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NUMERICAL ISSUES RELATED TO THE CALCULATION OF CONSOLIDATION SETTLEMENTS

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ABSTRACT

The evaluation of settlements due to consolidation is one of the most common computational procedures in geotechnical engineering. Recent research on this topic has indicated that current computational procedures underestimate consolidation settlements by as much as 70%. These errors are believed to result from the over-simplified manner by which the strains are numerically integrated. In this paper, the magnitude and nature of these numerical errors is investigated. A series of numerical experiments are performed to study the effects of load intensity and type, depth of foundation, thickness of soil layer, and preconsolidation pressure on these errors. Numerical errors are evaluated by comparing results obtained using the aforementioned procedures to either closed-form analytical solutions or numerical solutions using a high precision adaptive quadrature. Results of the numerical experiments indicate that such the underestimation of consolidation settlements is more pronounced in normally consolidated as well as heavily overconsolidated soils as compared to either lightly overconsolidated or underconsolidated soils. Recommendations are made regarding the proper use of the above-mentioned procedures in order to guarantee as sufficient degree of accuracy in the calculations.

I. INTRODUCTION

The evaluation of settlement due to consolidation is one of the most common computational procedures in geotechnical engineering. Traditionally, research on this topic has focused primarily on accurately estimating the time rate at which such settlements occur. However, recent study of this topic has indicated that present methods used for calculating consolidation settlements have the general tendency to underestimate such settlements by as much as 70%. Generally, the settlements due to surface loads are calculated by integrating the vertical strains resulting from such loads over the depth of the compressible soil layer. This process of integration is often approximate by dividing the entire soil layer into a finite number of sublayers, calculating the settlement in each layer based on the stress condition at the middle of the layer, and summing up the incremental settlements to obtain the settlement of the entire soil layer. This method shall be referred to in this paper as the finite sublayer method. This procedure is reduced to the conventional one-point method when one layer is used. While, the use of the one-point method and finite sublayer method for calculating consolidation settlements is described in most books on geotechnical engineering, none of these discussions provide clear guidelines for determining the number of sublayers required to achieve a prescribed level of accuracy. The limitations and inaccuracies of the conventional one-point method as well as the need for more rigorous methods for calculating consolidation settlements was first pointed out by McPhail et al. [2000].

In this paper, the magnitude and nature of these numerical errors is investigated. A series of numerical experiments are performed to study the effects of load intensity and type, depth of foundation, thickness of soil layer, and preconsolidation pressure on these errors. Numerical errors are evaluated by comparing results obtained using the aforementioned procedures to either closed-form analytical solutions or numerical solutions using a high precision adaptive quadrature. Results of the numerical experiments indicate that the underestimation of consolidation settlements is more pronounced in normally consolidated soils as compared to either overconsolidated or underconsolidated soils. Recommendations are made regarding the proper use of the above-mentioned procedures in order to guarantee as sufficient degree of accuracy in the calculations.

II. THEORY

The following discussion summarizes the conventional method used in geotechnical engineering for computing consolidation settlements within a soil mass due to the application of a surface load q. If the corresponding change in stress $\Delta\sigma(z)$ at a depth z is given by the expression

$$\Delta\sigma(z) = q \cdot I(z) \tag{1.1}$$

where $I(z)$ is the appropriate influence coefficient corresponding to the type of load applied, and assuming the soil is normally consolidated with a compression index of $C_{\varepsilon c}$, the resulting vertical strain ε_z is given by

$$\varepsilon_z = C_{\varepsilon c} \log\left(\frac{p_f(z)}{p_o(z)}\right) \tag{1.2}$$

where $p_o(z)$ is the initial effective overburden pressure, and $p_f(z) = p_f(z) + \Delta\sigma(z)$ is the final effective overburden pressure after application of the load. If the soil is overconsolidated, with a preconsolidation pressure of $p_p(z) > p_o(z)$, a compression index of $C_{\varepsilon c}$ and a recompression index of $C_{\varepsilon r}$, then the corresponding vertical strain ε_z is given by

$$\varepsilon_z = C_{\varepsilon r} \log\left(\frac{p_f(z)}{p_o(z)}\right) \tag{1.3}$$

for the case where the preconsolidation pressure is greater than the final pressure p_f, and

$$\varepsilon_z = C_{\varepsilon r} \log\left(\frac{p_p(z)}{p_o(z)}\right) + C_{\varepsilon c} \log\left(\frac{p_f(z)}{p_p(z)}\right) \tag{1.4}$$

for cases where $p_o(z) \le p_p(z) \le p_f(z)$. For a given strain profile $\varepsilon_z(z)$, the resulting settlement is obtained by integrating the strain over the entire thickness of the compressible layer

$$\Delta = \int_0^H \varepsilon_z(z)\,dz \tag{1.5}$$

In the *one-point* method, the integral in (1.5) is approximated by

$$\Delta_1 = H \cdot \varepsilon_z(H/2) \tag{1.6}$$

For the finite sublayer method, the integral in (1.5) is approximated by

$$\Delta_n = \frac{H}{n}\sum_{i=1}^{n} \varepsilon_z\left(\frac{2i-1}{2n}H\right) \tag{1.7}$$

where n is the number of sub-layers assumed. The need to resort to approximate methods for integrating (1.5) arises because $\varepsilon_z(z)$ is often difficult, if not impossible, to integrate analytically. Equation (1.7) can be considered equivalent to the generalized mid-point quadrature, in which the numerical error vanishes with increasing values of n and with (1.6) corresponding to the specific case where $n = 1$.

It should be emphasized numerical techniques for accurately integrating (1.5) abound. These include schemes such as the Newton-Cotes formulas that include the widely used Trapezoidal and Simpsons rule, Romberg integration which combines the Trapezoidal rule with Richardson's extrapolation technique, as well as Gauss quadrature (Chapra and Canale, [1998]). Of particular interest is the class of numerical procedures falling under general category of "adaptive scheme". In these procedures, the number of functional evaluations is systematically increased until the estimated errors are within prescribed limits (Press [1992]).

III. METHODOLOGY

Five numerical experiments where formulated to investigate the magnitude and nature of the numerical errors in the conventional one-point and finite sublayer method. The scope of this study was limited to homogenous soils. In each problem, the functional variation in the initial effective overburden pressure $p_o(z)$, preconsolidation pressure $p_p(z)$, and final effective overburden pressure $p_f(z)$ together with the resulting expression for strain $\varepsilon_z(z)$ was derived. The corresponding consolidation settlement was then evaluated using the conventional one-point and finite sublayer method, and the results were compared with the assumed exact solution. In cases where no analytical form of the exact solution was available, the exact solution was obtained by numerically integrating $\varepsilon_z(z)$ using an adaptive quadrature. For this purpose, a FORTRAN 77 program CONSETTLE was written to provide accurate consolidation settlement calculations. The program uses the IMSL subroutines QDAG and QDAGS for integrating the strains over the thickness of the compressible soil layer. Both routines employ an adaptive general-purpose quadrature based on the Gauss-Kronrod rules. For integrands without end-point singularities, the routine QDAG is employed. In special cases where the integrand is singular at one of the endpoints, the routine QDAGS is used.

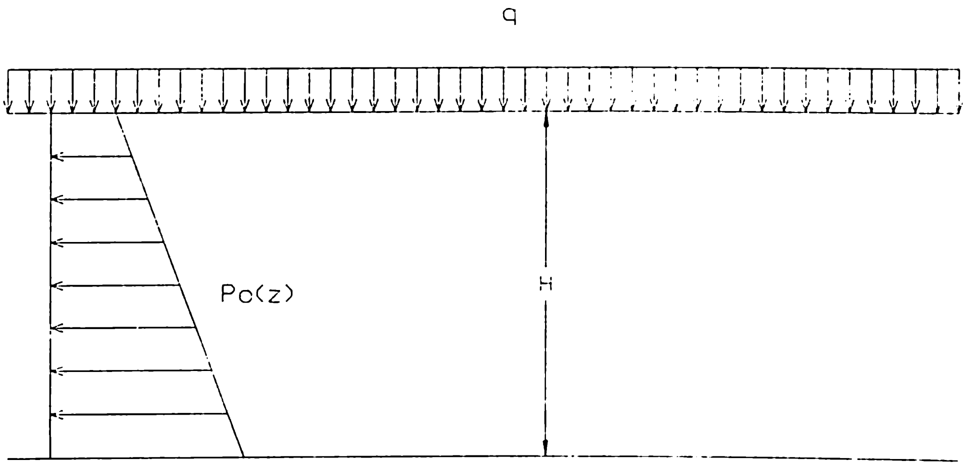

Figure 1. Geometry of fill problem

IV. DISCUSSION

CASE 1: Fill Underlain by Normally Consolidated Soil

The geometry of this case is illustrated in **Figure 1**. It consists of a layer of compressible soil that is normally consolidated with a thickness H, where the initial effective overburden pressure prior to the placement of a fill is assumed to be

