reservoir water allocation study that the author is aware of was done by ADB (1996) but this was not an optimization-simulation operations study. Also, since longer data is now available since the Angat Reservoir was built in 1967, this study is timely to revisit its reservoir operating policies. Also presented in this paper is a monthly forecasting model using a seasonal autoregressive (AR) time series model employed with sequential estimation for efficient updating of model parameters. A comparison of the performance of this reservoir operation optimization-simulation model and the existing and proposed rule curve-based procedure is also presented using the 58 years of historical Angat inflow data and the application of this model for water allocation of Angat Reservoir with forecasted inflows during year 2008.

2. EXISTING ANGAT RESERVOIR ALLOCATION PROCEDURE

Currently, to satisfy the water demands and power generation requirements derived from Angat Reservoir, the reservoir release policy is based on the reservoir level called *rule curve* defined as the an upper rule curve when the reservoir water level is at 212 m during the dry season or 210 m during the wet season, and the lower rule curve when the reservoir level is at 180 m. Based on these rules curves, the reservoir releases are made such that: 1) *when the water level is above the upper rule curve* - all domestic water supply and irrigation water demands including instream riverflow requirements are satisfied and power generated at the capacity of the hydropower plants; 2) *when the water level is between the upper and lower rule curves* - all domestic water supply and irrigation water demands are satisfied, however, power generation is limited to the releases made for domestic water supply and irrigation water; and, 3) *when the water level is below the lower rule curve* - domestic water supply is satisfied first by whatever water is available in the reservoir and that power generation is limited to the releases made from the auxiliary hydropower plants.

In the existing rule curve-based procedure, to account for hedging to avoid demand shortages or for the beneficial use of water in the future, the monthly reservoir release should not result in a reservoir storage below the specified desirable or target end-of-month storages (elevations) also referred to as lower rule curve that is varied monthly but the minimum lower rule curve is at the reservoir elevation of 180 m. During critical periods however, the reservoir storage is allowed to go below this lower rule curve up to the reservoir dead storage at elevation 160 m. Likewise, the upper rule curve is varied on monthly basis to relax or lower the flood control storage (elevation) requirement during the dry months of the year. Lately, there is a new rule curve (i.e., upper and target storages) proposed by technical working group of Angat Reservoir to improve reservoir operations. (The existing and proposed upper and target lower rule curves are shown in Fig. 2 below).

3. OPTIMIZATION-SIMULATION MODEL

The ability and reliability of Angat reservoir to supply domestic water and irrigation water including generation of hydropower is investigated in this study through a reservoir simulation and optimization model. Reservoir reliability studies can be conducted by simulation studies alone. In the case of normal reservoir operations for instance, the release rules must be specified *apriori* using a set of rules in the simulation model so that given the long-term sequences of watershed inflows as input to the model, reservoir releases are calculated. The resulting reservoir releases are then subjected to statistical analysis to calculate the reliability and dependability water yields. On the other hand, with combined simulation and optimization studies, the optimization model decides when and how much reservoir releases for a specified objective and constraint functions and the simulation model calculates the resulting reservoir storages, other outflows including uncontrolled outflows for given reservoir inflows.

In this study, the WATPOW model originally developed by Tabios and Shen (1993) is used. This model is described in the Appendix A. The model consists of a simulation module to represent the physical processes involve in the reservoir operations and an optimization module to specify the reservoir operating policies such as release schedules. The model may be used for pure simulation studies or for combined simulation optimization studies. There are three optimization procedures available in the WATPOW model, namely: 1) the constrained simplex algorithm called COMPLEX; 2) incremental dynamic programming procedure; and, 3) genetic algorithm. In this study, the COMPLEX optimization method is utilized since it is fast and efficient method compared to the other techniques. The use of the COMPLEX algorithm in reservoir operations had been demonstrated by Ford et al (1981) and its application to wastewater treatment design was presented by Craig et al (1978).

Figure 1 shows the network representation of the Angat Reservoir system and components. The major physical components of this system include the inflow points Angat River (ANGAT.INF) and Umiray River (UMIRAY.INF), the reservoir itself (ANGAT.RES), the main (MAIN.HPP) and auxiliary (AUX.HPP) hydropower plants, and the demands nodes, NIA-AMRIS irrigation system for Bulacan (NIA.DMD) and MWSS of Metro Manila (MWSS.DMD). Also indicated in Fig. 1 are names of links or arcs connecting the various nodes which are watercourses that may be rivers or conduits (tunnel and aqueducts).

4. OBJECTIVE FUNCTION AND CONSTRAINTS

For long term planning and operations and reservoir reliability and economic studies, the objective would generally be to maximize benefits derived from power generation with perhaps a penalty term to account for some soft constraints such as penalty for water delivery or demand violations as well as deviation from the desirable or target end of month reservoir storages. The associated constraints would be the contractual flow requirements such as for water supply and irrigation, as well as the hard system constraints defining the reservoir water balance.

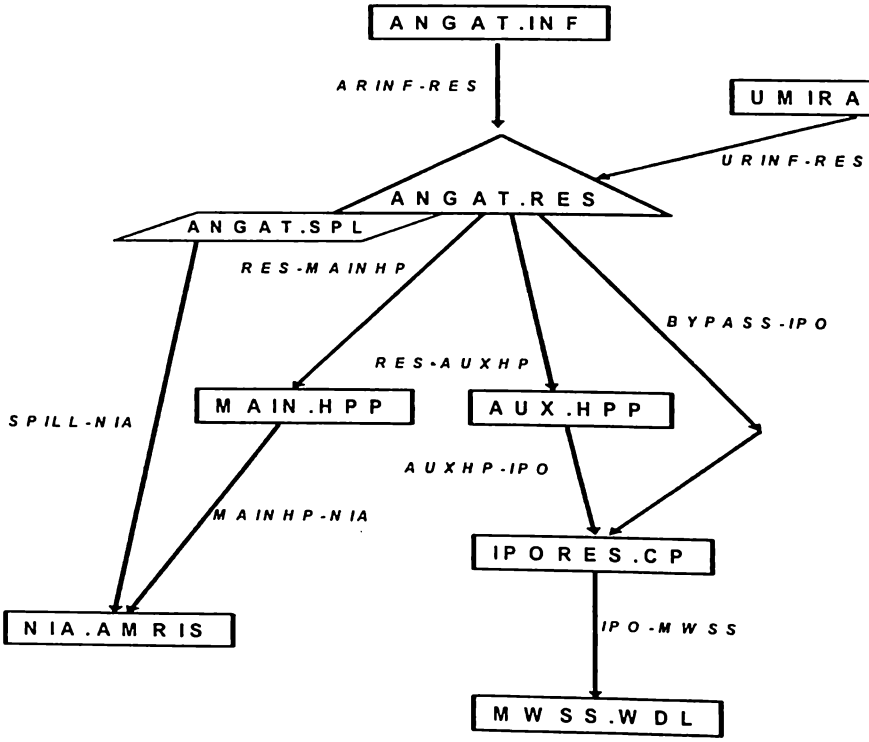

Fig. 1- Schematic of Angat Reservoir system indicating the various nodes: inflows (ANGAT.INF & UMIRAY.INF); reservoir (ANGAT.RES); hydropower plants (MAIN.HPP & AUX.HPP); spillway (ANGAT.SPL); combination point (IPORES.CP); demand nodes (NIA.AMRIS & MWSS.WDL). Also indicated are the names of links.

For this Angat Reservoir operation study, the objective function is a weighted, multi-objective to minimize water supply deficits (demand violations), maximize hydropower generation and minimize spills. The constraints include capacities of release gates, storage volumes, operational constraints and all other system constraints imposed implicitly in the simulation model. The objective function to be maximized in particular is given by:

$$F = p_{WS}\min(QWS_t - QWD_t, 0.0) + p_{IR}\min(QIR_t - QID_t, 0.0) - p_{SP}QSP_t +_t \\ b_M EM + b_A EA_t + b_F(S_t + I_t) \qquad (1)$$

where QWS_t and QWD_t are MWSS water supply release (through auxiliary hydropower plant) and MWSS water demand, respectively at time t with penalty coefficient p_{WS}, QIR_t and QID_t are the NIA-AMRIS irrigation release (through main hydropower plant) and irrigation demand, respectively with penalty coefficient p_{IR}, QSP_t is the reservoir spill with penalty coefficient p_{SP}, EM_t is the generated at the auxiliary hydropower plant with benefit coefficient b_M, and EA_t is the energy derived from hydropower production, it may be desirable to have multiple rate structure as a function of reservoir releases such as the distinction of on peak and off peak prices associated to on peak and off peak releases, respectively.