$$p_o(z) = \gamma z + q_0 \tag{1.8}$$

and, where γ is the buoyant weight of the soil in the compressible layer, and q_0 is a pre-existing surcharge at the top of the compressible soil layer. The preconsolidation pressure $p_p(z)$ is equal to $p_o(z)$ since the entire layer is normally consolidated. After placement of the fill, the final effective overburden pressure is assumed to be

$$p_p(z) = q + q_0 + \gamma z \tag{1.9}$$

where q is the increase in overburden pressure through the entire thickness of the compressible layer, arising from placement of the fill. Substitution of (1.8) and (1.9) into (1.2) yields after some simplification

$$\varepsilon_z(z) = C_{\varepsilon c} \log\left(1 + \frac{q}{\gamma z + q_0}\right) \tag{1.10}$$

Equation (1.10) can be non-dimensionalized through a change of variable from z to $\xi = z / H$, resulting in the following expression

$$\varepsilon_z(z) = C_{\varepsilon c} \log\left(1 + \frac{\alpha}{\xi + \kappa}\right) \tag{1.11}$$

where $\alpha = q / \gamma H$ and $\kappa = q_0 / \gamma H$. The corresponding expression for settlement is given by

$$\Delta = C_{\varepsilon c} H \int_0^1 \log\left(1 + \frac{\alpha}{\xi + \kappa}\right) d\xi \tag{1.12}$$

Integration of (1.12) gives

$$\Delta = C_{\varepsilon c} H \Big[\kappa\left(\log(\kappa) - \log(\kappa + 1)\right) - (\alpha + \kappa)\left(\log(\alpha + \kappa) - \log(\alpha + \kappa + 1)\right) + \log\left(1 + \frac{\alpha}{1 + \kappa}\right)\Big] \tag{1.13}$$

For the case where $\kappa = 0$, equation (1.13) simplifies to

$$\Delta = C_{\varepsilon c} H \left[(\alpha + 1)\log(1 + \alpha) - \alpha \log(\alpha)\right] \tag{1.14}$$

By using dimensionless quantities, the results of the following analysis can be generalized for problems with varying geometries. In this case, it can be seen that the problem depends on only three parameters, namely the layer thickness H, the load intensity factor α and the surcharge intensity factor κ. It should also be noted, only α and κ affect the computational errors arising from the approximate manner in which the strains are integrated.

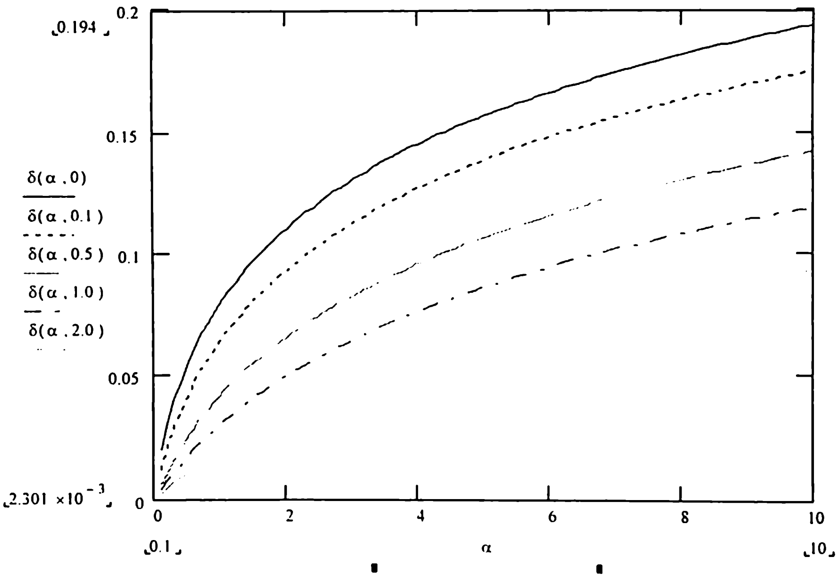

Figure 2. Effect of load and surcharge intensity on settlement of normally consolidated soil under a fill.

Figure 2 shows the computed normalized settlements Δ / H obtained using the numerical solution for different load intensity factors α and surcharge intensity factors κ. Results obtained agree within five decimal places to the analytical solution in (1.13) and (1.14). As shown, the computed settlements increase with increasing values of α and decreasing κ. This behavior can be explained by (1.11) in that at any given depth, increasing α increases ε_z, while increasing κ decreases ε_z. This rate of increase in the computed settlements is greatest for value of values of α close to 0, and decreases as α increases.

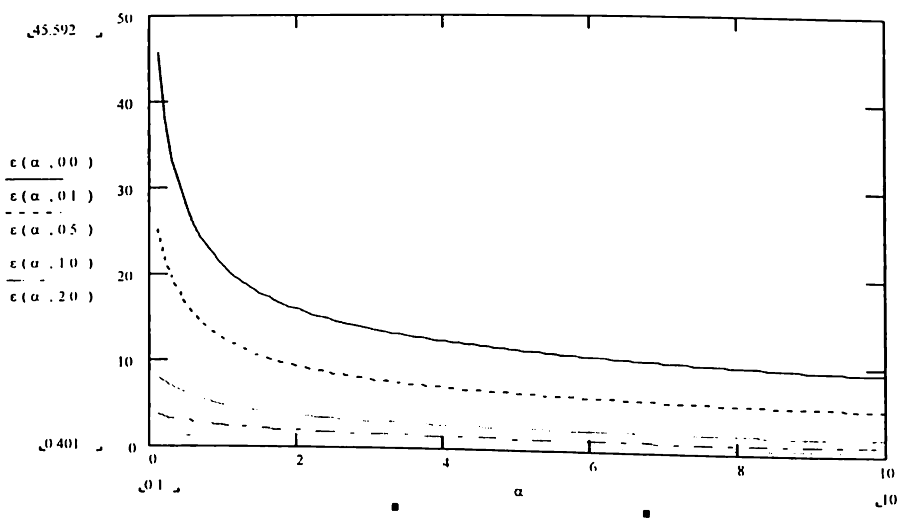

Figure 3. Computational error in consolidation settlement calculations (conventional one-point method) for fill underlain by normally consolidated soil.

Figure 3 shows the computational error in percentage resulting from the conventional one-point method. The graph indicates that as both the load intensity factor α and surcharge intensity factor κ increase, the error decreases. Increasing α resulted in a strain distribution that changed less abruptly with depth, resulting in a value at the mid point more representative of the strain throughout the thickness of the compressible soil layer. Increasing κ also resulted in a more uniform strain profile by decreasing the strain at the top of the soil layer, while maintaining approximately the same strain at the bottom of the layer. The under-prediction in the settlements was found to range from between 1 percent to 20 percent of the exact solution.

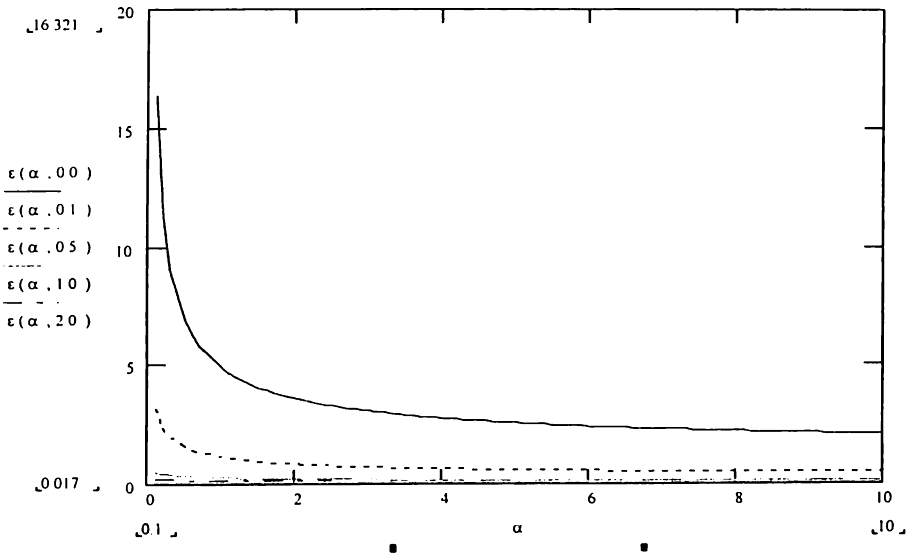

Figure 4. Computational error in consolidation settlement calculations (finite sublayer method using 2 meter thick layers) for fill underlain by normally consolidated soil.

Figure 4 shows the computational error in percentage resulting from the finite sublayer method with the thickness of each sublayer limited to 2 meters. Observed behavior with respect to the effects of α and κ were the same as those for the conventional one-point method. It was also noted that except for values of $\alpha < 0.5$, most of the resulting errors were within 5 percent of the exact value.

CASE 2: Fill Underlain by Under Consolidated Soil

This case is similar to the previous one with the exception that soil layer is still in the process of consolidating due to the prior placement of a fill with a corresponding uniform surcharge of q_0. Under these conditions, the initial effective overburden pressure prior to the placement of the new fill is assumed to be

$$p_o(\xi) = \gamma H\left[\xi + \kappa I_c(\xi,\tau)\right] \qquad (1.15)$$

where $I_c(z)$ is the influence coefficient function based on the Terzaghi one-dimensional consolidation theory given by the expression

$$I_c(\xi,\tau)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{(2n+1)\pi}{2}\xi\right)\exp\left(\frac{(2n+1)^2\pi^2}{4}\tau\right)}{2n+1} \tag{1.16}$$

Given the circumstances previous mentioned, the preconsolidation pressure $p_p(z)$ is equal to $p_o(z)$. After placement of the fill, the final effective overburden pressure is assumed to be

$$p_p(\xi)=\gamma H\left[\alpha+\kappa I(\xi,\tau)+\xi\right] \tag{1.17}$$

where q is the increase in overburden pressure through the entire thickness of the compressible layer, arising from placement of the fill. Substitution of (1.8) and (1.9) into (1.2) yields after some simplification

$$\varepsilon_z(\xi)=C_{\varepsilon c}\log\left(1+\frac{\alpha}{\xi+\kappa I_c(\xi,\tau)}\right) \tag{1.18}$$

Due to the complex nature of $I_c(\xi,\tau)$, it is not possible to integrate (1.18) analytically, and thus the settlement Δ can only be obtained through numerical integration.

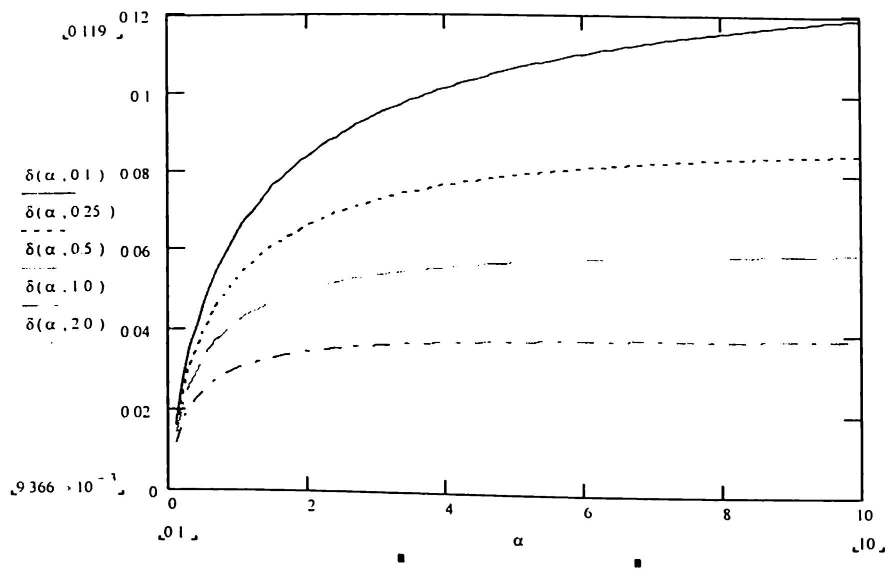

Figure 5. Effect of load and surcharge intensity on settlement of underconsolidated fill.

Figure 5 shows the computed normalized settlements Δ/H obtained using the numerical solution for different load intensity factors α and surcharge intensity factors κ. As shown the computed settlements increase with increasing values of α and decreasing κ as in the normally consolidated case. This behavior can be explained in the same manner as the previous case, the strain profiles of these two cases being very similar.

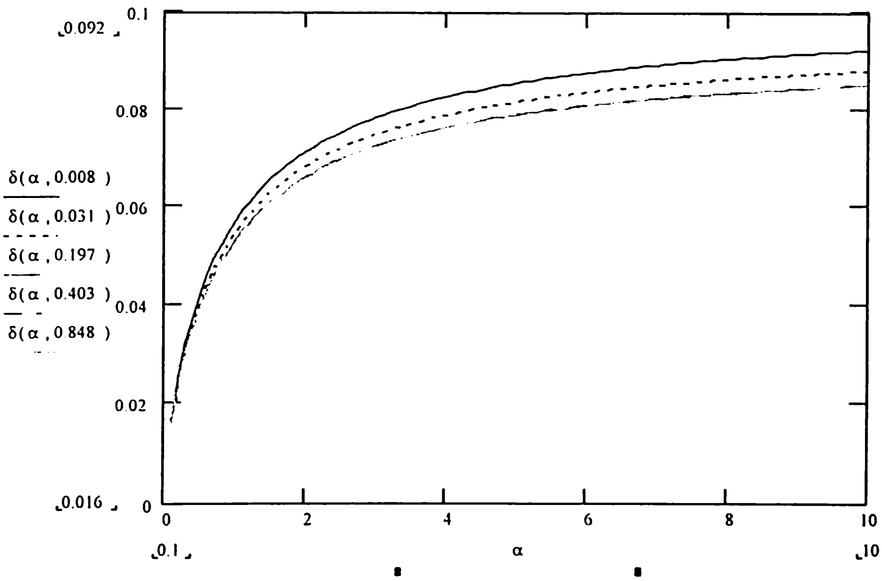

Figure 6. Effect of time factor on consolidation settlement of underconsolidated fill.

Figure 6 shows the effect of the time factor τ on the settlement. This figure shows that the settlement increases with decreasing degree of consolidation.

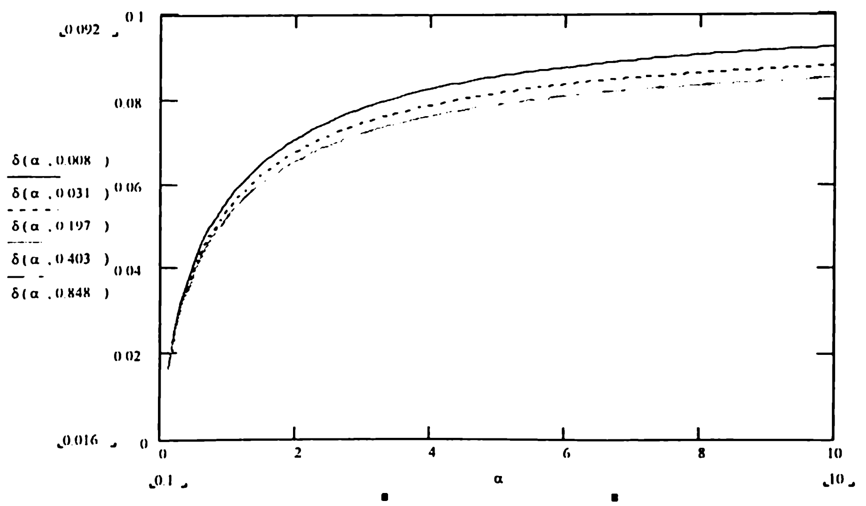

Figure 6. Effect of time factor on consolidation settlement of underconsolidated fill.