The quantity $(S_t + I_t)$ with benefit coefficient b_F represents the future benefit function where S_t and I_t are the reservoir storage and total reservoir inflow, respectively. This future benefit function accounts for the hedging operation rule. However, as discussed in the use of COMPLEX optimization algorithm (see Appendix A), the foresight or hedging into future flow conditions may accounted through a month-to-month optimization with a *future benefit function* or through a multiple-month *moving planning horizon* or *window* scheme. In this latter scheme, for a 6-month or longer window, the future benefit function is unnecessary.

The major constraint function imposed in the Angat Reservoir operation study is the hard system constraint described by the reservoir water balance computations, maximum flow capacities of controlled releases and the flood control storage requirement. Other constraints inherent in the COMPLEX optimization method such as ranges of values of decision variables as well as implicit constraints discussed in Appendix A are also imposed. Note that the water supply and irrigation demand violations as well as spillway flows are imposed as soft constraints which are incorporated in the objective function as penalty functions.

5. OPTIMIZATION-SIMULATION STUDIES AND RESULTS

For the optimization-simulation studies, several trial runs where made to arrive at some reasonable penalty and benefit coefficients in the objective function in Eq. (1). From these trials, 4 cases, referred to as OS1, OS2, OS3 and OS4, are selected in particular as shown in Table 1 below. The future benefit coefficient is set equal to zero since in this study, the anticipatory or hedging operating rule is through the moving planning horizon scheme with a 6-month window. (See discussion of hedging rule options of WATPOW model in Appendix A.)

The results of the optimization-simulation studies based on 58 years of monthly (1946-1963 and 1968-2007) of Angat River inflows, are summarized in Table 2. Note that the Umiray River inflows are taken equal to the 12 monthly means (based on 2001 to 2005 data) so that these 12 values are recycled over the 58 years simulation period. Also shown are the results of reservoir operations based on existing rule curve as well as proposed rule curve procedures employed in the Angat Reservoir monthly water allocation to various uses. In the existing and proposed rule curve procedures, cases ERC1 and PRC1, the amount of MWSS reservoir release can be as large as possible to satisfy the MWSS demand and constrained only to reservoir dead storage elevation at 160 m while the NIA release (after the MWSS demand is satisfied) is constrained such that the resulting end-of-month reservoir storage elevation does not go below the specified lower rule curve storage elevation for that month. In Cases ERC2 and PRC2, both the MWSS and NIA reservoir releases are constrained such that the resulting end-of-month reservoir elevation does not go below the specified lower rule curve.

In Table 2, under optimization-simulation cases, case OS1 results in the least MWSS demand violations of 6 months with 152 NIA demand violations out of a total of 696 months optimization-simulation period. However, in case OS4 when the NIA demand is given more importance (i.e., the NIA and MWSS penalty coefficients are set to 100 and 50, respectively), it results in the least NIA demand violations of 43 months but with 186 months of MWSS demand violations. In terms of MWSS demand violations, the existing rule curve case ERC1 resulted in 5 months demand violations and the proposed rule curve case PRC1 resulted in 6 months demand violations, both of which are comparable to cases OS1 and OS2 with 9 demand violations. On the other hand, the NIA demand violations in ERC1 and PRC1 cases resulted in 325 and 271 months in violations, respectively which are twice as much as cases OS1 and OS2 with only 154 and 166 violations, respectively. Another demand violation statistics is the resulting overall monthly average demand deficits. Case ERC1 results in the lowest MWSS average demand deficit of 21806 TCM (1000 m^3) and with NIA average demand deficit of 44739 TCM. These values may be compared to cases OS1 and OS2 with MWSS average demand deficits of 28996 and 29411 TCM, respectively with corresponding NIA average demand deficits of 51400 and 46649 TCM, respectively. Note that a 31-day month, the MWSS demand is about 123206 TCM or 46 m^3/sec and largest NIA demand is 96422 TCM. The average demand deficit statistics among the different cases may not be comparable since they are tied down to the absolute count or total number of months of demand violations.

In terms of hydropower generation, Table 2 shows that both the optimization-simulation cases OS1 and OS2 result in annual energy generation of about 627 GWH compared to the existing and proposed rule curve cases ERC1 and PRC1 of 559 and 563 GWH, respectively. Cases OS3 and OS4 result in even higher generation of 632 and 637 GWH, respectively compared to all cases. The optimization-simulation cases OS1 and OS2 in particular result in about 65 GWH more energy generation annually compared to the existing and proposed rule curve cases ERC1 and PRC1. The existing and proposed rule curve cases ERC2 and PRC2 result in higher annual energy generation of 568 and 572 GWH, respectively, higher by about 10 GWH compared to cases ERC1 and PRC1.

Generally, these results show a significant improvement in Angat Reservoir water and power yields with the optimization-simulation model compared to the existing or proposed rule curve procedures. The applications to Angat Reservoir monthly water allocation problem of these optimization-simulation cases OS1 and OS3 including the existing and proposed rule curve cases ERC1 and PRC1 are presented later with forecasted inflows for year 2008.

Figure 2 below shows the time series plots the average and monthly reservoir elevations over 58 years optimization-simulation period for cases OS1 and OS3 including those for the existing and proposed rule curve cases ERC1 and PRC1. Superposed in these plots are the existing and proposed upper and lower rule curves (or target storage elevations). Cases OS1 and OS3 result in average reservoir elevations that are closer to the proposed lower rule curve than the existing lower rule curve. In the existing and proposed rule curve cases ERC1 and PRC1, the end-of-month storage elevations also fall below their respective lower rule curves to ensure that the MWSS water demands are satisfied but not below the dead storage level of 160 m. It may be noted that the resulting actual end-of-month storage elevations go below the lower rule curves but not above the upper rule curve since these flood control storage elevations are imposed as hard constraints in the model and that the reservoir is forced to spill to satisfy the flood storage rule curve. In the optimization-simulation cases, the proposed upper rule curve is used but a flood control operations strategy should be developed separately under this optimization-simulation approach.

Figure 3 shows the average and monthly reservoir elevations for the existing and proposed rule curve cases ERC2 and PRC2. In these two latter cases, the reservoir levels do not go below the specified lower rule curve reservoir elevations.

Table 1. Penalty coefficients for MWSS demand violations, NIA demand violations and reservoir spills as well as hydropower generator benefit coefficient applied in the objective function for 4 optimization-simulation (OS) cases.

Cases	Penalty Coefficient for MWSS Demand Violations	Penalty Coefficient for NIA Demand Violations	Penalty Coefficient for Spills	Hydropower Generator Benefit Coefficient
OS1	100	10	10	4000
OS2	100	50	10	4000
OS3	100	100	10	4000
OS4	50	100	10	4000

Table 2. Results of optimization-simulation runs (OS cases) for 58 years (1946-1963, 1968-2007) of historical data in terms of MWSS demand violations, NIA demand violations and hydropower generator for various runs (cases). Also given are results for the pure simulation with existing rule curve-based (ERC case) and proposed rule curve-based (PRC case) operating procedures.

Cases	MWSS Demand Violations		NIA Demand Violations		Overall Monthly Average Surplus (TCM)		Annual Energy Generation GWH		
	Total Months	Overall Monthly Average (TCM)	Total Months	Overall Monthly Average (TCM)	MWSS	NIA	Aux HPP	Main HPP	Total
Optimization-Simulation Cases									
OS1	9	-28996	154	-51400	0	38131	326.1	300.9	626.9
OS2	9	-29411	166	-46649	0	36430	327.3	300.1	627.4
OS3	131	-32306	129	-25193	0	33800	313.5	318.5	632.0
OS4	180	-32170	48	-29302	0	30287	308.8	328.5	637.3
Existing Rule Curve Cases									
ERC1	5	-21806	325	-44739	0	30573	332.1	226.9	559.0
ERC2	148	-32002	276	-39197	0	29110	317.0	251.0	568.0
Proposed Rule Curve Cases									
PRC1	6	-22973	271	-43771	0	27511	327.3	236.4	563.7
PRC2	115	-34763	242	-38160	0	26464	315.9	256.4	572.3