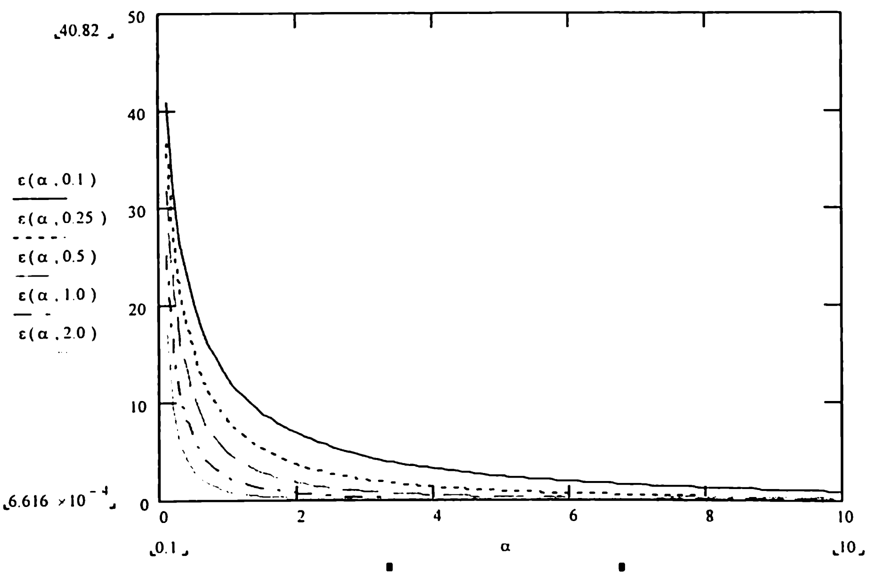

Figure 7. Effect of surcharge intensity on computational error in consolidation settlement calculations (conventional one-point method) for underconsolidated fill.

Figure 7 shows the computational error in percentage resulting from the conventional one-point method. The graph indicates that as both the load intensity factor α and surcharge intensity factor κ increase, the error decreases very much in the same manner as the normally consolidated case.

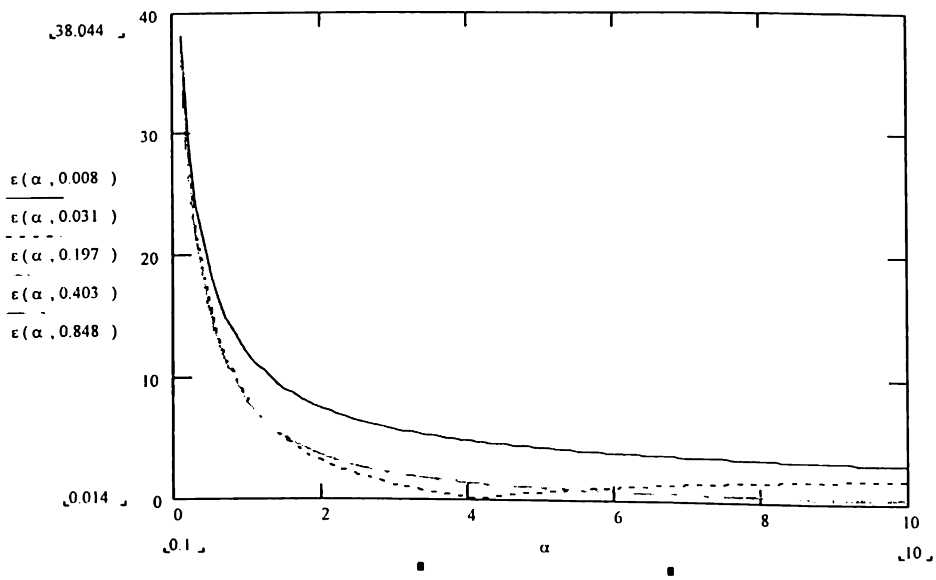

Figure 8. Effect of time factor on computational error in consolidation settlement calculations (conventional one-point method) for underconsolidated fill.

Figure 8 shows effect of the time factor τ on the computational error resulting from the conventional one-point method. It can be seen that with increasing α, the error decreases in an asymptotic manner. Differences in the computational error become negligible for when the degree of consolidation is equal to 50 percent or greater.

Computational errors for the aforementioned method were generally less than those obtained for normally consolidated soils, particularly with increasing values of α. In general, the under-prediction in the settlement was found to range from 1 percent to 5 percent.

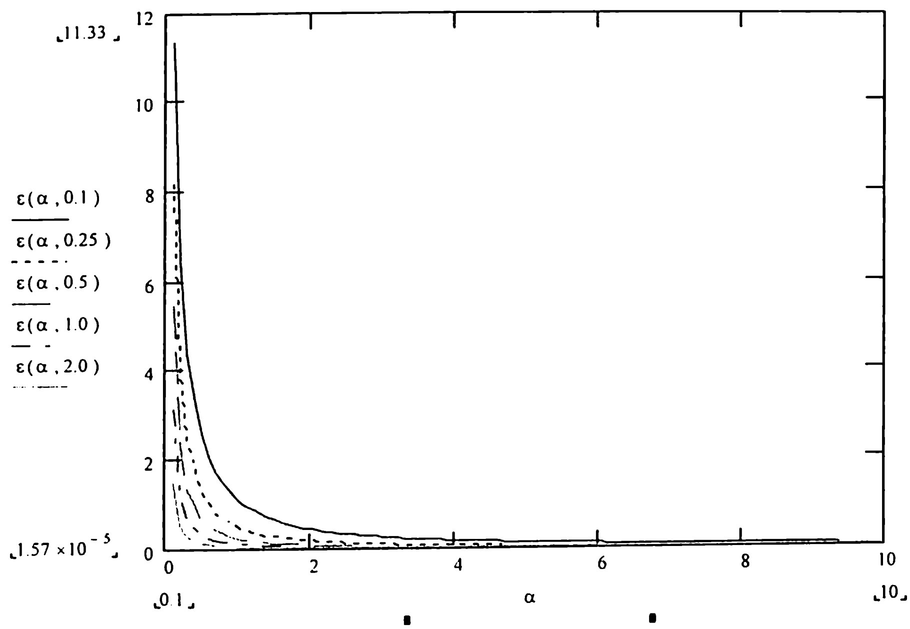

Figure 9. Computational error in consolidation settlement calculations (finite sublayer method using 2 meter thick layers) for underconsolidated fill.

Figure 9 shows the computational error in percentage resulting from the finite sublayer method with the thickness of each sublayer limited to 2 meters. Observed behavior with respect to the effects of α and κ were the same as those for the conventional one-point method. It was noted that except for values of $\alpha < 0.5$, all of the resulting errors were within 2 percent of the exact value.

CASE 3: Fill Underlain by Over Consolidated Soil

This case involves a compressible soil layer of thickness H, with an initial effective overburden pressure distribution as described in(1.8), and a final effective overburden pressure distribution as given by(1.9). In order to simplify the problem, the initial surcharge q_0 was assumed equal to zero. However, it was also assumed that the soil within a depth of z_p from the top of the layer was over consolidated due to desiccation according to the following quadratic function

$$p_p(z) = \frac{p_t - \gamma z_p}{z_p^2}\left(z - z_p\right)^2 + \gamma z_p, \quad z \le z_p, \tag{1.19}$$

where p_t is the preconsolidation pressure at the top of the soil layer. For value of $z > z_p$, the preconsolidation and initial overburden pressures are the same. In non-dimensional form, the preconsolidation pressure can be written as

$$p_p(\xi) = (\alpha_t - \xi_p)\left(\frac{\xi}{\xi_p} - 1\right)^2 + \gamma\xi_p \tag{1.20}$$

where $\alpha_t = \frac{q_t}{\gamma H}$, and $\xi_p = \frac{z_p}{H}$. For cases where $p_t \le q$, the resulting settlement was given by the expression

$$\begin{aligned}\Delta = H\Bigg(& C_{\varepsilon r}\int_0^{\xi_p} \log\left(\frac{p_p(\xi)}{p_o(\xi)}\right)d\xi + C_{\varepsilon c}\int_0^{\xi_p} \log\left(\frac{p_f(\xi)}{p_p(\xi)}\right)d\xi \\ & + C_{\varepsilon c}\int_{\xi_p}^{1} \log\left(\frac{p_f(\xi)}{p_o(\xi)}\right)d\xi \Bigg)\end{aligned} \tag{1.21}$$

For cases where $p_t > q$, it was first necessary to find the intermediate point ξ_m where $p_p(\xi_m) = p_f(\xi_m)$. Consequently the resulting settlement was given by the expression

$$\begin{aligned}\Delta = H\Bigg(& C_{\varepsilon r}\int_0^{\xi_m} \log\left(\frac{p_p(\xi)}{p_o(\xi)}\right)d\xi + C_{\varepsilon r}\int_{\xi_m}^{\xi_p} \log\left(\frac{p_p(\xi)}{p_o(\xi)}\right)d\xi \\ & + C_{\varepsilon c}\int_{\xi_m}^{\xi_p} \log\left(\frac{p_f(\xi)}{p_p(\xi)}\right)d\xi + C_{\varepsilon c}\int_{\xi_p}^{1} \log\left(\frac{p_f(\xi)}{p_o(\xi)}\right)d\xi \Bigg)\end{aligned} \tag{1.22}$$

It should be emphasized that determination of ξ_m was only necessary in cases where the settlement was computed using the conventional one-point method and finite sublayer method. In the adaptive quadrature, simple application of equations (1.3) and (1.4) was required, as well as for cases where value of n assumed in the finite sublayer method was fairly large.

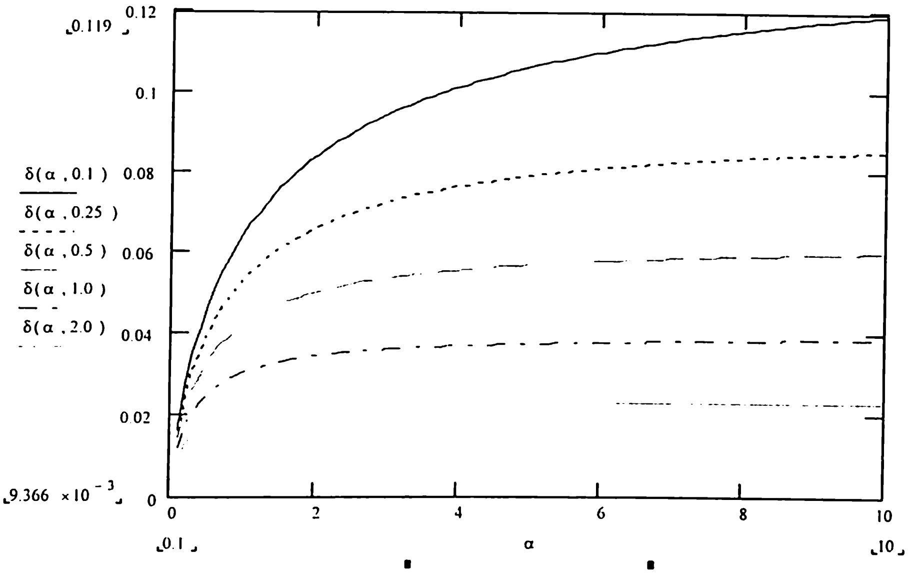

Figure 10. Effect of load intensity and pre-consolidation pressure on settlement of overconsolidated fill.

Figure 10 shows the effect of the load intensity factor α and preconsolidation pressure intensity factor κ on the computed settlements. The behavior in this case is the same as the previous two cases. In the case of increasing κ, the decrease in settlement occurs due to the fact a larger portion of the settlement occurring in the soil near the top of the layer is due to recompression.

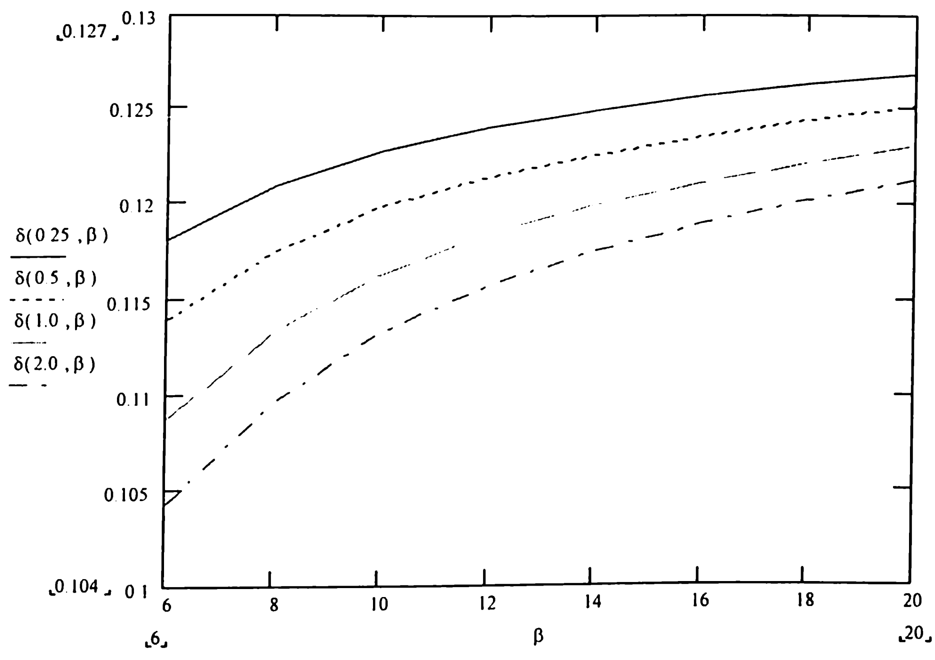

Figure 11. Effect of thickness of overconsolidated layer on settlement.

Figure 11 illustrates the effect of the thickness of the over consolidated portion of the soil layer, in relation to the overall thickness of the entire soil layer. A parameter β representing the thickness of the entire layer with respect to the thickness of the overconsolidated portion is used. The graph shows that as the thickness of the overconsolidated portion increases, as characterized by decreasing β, the settlement decreases due to the fact that more of the settlement occurs in recompression.

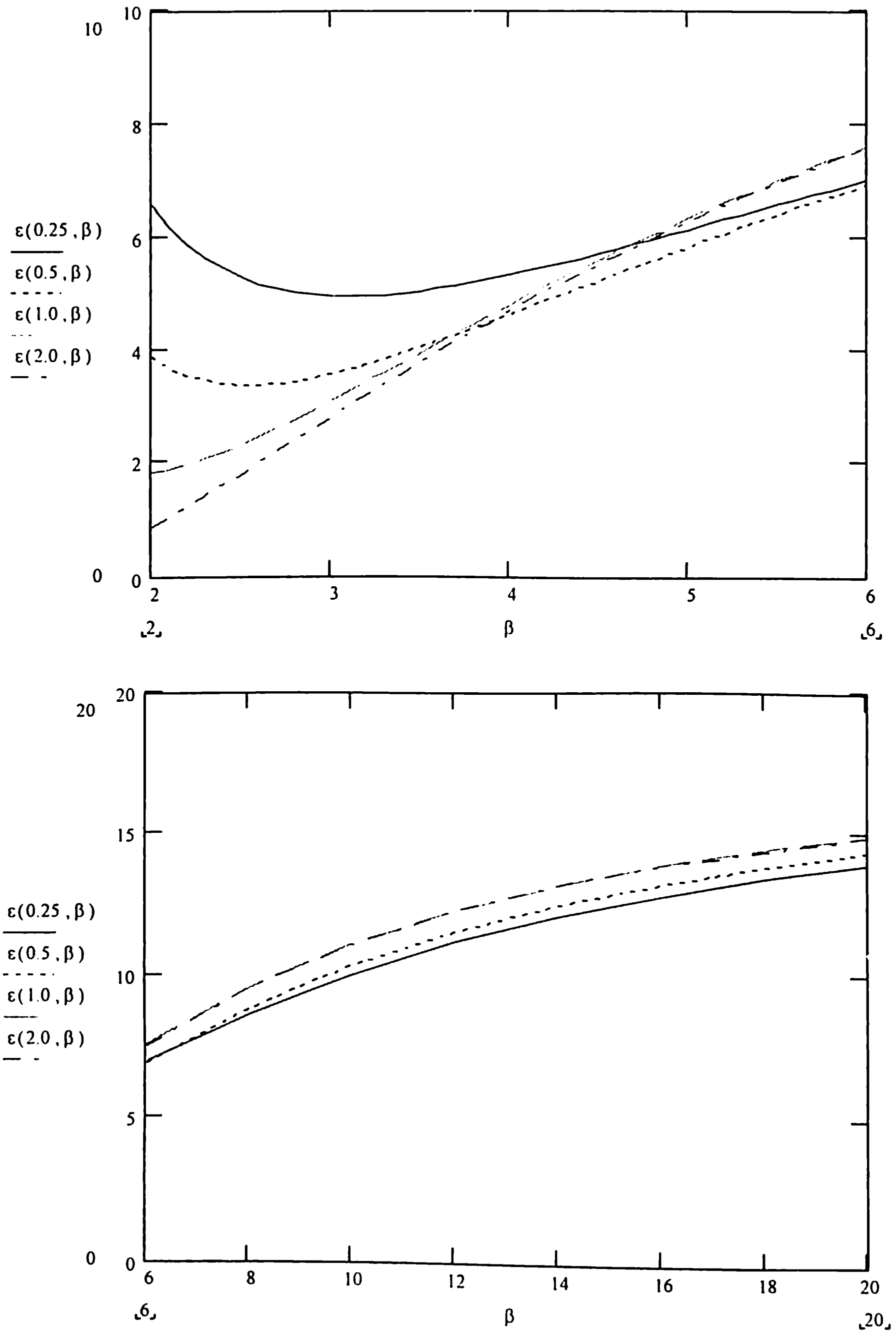

Figure 12. Computational error in consolidation settlement calculations (conventional one-point method) for overconsolidated fill.