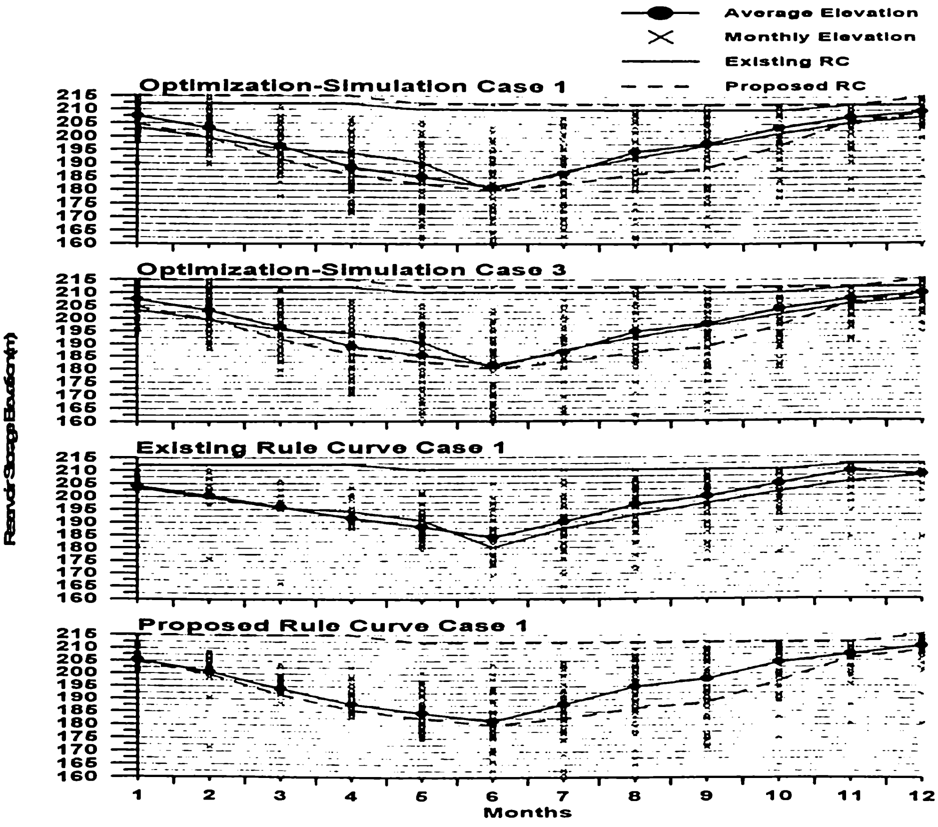

Fig. 2 - Average and monthly reservoir storage elevations of cases: optimization-simulation cases OS1 and OS3; existing rule curve case 1 (ERC1) and proposed rule curve case 1 (PRC1). Envelope curves are upper and lower target storage elevations of existing and proposed rule curve (RC) operating procedures.

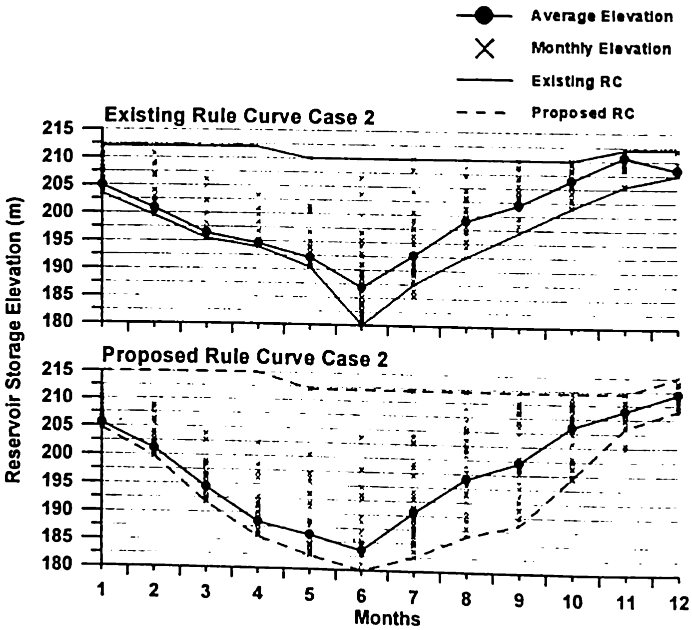

Fig. 3 - Average and monthly reservoir storage elevations of existing rule curve case 2 (ERC2) and proposed rule curve case 2 (PRC2). Envelope curves are upper and lower target storage elevations of existing and proposed rule curve (RC) operating procedures.

6. FORECASTING MONTHLY INFLOWS WITH SEASONAL AUTOREGRESSIVE MODEL VIA SEQUENTIAL MODEL PARAMETER ESTIMATION

As mentioned earlier, the current practice in allocating the Angat Reservoir water for various uses is done by a technical working group composed of personnel from relevant government agencies (PAGASA, NAPOCOR, NIA, MWSS, NWRB, and among others). The group meets every month to update and decide these reservoir monthly allocations using a simulation model that requires forecasting the Angat Reservoir inflows. Based on information from PAGASA on outlook of weather and climate in the near medium-term, the group forecasts the Angat River flows for the next 6 months as some percentage (e.g., 60%, 80%, 100% and 120%) of the historical monthly mean for the month under consideration. This procedure is ad hoc and somehow arbitrary since it is based on insights, experience and feel of the group. However, since the Angat inflows exhibit an autoregressive-moving average (ARMA) temporal dependence (Dizon, 1987), this ARMA model may be used for this purpose.

In this study, the first-order and second-order autoregressive model are tried as candidate models to be used in forecasting the Angat River inflows but not the Umiray River flows. These two models are formulated with seasonal (monthly) parameters which are estimated via sequential parameter estimation method so that as new observations become available, the model parameters can be updated.. With this sequential estimation technique, implementing the seasonal model for forecasting variables in real-time is efficient.

The seasonal first-order autoregressive SAR(1) model is written as:

$$Q_{y,m} = \overline{Q}_m + [\phi_{1,m}(Q_{y,m-1} - \overline{Q}_{m-1}) / S_{m-1} + \varepsilon_{y,m}]S_m \qquad (2)$$

The seasonal AR(2) model is written as:

$$Q_{y,m} = \bar{Q}_m + [\phi_{1,m}(Q_{y,m-1} - \bar{Q}_{m-1})/S_{m-1} + \phi_{2,m}(Q_{y,m-2} - \bar{Q}_{m-2})/S_{m-2} + \varepsilon_{y,m}]S_m \tag{3}$$

where $Q_{y,m}$ is the monthly flow at year y and month m with associated mean $\bar{Q}_m$ and standard deviation $S_m = \sqrt{Var(Q_m)}$, $\phi_{1,m}$ and $\phi_{2,m}$ are first- and second-order autoregressive parameters and $\varepsilon_{y,m}$ is the error term.

In the sequential estimation method, the model parameters including the relevant statistics are estimated using the following equations.

The seasonal AR(1) parameter is estimated as:

$$\phi_{1,m} = \rho_{1,m} \tag{4}$$

With corresponding error term variance given by:

$$Var(\varepsilon_{y,m}) = \left[1 - \phi_{1,m}^2\right]^2 \tag{5}$$

The seasonal AR(2) parameters are estimated as:

$$\phi_{1,m} = (\rho_{1,m} - \rho_{1,m-1}\rho_{2,m})/(1 - \rho_{1,m-1}^2) \tag{6}$$

$$\phi_{2,m} = (\rho_{2,m} - \rho_{1,m-1}\rho_{1,m})/(1 - \rho_{1,m-1}^2) \tag{7}$$

With corresponding error term variance given by:

$$Var(\varepsilon_{y,m}) = \left[1 - \phi_{1,m}\rho_{1,m} - \phi_{2,m-1}\rho_{2,m}\right]^2 \tag{8}$$

The autocorrelation functions are given by:

$$\rho_{1,m} = Cov(Q_{y,m}, Q_{y,m-1})/\sqrt{Var(Q_{y,m}) \cdot Var(Q_{y,m-1})} \tag{9}$$

$$\rho_{2,m} = Cov(Q_{y,m}, Q_{y,m-2})/\sqrt{Var(Q_{y,m}) \cdot Var(Q_{y,m-2})} \tag{10}$$

Finally, the sequential estimators of the mean, variance and covariances are given by:

$$\bar{Q}_m^{(t)} = \frac{t-1}{t}\bar{Q}_m^{(t-1)} + \frac{1}{t}Q_{y,m}^{(t)} \tag{11}$$

$$Var(Q_{y,m})^{(t)} = \frac{t-1}{t}Var(Q_{y,m})^{(t-1)} + \frac{1}{t}\left[Q_{y,m}^{(t)} - \bar{Q}_m^{(t)}\right]^2 \tag{12}$$

$$Cov(Q_{y,m}Q_{y,m-1})^{(t)} = \frac{t-1}{t}Cov(Q_{y,m}Q_{y,m-1})^{(t-1)} + \frac{1}{t}\left[Q_{y,m}^{(t)} - \bar{Q}_m^{(t)}\right] \cdot \left[Q_{y,m-1}^{(t)} - \bar{Q}_{m-1}^{(t)}\right] \tag{13}$$

$$Cov(Q_{y,m}Q_{y,m-2})^{(t)} = \frac{t-1}{t} Cov(Q_{y,m}Q_{y,m-2})^{(t-1)} + \frac{1}{t}\left[Q_{y,m}{}^{(t)} - \bar{Q}_m{}^{(t)}\right]\cdot\left[Q_{y,m-2}{}^{(t)} - \bar{Q}_{m-2}{}^{(t)}\right] \tag{14}$$

The superscript *(t)* signifies the estimate of that particular statistic at that month *m* when the observed flow $Q_{y,m}{}^{(t)}$ becomes available so that *t* is essentially the count of the total number of observed values at that concurrent month which is equal to the number of years of data.