Figure 12 shows the computational error resulting from the conventional one-point method. For values of $\beta < 4$, the error decreases with increasing α, whereas, for value of $\beta > 4$, the error increased with increasing α.

The under-prediction in settlements obtained ranged from 1 percent to 5 percent for cases of lightly overconsolidated soils. For more heavily consolidated soils, the under-prediction approach that similar to that of normally consolidated soils. This behavior could be explained by the fact the most of the under-prediction takes places due to the inability of the one-point method to account for the very large strains occurring near the top of the compressible layer. In lightly overconsolidated soils, smaller strains occur due as the settlement involves both virgin compression as well as recompression. This results in a strain profile that is more uniform, for which the one-point method yields more accurate results. For cases involving heavily overconsolidated soils where the entire thickness of the compressible layer is overconsolidated, the strain profile is similar to that for the normally consolidated case with the exception that the strains are reduced by the same factor. Consequently, the under-prediction in the settlements is similar to that which occurs in the normally consolidated case.

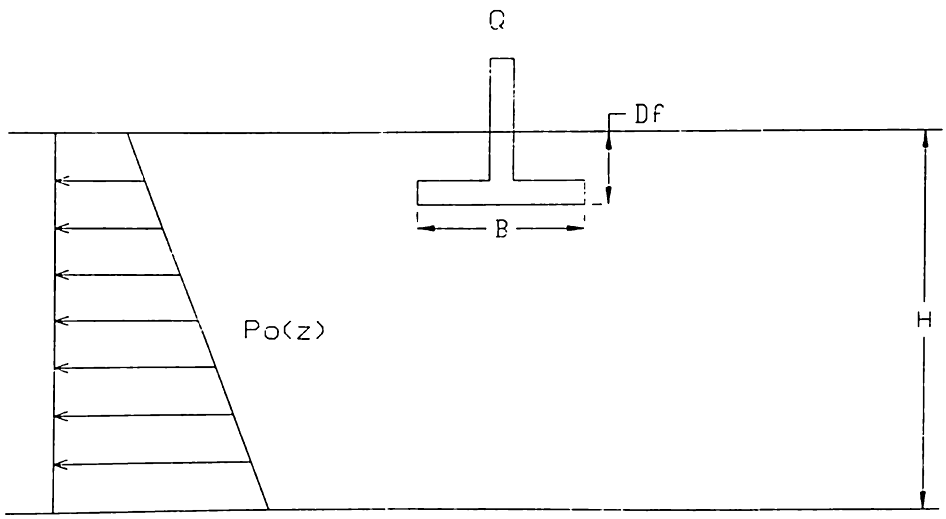

Figure 13. Geometry of footing problem.

CASE 4: **Isolated Footing Underlain by Normally Consolidated Soil**

The case geometry of this case is illustrated in Figure 13. It consists of a spread footing of width B founded at a depth of D_f within a compressible, normally consolidated soil layer of thickness H. Assuming that the footing exerts a uniform bearing pressure of q, the corresponding stress change is given by (Poulos and Davis [1974])

$$\Delta\sigma = qI_B(\xi) \tag{1.23}$$

where $I(\xi)$ is the appropriate Boussinesq influence factor in terms of the non-dimensional parameter $\xi = \frac{z}{b}$, with $b = \frac{B}{2}$. In this particular case, ξ is measured from the base of the footing. For a square footing

$$I_B(\xi) = \frac{2}{\pi}\left[\arctan\left(\frac{1}{\sqrt{\xi^2+2}}\right) + \frac{2\xi}{\sqrt{\xi^2+2}\left(\xi^2+1\right)}\right] \tag{1.24}$$

For a circular footing, $I(\xi)$ is given by the expression

$$I_c(\xi) = 1 - \left[\frac{1}{1+\xi^{-2}}\right]^{3/2} \tag{1.25}$$

For a strip footing, $I(\xi)$ is given by the expression

$$I_s(\xi) = \frac{1}{\pi}\left[\theta + \sin(\theta)\cos(\theta + 2\phi)\right] \tag{1.26}$$

where $\phi = \arctan\left(\frac{\eta - 1}{\xi}\right)$, $\theta = \arctan\left(\frac{\eta + 1}{\xi}\right) - \delta$, and $\eta = \frac{\xi}{b}$.

The initial overburden pressure is given by the expression

$$p_o(\xi) = \gamma D_f(\kappa\xi + 1) \tag{1.27}$$

where $\kappa = \frac{b}{D_f}$. The final overburden pressure is given by the expression

$$p_f(\xi) = \gamma D_f(\kappa\xi + 1) + (\alpha - 1) I_B(\xi) \tag{1.28}$$

where $\alpha = \frac{q}{\gamma D_f}$. The corresponding settlement in terms of non-dimensional terms is given by

$$\Delta = bC_{\varepsilon c}\int_0^\beta \log\left(1 + \frac{\alpha - 1}{\kappa\xi + 1} I_B(\xi)\right) d\xi \tag{1.29}$$

where $\beta = \frac{H}{b}$.

In this particular case, it was discovered that the problem depends on four parameters, namely, the footing width B, the load intensity factor α, the surcharge intensity factor κ, and the relative thickness β. Only α, β and κ were noted to affect the computational error.

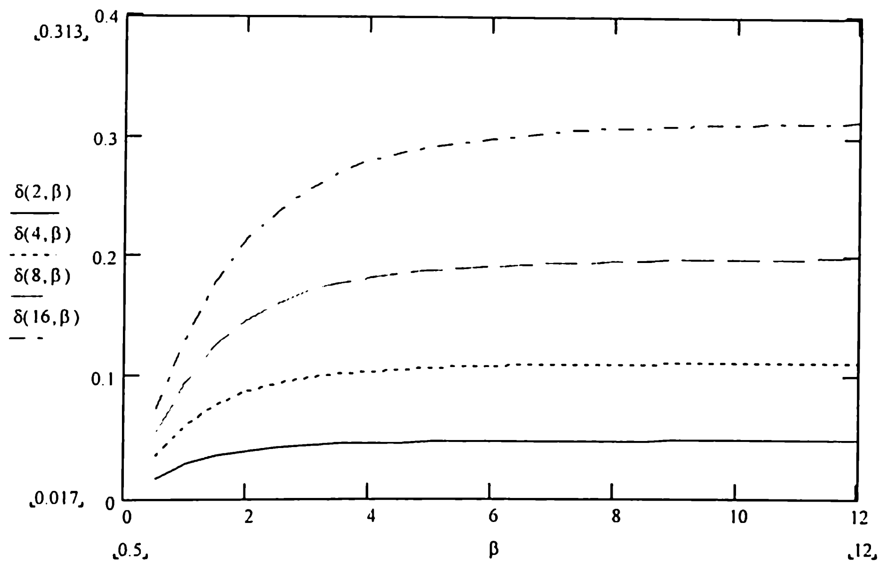

Figure 14. Effect of load intensity on settlement of square footing on normally consolidated soil.

Figure 14 shows the consolidation settlement as a function of load intensity α and relative thickness β. As was observed previously, the settlement increases with increasing α. While the settlement also increases with increasing β, the rate at which this increase takes places decreases rapidly with increase β such that for values of $\beta > 6$, this increase is negligible.