To choose between a SAR(1) and SAR(2) model, Figs. 4 and 5 show the bias and mean-square errors (MSE), respectively for 12 months (January is month 1 and December is month 12) of implementing the models using the historical data starting 1951 to 2008. Note that units of bias and MSE are both in MCM (million m^3). Data for years 1946 to 1950 are used as training or warm-up data to relatively stabilize the forecasting model parameters before implementing and evaluating the forecast errors. The forecasts errors are evaluated from 1- to 6-step ahead. The 1-step ahead forecast or 1 month lead time forecast is forecasting for instance the August (month 8) flow given the previous July (month 7) flow and also June flow in the case of SAR (2) model. For the 6-month ahead forecast in the case of SAR(1) model, the August (month 8) flow is forecasted by first forecasting the March flow given the previous February (month 2) flow, then the April flow is forecasted as a function of forecasted March flow, then May flow is forecasted, and so on up to forecasting the August flow.

With regard to the resulting forecast error bias, the SAR(1) model forecasts generally underestimate the observed values compared to those of the SAR(2) model. Both models result in relatively large negative bias at months of October, September and both August and September for the 1-step, 2-step and 3-step ahead forecasts, respectively. The worst bias is -11 MCM in the 6-step ahead forecast during the month of May and the average bias is about -5 MCM for all months for all forecast lead times which is reasonable considering that the range of Angat inflows is from about 0 to 300 MCM.

In terms of the mean-square errors (MSE), the SAR(1) model provides slightly better MSE's compare to SAR(2) model. At the different forecast lead times, the highest MSE's occur during the months of June though September but most especially June. It appears that with the results of the MSE's that the SAR(1) model is a better model than SAR(2) model.

Figures 6 and 7 show the implementation of the SAR(1) and SAR(2) models in forecasting Angat inflows for years 1997 and 2008, respectively. The year 1997 is especially illustrated here since it is the start of a 2-year dry or low flow year considered a severe El Nino episode. The topmost time series plot in Fig. 5 shows the 6-month ahead forecasts of March through August flows using the SAR(1) and SAR(2) models given only observed flows (information available) at the end of February 1997. The time series plot at the bottom of Fig. 6 shows the corresponding 6-month ahead forecasts given only information at the end of May 1997. Figure 7 likewise shows comparison of observed and 6-month ahead forecasted inflows using the SAR(1) and SAR(2) models for year 2008 except that the observed data available is up to end of April' 2008. It is shown in these two figures (6 and 7) that the forecasted inflows using the SAR(2) model are closer to the observed values compared to SAR(1) forecasts. The SAR(1) model forecasts consistently underestimate the observed values especially in the 2008 data. For relatively low values of observed inflows which is the case of year 1997, it is shown that the SAR(1) model forecasts can keep up with the SAR(2) model forecasts.

Based on the bias, MSE and especially model implementation for actual forecasting, the SAR(2) model is the more appropriate forecasting model compared to the SAR(1) model.

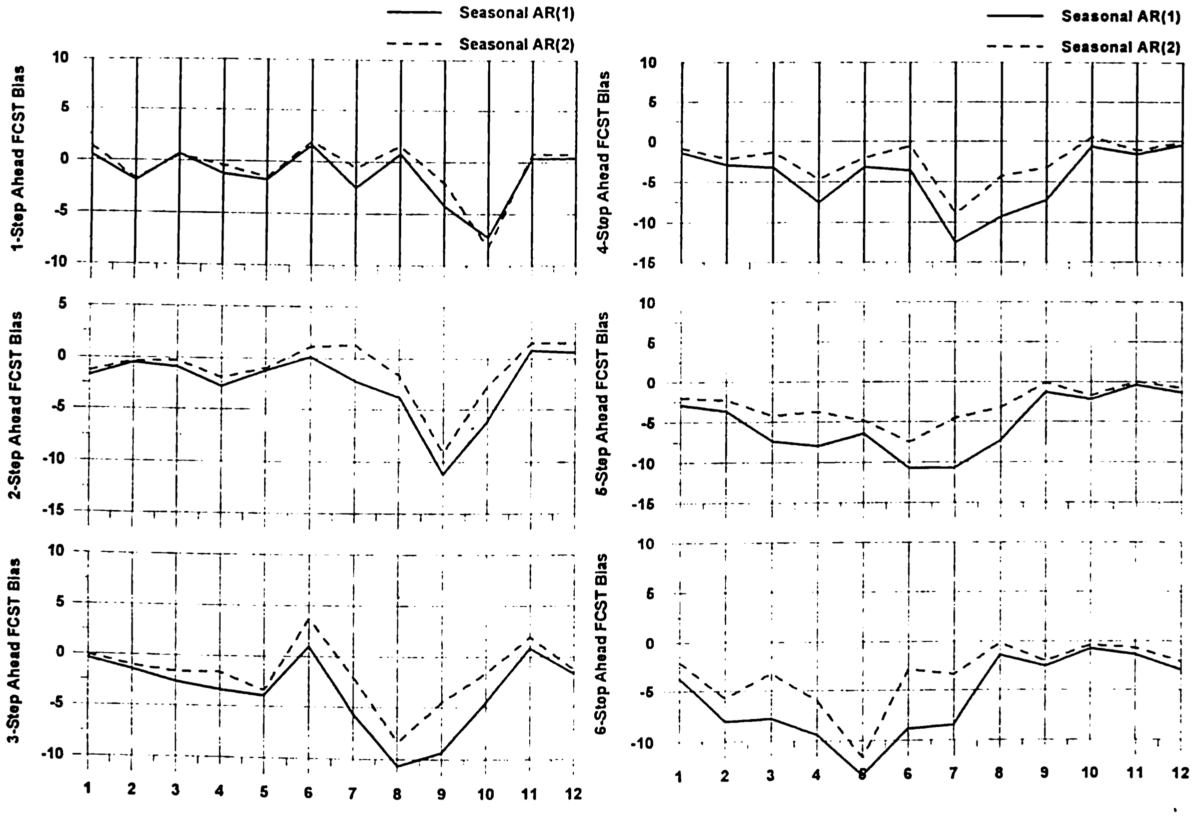

Fig. 4 Monthly bias for seasonal AR(1) and seasonal AR(2) models at different time-step ahead forecasts.