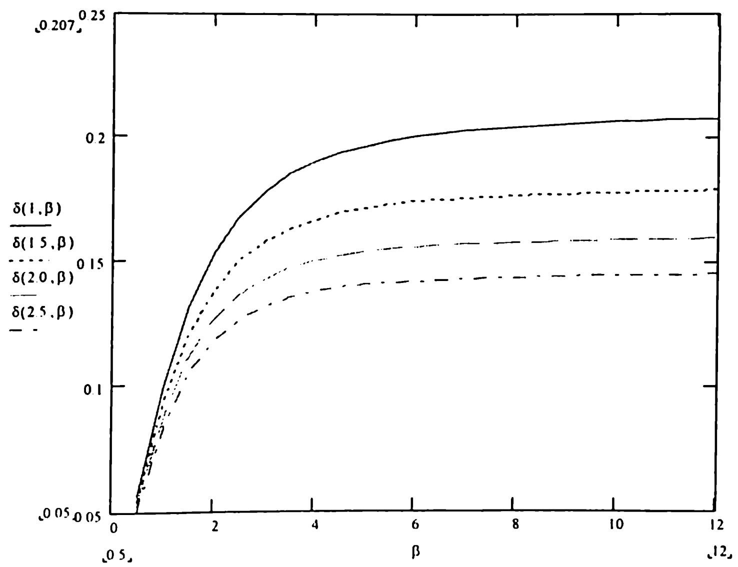

Figure 15. Effect of foundation depth on settlement of square footing on normally consolidated soil.

Figure 15 shows the consolidation settlement as a function of the foundation depth factor κ and relative thickness β. The graph shows that with increasing κ, the resulting settlement decreases due to the increase in the overburden pressure through out the entire compressible layer.

Settlement for this problem obtained using the conventional one-point method was compared to those numerically obtained using the settlement ratio ρ defined as the ratio between settlements obtained from the one-point method and those obtained using the numerical procedure. A value of $\rho < 1$ indicated the exert of the under prediction in the one-point method.

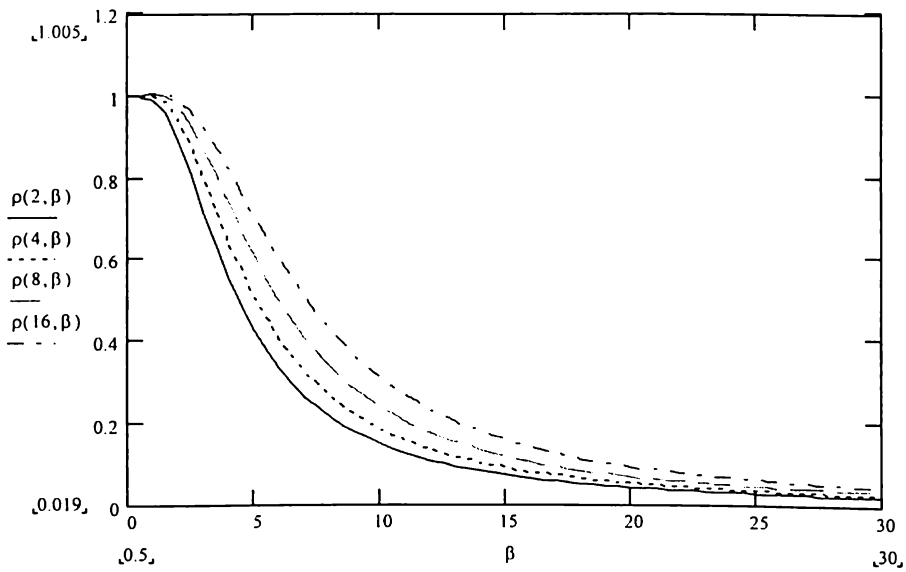

Figure 16. Effect of load and surcharge intensity on settlement ratio (conventional one-point method) of square footing on normally consolidated soil.

Figure 16 shows the settlement ratio as a function of load intensity α and relative thickness β. The graph shows the general tendency of the conventional one-point method to underpredict the settlement. This under prediction ranges from between 20 percent and 2000 percent, as increases with increasing α.

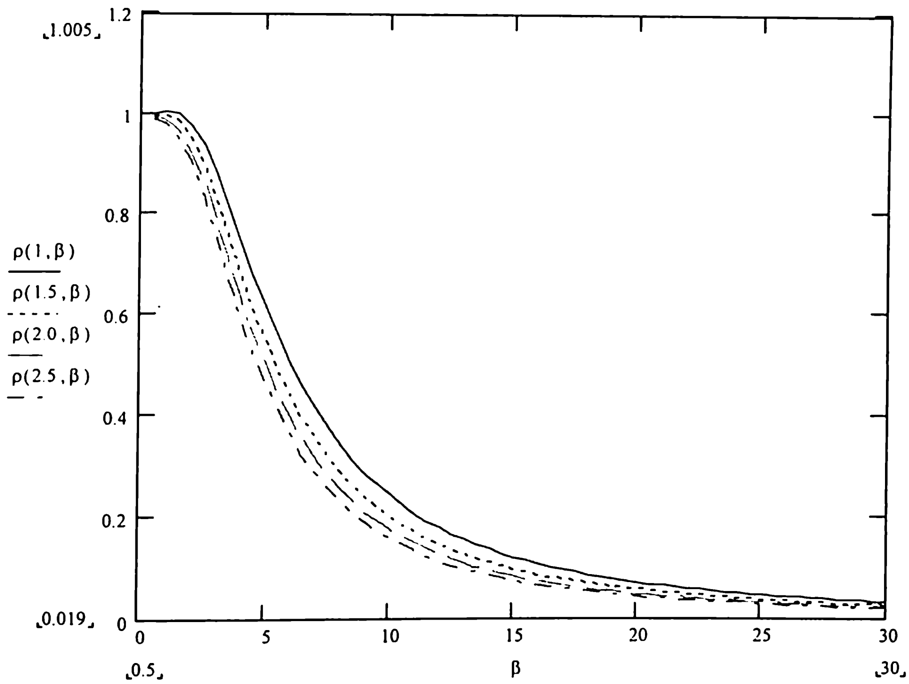

Figure 17. Effect of foundation depth on settlement ratio (conventional one-point method) of square footing on normally consolidated soil.

Figure 17 shows the settlement ratio as a function of foundation depth κ and relative thickness β. The graph shows the general tendency of the conventional one-point method to underpredict the settlement. This under prediction ranges from between 20 percent and 2000 percent, as increases with decreasing κ.