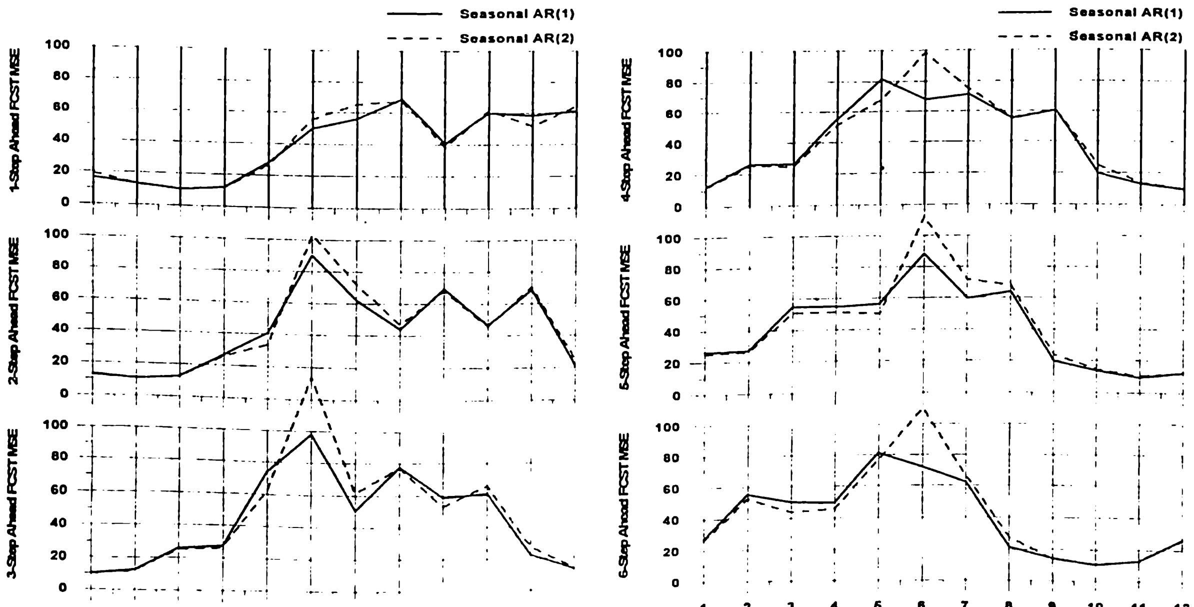

Fig. 5 Monthly mean-square errors (MSE) for seasonal AR(1) and seasonal AR(2) models at different time-step ahead forecasts.

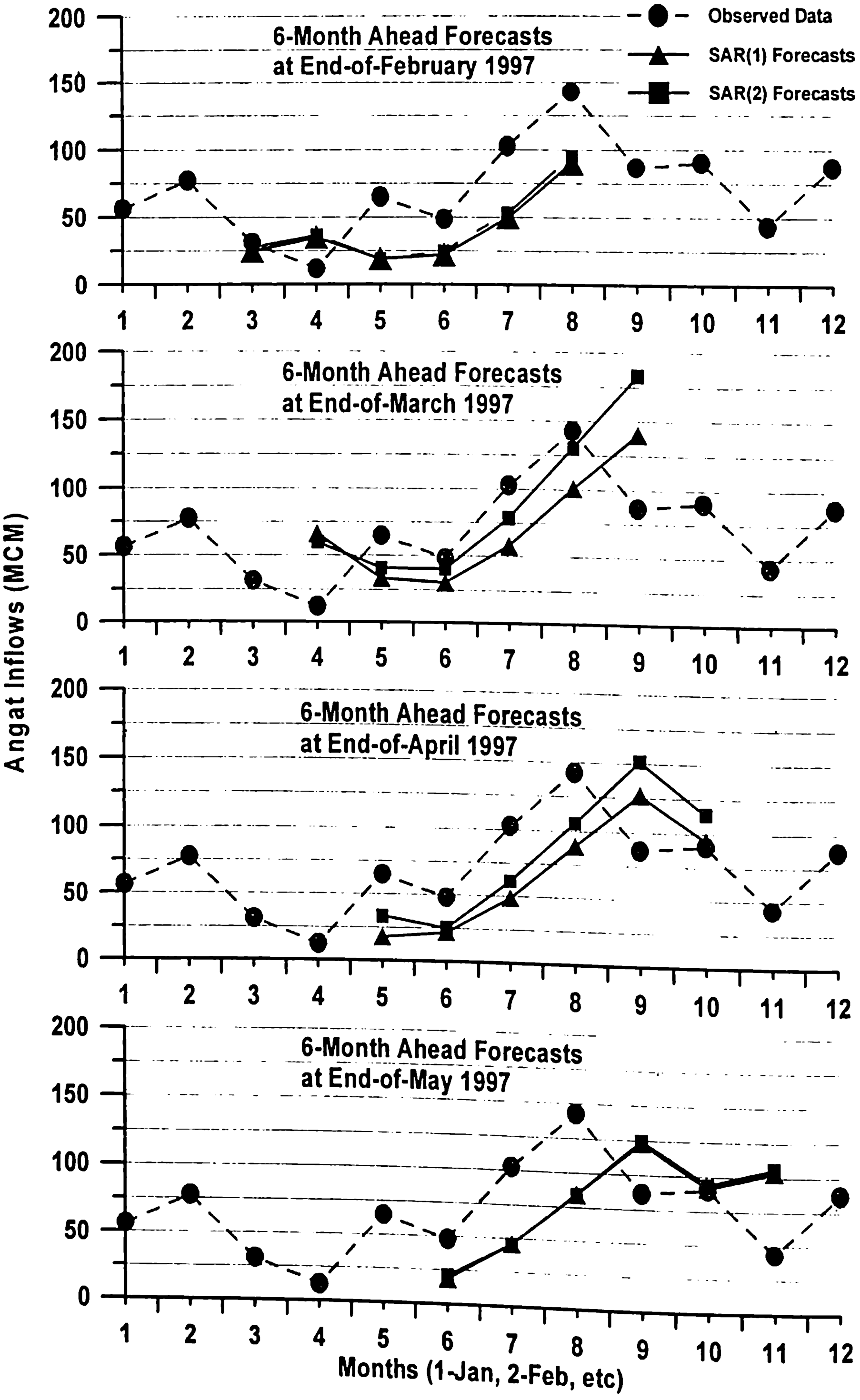

Fig. 6 6-month ahead forecasts using seasonal AR(1) and seasonal AR(2) models including observed data for 1997.

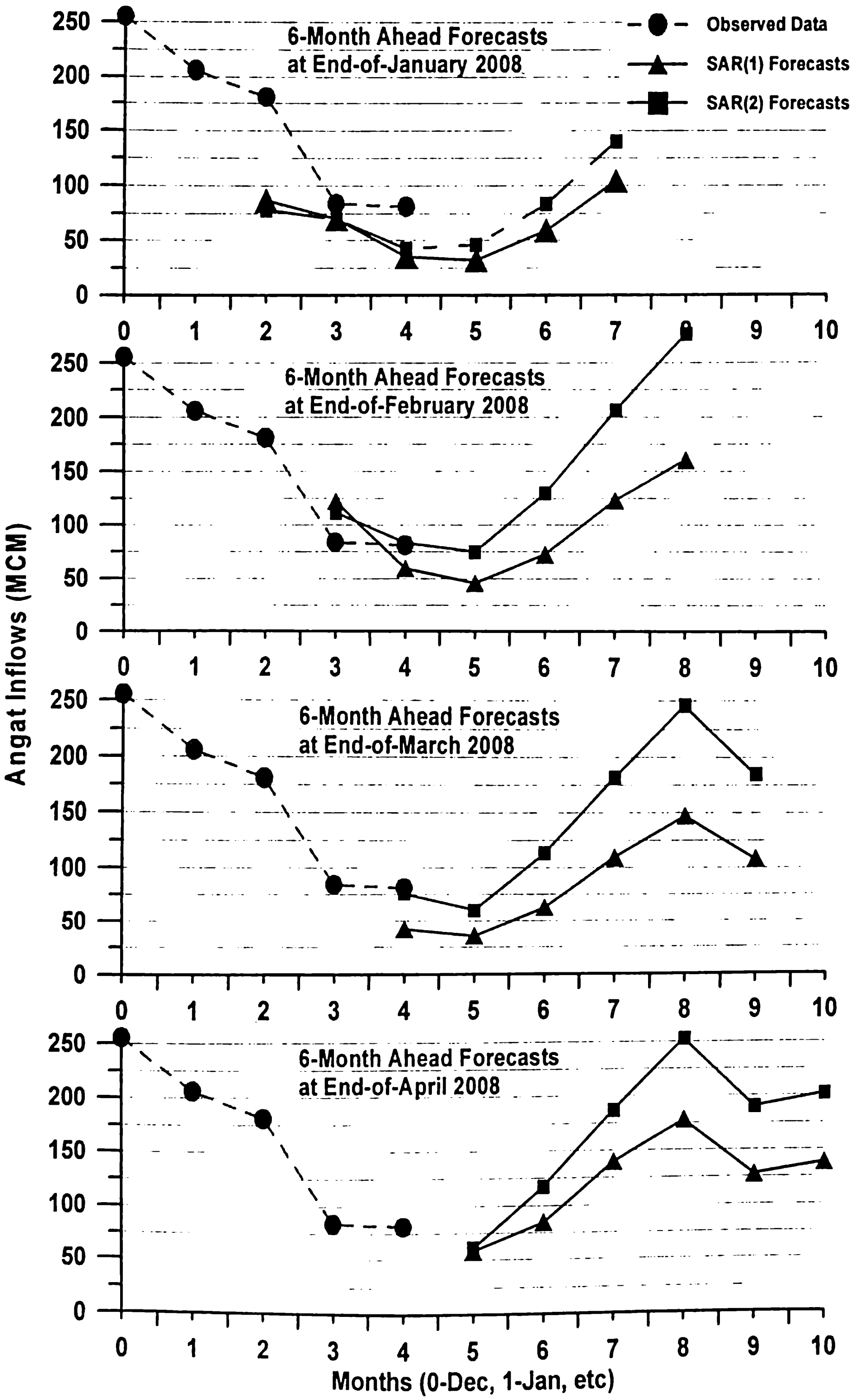

Fig. 7 6-month ahead forecasts using seasonal AR(1) and seasonal AR(2) models including observed data for 2008.

7. ANGAT RESERVOIR MONTHLY ALLOCATION WITH FORECASTED INFLOWS FOR YEAR 2008

With data only available up to the end of April 2008, the optimization-simulation model including the existing and proposed rule curve schemes were used for 6-month ahead monhtly water allocation of Angat Reservoir to various uses. In this case, the Angat River flows are forecasted flows from May to October 2008. As before, the Umiray River flows are recycled mean monthly flows based on 2001-2005 data.

Table 3 show selected reservoir variables such as Angat Reservoir inflows, releases, spills and storage levels for the optimization-simulation cases OS1 and OS3 as well as using the existing and proposed rule curve cases ERC1 and PRC1. In cases OS1 and OS3, the Angat Reservoir started off with a reservoir elevation in January at nearby spilling levels carried over from continuously running the optimization-simulation model starting with the 1946 data. On the other hand, the existing and proposed rule curve cases ERC1 and PRC1 started off with slightly lower reservoir elevations in January 2008 which was likewise carried over from continuously running the procedure from 1946.

Table 4 shows the corresponding hydropower generation and demand violations (or surplus) for the various cases. In terms of hydropower generation, the optimization-simulation cases OS1 and OS3 resulted in the same total of 597 GWH (308 from main hydropower plant plus 289 GWH from auxiliary hydropower plant) for this 10-month period compared to those of existing and proposed rule curve-based cases ERC1 and PRC1 of 488 GWH and 547 GWH, respectively. Both optimization-simulation cases OS1 and OS3 are about 110 and 50 GWH higher that the rule curve-based procedures ERC1 and PRC1, respectively. During the period from January through October 2008, there was no demand violation in any month in the four cases. However, there are surpluses (excess reservoir releases) for NIA irrigation through the main hydropower plants especially in cases OS1 and OS3 as shown in Table 4.

Figure 8 shows the time series plots of reservoir elevations for 2008 for optimization-simulation cases OS1 and OS3 and for existing and proposed rule curve schemes ERC1 and PRC1. Superposed in the plots of OS1 and OS3 cases are the upper and lower rule curves of the existing and proposed rule curve schemes. In the cases of plots reservoir elevations resulting from existing and proposed rule curve schemes, their associated upper and lower rule curves are also plotted. It is seen that in the OS1 and OS3 cases, the resulting reservoir storage elevations are high since the year started off at high reservoir levels as mentioned earlier. It may be noted that year 2008 is a fairly wet year thus the ample reservoir inflows this year.

Table 3. Selected reservoir variables for year 2008 for the four cases: i) OS1; ii) OS3; iii) existing rule curve ERC1 case; and, iv) proposed rule curve PRC1 case. Note that the Angat River flows from January to April are observed data and those from May to October are forecasted inflows.

Month	Angat Inflow (TCM)	Umiray Inflow (TCM)	MWSS Releases (TCM)	NIA Releases (TCM)	Reservoir Spills (TCM)	End-of-Month Storage (TCM)	End-of-Month (M)
Optimization-Simulation Case 1 (OS1)							
JAN	205800	36920	123206	118821	0	814548	215.0
FEB	180800	36590	111283	106046	0	814609	215.0
MAR	83700	27020	123206	83032	0	719091	210.3
APR	81000	14360	119232	40176	0	655043	207.1
MAY	60990	20080	123206	0	0	612907	204.9
JUN	117100	20710	119232	72321	0	559164	202.0
JUL	186850	35110	123206	74995	0	582922	203.3

AUG	252690	38800	123206	66969	0	684237	208.5
SEP	189640	28430	119232	58925	0	724150	210.5
OCT	201860	35690	123206	266171	0	572323	202.7
Optimization-Simulation Case 1 (OS3)							
JAN	205800	36920	123206	119351	0	814609	215.0
FEB	180800	36590	111283	106106	0	814610	215.0
MAR	83700	27020	123206	83030	0	719093	210.3
APR	81000	14360	119232	40177	0	655045	207.1
MAY	60990	20080	123206	0	0	612908	204.9
JUN	117100	20710	119232	72319	0	559167	202.0
JUL	186850	35110	123206	74995	0	582925	203.3
AUG	252690	38800	123206	66962	0	684248	208.5
SEP	189640	28430	119232	58926	0	724160	210.5
OCT	201860	35690	123206	266181	0	572323	202.7
Existing Rule Curve Case 1 (ERC1)							
JAN	205800	36920	123206	96422	0	686752	208.7
FEB	180800	36590	111283	96422	0	696436	209.2
MAR	83700	27020	123206	83030	0	600920	204.3
APR	81000	14360	119232	40176	0	536872	200.8
MAY	60990	20080	123206	0	0	494736	198.4
JUN	117100	20710	119232	72317	0	440997	195.1
JUL	186850	35110	123206	74995	0	464755	196.5
AUG	252690	38800	123206	66960	0	566079	202.4
SEP	189640	28430	119232	58925	0	605992	204.6
OCT	201860	35690	123206	34819	0	685517	208.6
Proposed Rule Curve Case 1 (PRC1)							
JAN	205800	36920	123206	325569	0	608555	204.7
FEB	180800	36590	111283	96422	0	618240	205.2
MAR	83700	27020	123206	83030	0	522723	200.1
APR	81000	14360	119232	40176	0	458675	196.2
MAY	60990	20080	123206	0	0	416539	193.4
JUN	117100	20710	119232	72317	0	362800	189.7
JUL	186850	35110	123206	74995	0	386559	191.4
AUG	252690	38800	123206	66960	0	487882	197.9
SEP	189640	28430	119232	58925	0	527795	200.3
OCT	201860	35690	123206	34819	0	607320	204.6

Table 4. Hydropower generation and demand violations (or surplus if positive values) for year 2008 for cases: i) OS1; ii) OS3; iii) existing rule curve ERC1; and, iv) proposed rule curve PRC1. Note that Angat River flows from January to April are observed data and those from May to October are forecasted inflows

Month	Auxiliary Hydropower Plant		Main Hydropower Plant		Demand Violations (-) or Surplus (+)	
	Power (MW)	Energy (GWH)	Power (MW)	Energy (GWH)	MWSS (TCM)	NIA (TCM)
Optimization-Simulation Case 1 (OS1)						
JAN	44.8	33.3	54.3	40.4	0	22399
FEB	44.8	30.1	53.6	36.0	0	9624
MAR	43.9	32.7	37.3	27.8	0	2
APR	42.4	30.5	18.1	13.1	0	0
MAY	41.4	30.8	0.0	0.0	0	0
JUN	40.4	29.1	31.5	22.7	0	4
JUL	40.1	29.8	31.4	23.4	0	0

AUG	41.3	30.8	28.7	21.4	0	9
SEP	42.7	30.7	26.8	19.3	0	0
OCT	41.6	31.0	114.7	85.4	0	231352
	Total	**308.7**		**289.3**		
Optimization-Simulation Case 1 (OS3)						
JAN	44.8	33.3	54.5	40.6	0	22929
FEB	44.8	30.1	53.7	36.1	0	9684
MAR	43.9	32.7	37.3	27.8	0	0
APR	42.4	30.5	18.2	13.1	0	1
MAY	41.4	30.8	0.0	0.0	0	0
JUN	40.4	29.1	31.5	22.7	0	2
JUL	40.1	29.8	31.4	23.4	0	0
AUG	41.3	30.8	28.7	21.4	0	2
SEP	42.7	30.7	26.8	19.3	0	1
OCT	41.6	31.0	114.7	85.4	0	231362
	Total	**308.7**		**289.5**		
Existing Rule Curve Case 1 (ERC 1)						
JAN	42.1	31.4	42.0	31.2	0	0
FEB	42.5	28.5	46.7	31.4	0	0
MAR	41.6	31.0	35.8	26.6	0	0
APR	40.0	28.8	17.4	12.5	0	0
MAY	38.9	29.0	0.0	0.0	0	0
JUN	37.8	27.2	29.9	21.5	0	0
JUL	37.5	27.9	29.8	22.2	0	0
AUG	38.9	28.9	27.4	20.4	0	0
SEP	40.4	29.1	25.6	18.5	0	0
OCT	41.6	30.9	15.0	11.2	0	0
	Total	**292.7**		**195.5**		
Proposed Rule Curve Case 1 (PRC 1)						
JAN	42.9	31.9	143.6	106.8	0	229147
FEB	41.0	27.5	45.4	30.5	0	0
MAR	40.1	29.8	34.7	25.8	0	0
APR	38.3	27.6	16.8	12.1	0	0
MAY	37.1	27.6	0.0	0.0	0	0
JUN	35.8	25.8	28.7	20.7	0	0
JUL	35.4	26.4	28.6	21.3	0	0
AUG	37.1	27.6	26.4	19.6	0	0
SEP	38.7	27.9	24.8	17.9	0	0
OCT	40.0	29.8	14.6	10.8	0	0
	Total	**281.8**		**265.5**		

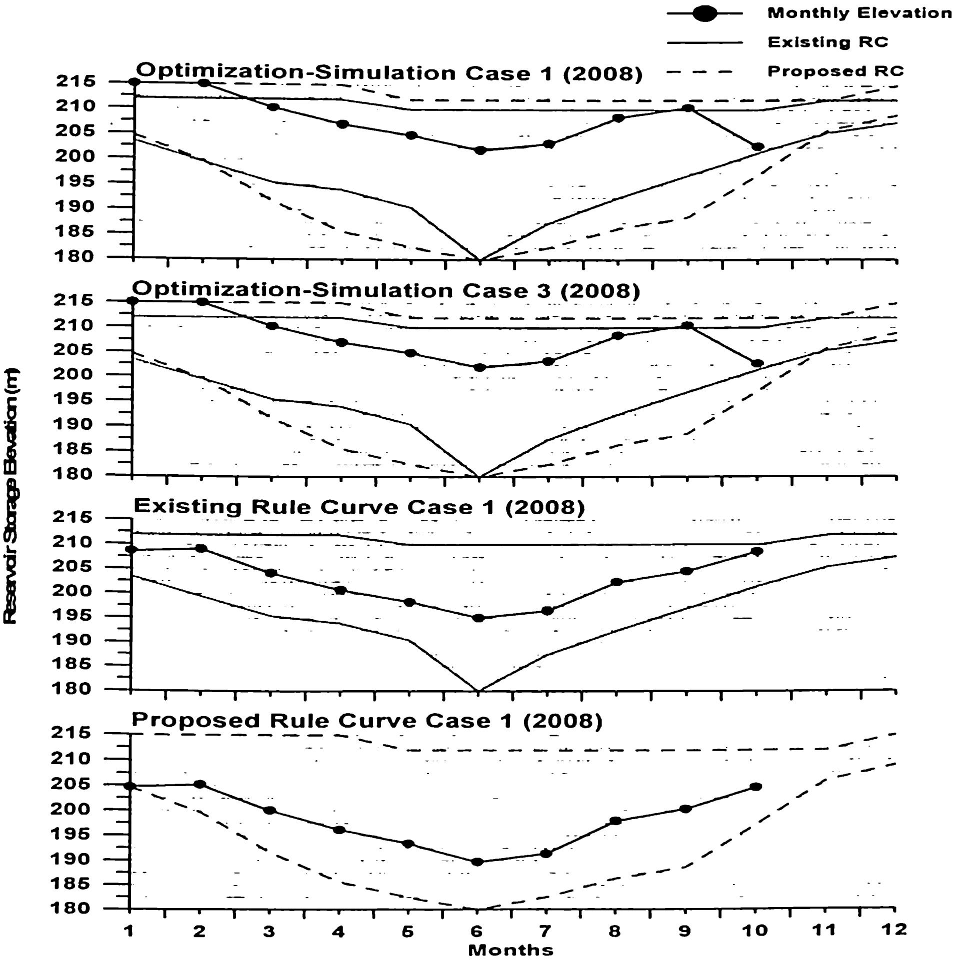

Figure 8. Monthly reservoir storages of cases: Optimization-simulation cases OS1 and OS3, existing rule curve ERC1 case and proposed rule curve PRC1 case in 2008 with forecasted Angat River inflows from May (month 4) to October. Envelope curves are upper and lower target storage elevations of existing and proposed rule curve operating procedures.

8. CONCLUSIONS AND RECOMMENDATIONS

This study demonstrated the use of a reservoir operations optimization-simulation model for the monthly allocation of the multi-purpose Angat Reservoir for water supply to MWSS for Metro Manila consumption, irrigation water supply to NIA for Bulacan farms, hydropower generation to NAPOCOR for the Luzon electric power grid and for flood control purposes. In this combined optimization-simulation method, the simulation module accurately depicts the operation of the entire water resource system while the optimization module is used to determine the optimal operating policies for the water resource system. An anticipatory or hedging operating policy is employed in the model using a 6-month moving planning horizon. This paper also presented a monthly forecasting model of reservoir inflows using a seasonal autoregressive (streamflow-to-streamflow) model implemented with sequential model parameter estimation method. The sequential estimation technique is employed to efficiently update the autoregressive model parameters as new, observed streamflow data becomes available.

The performance of the reservoir operations optimization-simulation model was evaluated based on 58 years of historical monthly data of Angat Reservoir as well as forecasted data in year 2008. The resulting reservoir operations were compared to those using the existing as well as proposed procedure of Angat Reservoir where the reservoir releases are based on target storage (elevations) or rule curves. It is demonstrated in this study that the Angat Reservoir monthly allocation procedure using an optimization-simulation model performs better than the currently employed rule curve-based operation in terms of satisfying water demand and hydropower generation based on 58-years of historical monthly reservoir inflows as well as forecasted inflows in year 2008. From results of this study, it is strongly advocated and recommended that a dynamic, anticipatory reservoir operating strategy using an optimization-simulation model with reservoir inflow forecast methodology be used in actual, real-time reservoir operations in lieu of the rule curve-based reservoir operations procedure.

The optimization-simulation model developed in this study can be enhanced and implemented for emergency or flood reservoir operations. Sometime in 1978, the area downstream of Angat Reservoir was flooded causing significant physical damage and some loss of lives downstream of the reservoir. The problem may have been the failure to anticipate the time of arrival and magnitude of floodwater inflows into the reservoir so that when it finally came, immediate evacuation of floodwaters has to be made from the controlled release gates of the reservoir in addition to spillway releases to prevent overtopping and possibly damaging the dam. It may be noted that the current practice in Angat Reservoir flood control operations is that when a typhoon or storm rainfall event is expected in the Angat watershed area, regardless of the rainfall or corresponding runoff amount, the reservoir level is brought down to the specified flood control storage elevation referred to as the upper rule curve. Under emergency or flood conditions, the decision whether to evacuate the flood water to avoid risks of overtopping the dam or to hold or save the water for future water supply reliability can only be accomplished through a fast and efficient dynamic, real-time control, reservoir optimization-simulation model and real-time inflow forecast methodology.

The forecasting model based on the autoregressive (AR) or autoregressive-moving average (ARMA) model in general, can also be improved to include an exogenous variable (called ARMAX model) such as rainfall and formulated as a multivariate model for multi-site streamflow forecasting. For instance, the Angat River and Umiray River flows can be forecasted simultaneously with rainfall input (observed or forecasted) from several sites or subregions. For purposes of normal reservoir operations, the monthly rainfall forecast as input to this streamflow forecasting model may be obtained from long-term weather and climate outlook even with a simple statistical but operational model. For emergency reservoir operations on the other hand, the short-term rainfall forecast (as exogenous variable in the streamflow forecasting model) can be a function of observed rainfall at Angat watershed as well as say the rainfall data observed in Bicol Region since storm rainfall especially typhoon-related rainfall are generated from large-scale, synoptic weather systems with as much as 100 kms radius of influence. The use of observed typhoon-related rainfall data in Bicol Region can even provide a good, reasonable lead time to forecast rainfall at the Angat watershed since storm systems normally travel at a speed of 15 to 20 km/hour.

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APPENDIX A
WATPOW MODEL AND COMPLEX ALGORITHM

Brief Description of WATPOW Model

The WATPOW model originally developed by Tabios and Shen (1993) is a generalized optimization-simulation model. This model is formulated based on the network representation of the physical system. The model basically consists of a simulation module and an optimization module. The simulation module is nested in the optimization module. The model may be used for pure simulation studies or for combined simulation optimization studies. In implementing this scheme, both simulation and optimization models would interact by generating the inputs to each other sequentially (or iteratively). The results obtained from one model should converge with those of the other model to achieve the desired accuracy as the cycle of optimization-simulation calculations is repeated. At the end, an optimal solution should be obtained which does not violate any of the system idiosyncrasies. The combined optimization and simulation scheme has been advocated by Jacoby and Loucks (1972).

The major model components in the simulation module of WATPOW are: 1) inflow nodes, 2) demand nodes, 3) combination or junction nodes, 4) diversion nodes, 5) hydropower plants, 6) reservoirs, 7) pumped storage system, and 8) special nodes. These 8 model components constitute the nodes of the network. Another major model component is the links or arcs which connect the various nodes in the network. Essentially, the links represent watercourses such as closed conduits, diversion canals, fish links or rivers. Along with the major model components are processes such as reservoir evaporation, reservoir leakage and canal conveyance losses. The simulation model of WATPOW performs mass balance calculations from node to node throughout the network. The nodal calculations may be in any order, provided that when calculations are performed for a given node, all its inflows, demands, outflows and operation rules are known. The order or sequence of the nodal calculations is specified by the user.

There are three optimization procedures available in the WATPOW model, namely: 1) the constrained simplex algorithm called COMPLEX due to Box (1965); 2) incremental dynamic programming (IDP) procedure; and, 3) genetic algorithm. For purposes of this study, the COMPLEX optimization method is utilized since it is a fast and efficient method compared to the other techniques.

The COMPLEX method is a nonlinear optimization algorithm using a sequential search technique. To initiate the search technique, a set of alternative feasible solutions are randomly generated based on the constraints on the decision variables. The technique tends to find the global solution due to the fact that the initial set of solution are randomly scattered throughout the feasible region of the optimization problem. No derivatives are required in the technique and the technique is generally applicable in solving optimization problems with nonlinear objective function and nonlinear equality and inequality constraints. The decision variables are flow releases at designated release outlets of specified reservoirs as well as flow amounts at specified flow links in the network. In the WATPOW program, the subprogram to perform the COMPLEX optimization technique was adapted from Kuester and Mize (1973). The use of the COMPLEX algorithm in reservoir operations had been demonstrated by Ford et al (1981) and its application to wastewater treatment design was presented by Craig et al (1978).

Optimization with the COMPLEX technique can be conducted on a month-to-month or multi-month basis. For month-to-month or multi-month optimization, the COMPLEX optimization is conducted as follows. For a certain *planning horizon*, say a length of twelve months (from January to December of the first year), perform the combined simulation-optimization for this 12-month period. After this, discard the results for the months of February to December of this first year but keep the solution for the month of January. Then proceed with the optimization-simulation scheme for the months of February of the first year to January of the second year (i.e., moving or sliding forward the twelve-month planning horizon by a month) with initial conditions (such as beginning-of-month-storage) already computed for January of the first year. After this, discard the results for the months March (this year) to January (next year) but keep the solution for the month of February of this current year. The above process is repeated until the entire period of study is covered. The same procedure is applied for month-to-month optimization by setting the planning horizon equal to one month. Note that in month-to-month optimization, there is no month where the results are discarded.

Another scheme to account for foresight (i.e., future flow conditions) aside from the multiple-month *moving planning horizon* scheme is through month-to-month optimization with a *future benefit function*. An example of a future benefit function would be a function of end-of-month reservoir storage level plus current month's inflow to provide a trade-off between releasing water right now to satisfy the immediate demands (lower future benefit but higher immediate benefit) or, holding the water in storage (higher future benefit) for the future.

COMPLEX Optimization Algorithm

The COMPLEX algorithm is used to solve the optimization problem given by:

$$Maximize\ F(x_1, x_2, \ldots, x_N) \qquad \text{(A1)}$$

which is a general nonlinear function of N explicit (independent) variables $x_1, x_2, \ldots, x_N$ subject to M constraints of the form:

$$x_{\min} \leq x_i \leq x_{\max} \qquad \text{(A2)}$$

in which $x_{N+1}, x_{N+2},..., x_M$ are implicit variables and are functions of the explicit variables $x_1, x_2,..., x_N$; and the lower and upper constraints x_{min} and x_{max}, respectively are either constants or functions of $x_1, x_2,..., x_N$. Constraints on the variables $x_1, x_2,..., x_N$ are called explicit constraints and the constraints on $x_{N+1}, x_{N+2},..., x_M$ are called implicit constraints. Note that M should be greater or equal to N.

In the WATPOW model, the explicit variables can be any system variables computed in the simulation model such flow amounts in the links, reservoir flow releases and hydropower energy or capacity generated. Also, values of the objective function $F(x_1, x_2,..., x_N)$ and the implicit variables $x_{N+1}, x_{N+2},..., x_M$ as well as the lower and upper constraints x_{min} and x_{max}, respectively are calculated in user-supplied subroutines in the WATPOW model.

To find the solution of the above optimization problem using COMPLEX optimization, the algorithm proceeds as follows:

S1. Generate the initial ***P*** feasible sets of solution. A value of $P \geq 2N$ is suggested. It is suggested that one initial set of points $(x_{1,1}, x_{2,1},..., x_{N,1})$ that satisfies all M constraints must be known and entered as input. The remaining $P - 1$ points are generated by:

$$x_{i,k} = x_{\min} + (x_{\max} - x_{\min}) \cdot U \quad for\ i = 1,2,...,N; k = 2,3,...,P \tag{A3}$$

in which U is a uniformly distributed random number over the interval [0,1].

After generating each point, a check is made if it satisfies all the constraints. While the generating equation in (A3) assures that the explicit constraints are satisfied, the implicit constraints may not be satisfied. If the i-th generated point violates the constraints, calculate the new point:

$$x_{i,p} = x_{i,p} + (\overline{x}_i - x_{i,p})/2 \quad for\ i = 1,2,...,N \tag{A4}$$

where p is the number of points that have already been generated in which $2 \leq p \leq P$ and the centroid of the remaining points x_i is defined as:

$$\overline{x}_i = \frac{1}{p-1}\left[\sum_{j=1}^{p} x_{i,j} - x_{i,p}\right] \qquad for\ i = 1,2,...,N \tag{A5}$$

Equation (A4) is repeated as many times as necessary to satisfy all implicit constraints.

S2. For each feasible set, evaluate the objective function $F(..)$ for $p = 1,2,...,P$. Then, select the point having the lowest objective function and carry out the reflection step for this point. The reflection step is to compute a new value for this point using the following formula:

$$x_{i,p} = \overline{x}_i + \alpha[\overline{x}_i - x_{i,p}] \quad for\ i = 1,2,...,N \tag{A6}$$

and the centroid of the remaining points is computed as:

$$\overline{x}_i = \frac{1}{P-1}\left[\sum_{j=1}^{P} x_{i,j} - x_{i,p}\right] \qquad for\ i = 1,2,...,N \tag{A7}$$

In Eq. (A6) a value of 1.3 is recommended for α. Note that in Eq. (A7), the summation is over all the points P rather than small p as done in Eq. (A5).

After the above procedure, one of the following the action is taken:

a) If the new computed point is feasible and its corresponding objective function value is worse than its previous objective function value, then recompute a new point using Eq. (A4) until a better objective function value is obtained.

b) If the new computed point is feasible and its corresponding objective function value is better than its previous value, then go to step ***S4***.

c) If the new computed point is infeasible, go to step ***S3***.

S3. Adjust for feasibility as follows:

a) If an explicit constraint is violated, the point is moved inside the constraint by setting it equal to the upper or lower bound of the constraint whichever is applicable.

b) If an implicit constraint is violated, the point is moved one-half distance toward the centroid of the remaining points using Eq. (A4).

S4. Check for convergence. Convergence is attained if the objective function values at each point are within a specified small number for a specified number of consecutive iterations. An iteration is defined as the calculations required to select a new point that satisfies the constraints and does not repeat in yielding the lowest objective function value.