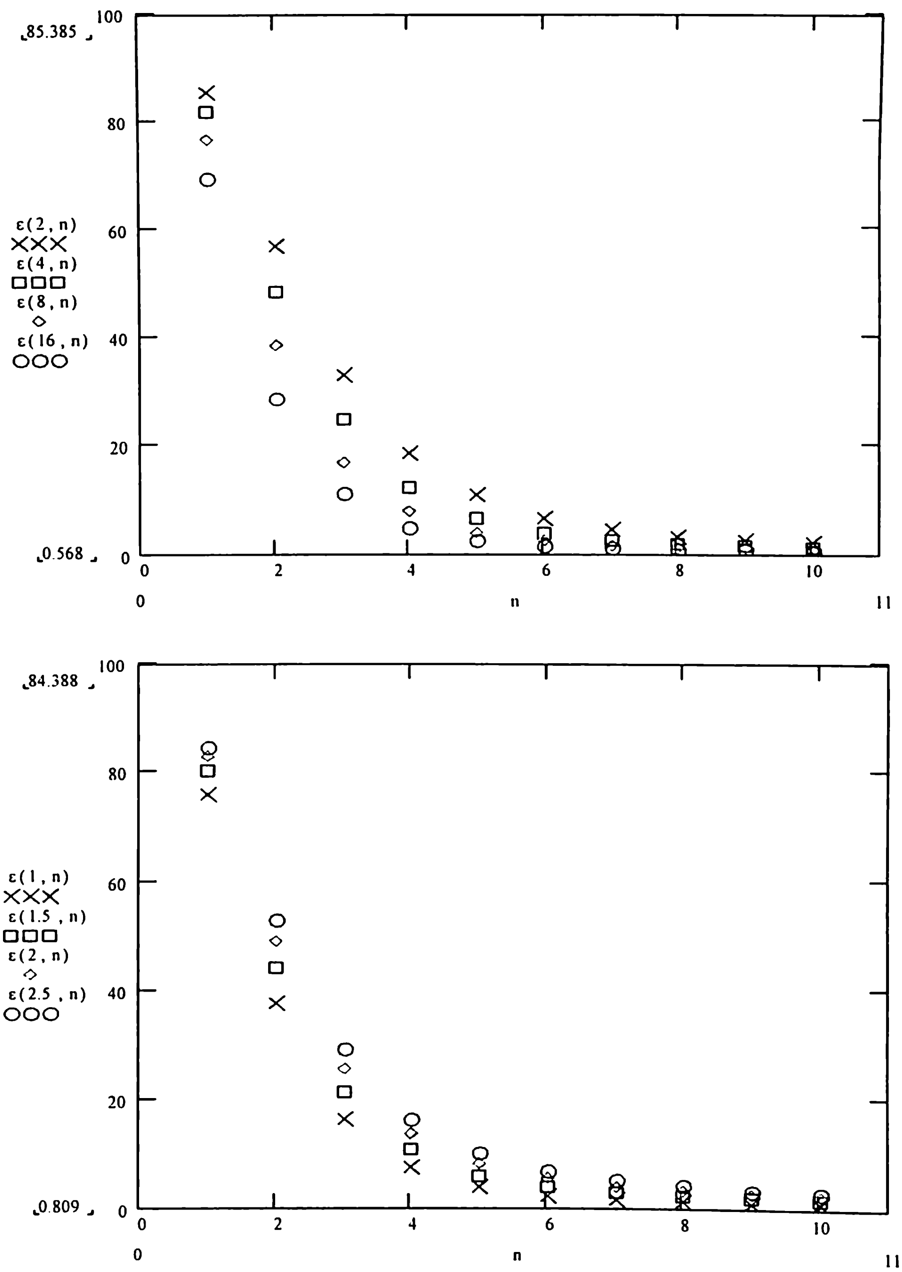

Figure 18. Effect of sublayer thickness on settlement computed by finite sublayer method for square footing on normally consolidated soil considering (a) load intensity (b) depth of compressible layer

Figure 18 shows the settlement computational error as a function of load intensity factor α using varying numbers of sublayers in the finite sublayer method. Figure 17 shows the settlement computational error as a function of foundation depth κ using varying numbers of sublayers in the finite sublayer method. These figures show that sublayer thickness of between 1 to 2 meters is required to obtain settlements within 10% of the exact value.

The following table illustrates the effect of the shape of the footing on the resulting settlement and corresponding computational error. Consolidation settlements were calculated for a 2 meter wide footing, underlain by a 10 m. layer of normally consolidated soil. Settlements were computed at the center of the footing assuming a square, circular and strip footing. For the strip footing, settlements were also computed at the footing edge. The compression index $C_{\varepsilon c}$ was assumed equal to 0.24 with a void ratio of 0.8 and a unit weight of soil equal to $\gamma = 14.52$ kN/m^3. In all cases analyzed, the footing was located at 1 meter below the ground surface.

Footing Shape	Exact Δ	One-point Method		Sublayer Method	
		Δ	ρ	Δ	ρ
Square	0.2054	0.0584	0.2843	0.19971	0.9723
Circle	0.1912	0.0473	0.2474	0.18501	0.9676
Strip (Center)	0.2896	0.1653	0.5708	0.28389	0.9803
Strip (Edge)	0.2342	0.1534	0.6545	0.22927	0.9789

Results of the above analyses indicate that for the one-point method, the under prediction is greatest in the circular footing and least in the strip footing. Settlements at the middle are under-predicted more than at the edge of the footing. In the sublayer method, a total of 5 sublayers were used for each analysis. The resulting under prediction was within 5% of the exact value for all types of footings analyzed.

CASE 5: Isolated Footing Underlain by Over Consolidated Soil

This case is similar to the previous case with the exception that prior to application of the footing load, the soil was pre-loaded by the application of a surcharge of $q_0 \le q$. For the conventional one-point method as well as the finite sub-layer method, it is necessary to determine the intermediate point ξ_m where the condition $p_p(\xi_m) = p_f(\xi_m)$ is satisfied, where the preconsolidation pressure is defined by the expression

$$p_p = \gamma D_f \left(\alpha_p + \kappa\xi + 1\right), \tag{1.30}$$

and where $p_f(\xi)$ is given by (1.28).

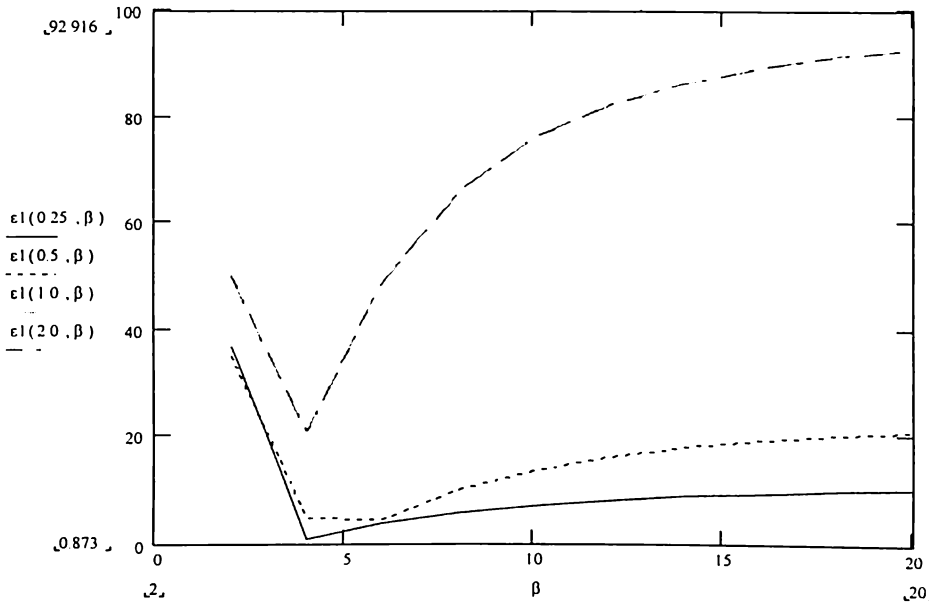

Figure 19. Effect of compressible layer thickness and preconsolidation pressure on computational errors in settlement calculations for square footing on pre-loaded soil using conventional one-point method.

Figure 19 illustrates the numerical error in the settlements computed using the conventional one-point method. The graph shows that these errors increase with increasing preconsolidation pressure and layer thickness. However, the magnitude of theses errors is significantly less than that for the normally consolidated soils for cases where the preconsolidation pressure is less than the bearing pressure of the footing. For cases where the preconsolidation pressure is greater than the bearing pressure exerted by the footing, the errors are the same as those for the normally consolidated case. The aforementioned behavior is due to the fact that in the latter case, the entire settlement results from recompression, whereas only part of the settlement results from recompression in the former case. This reduces the strains that occur near the top of the soil layer in the latter case, and results in a more uniform strain distribution.

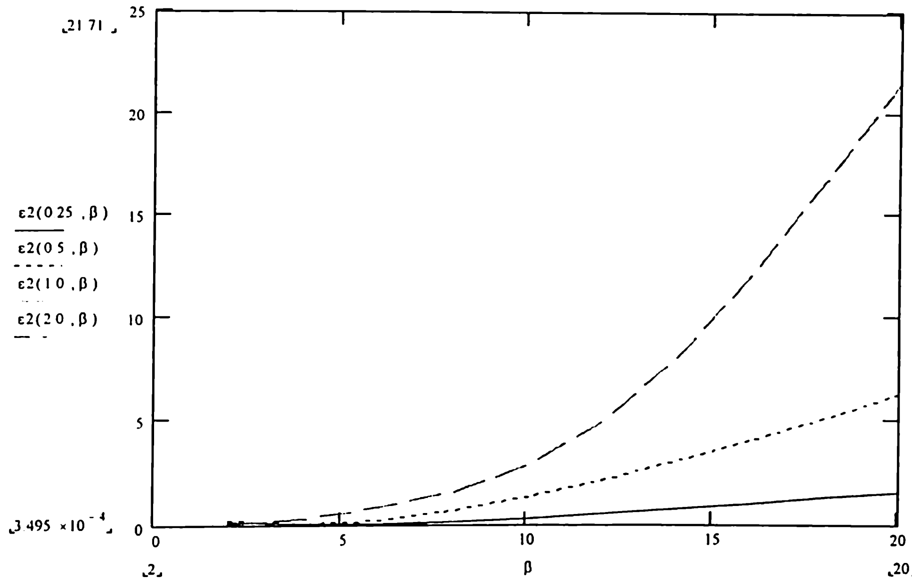

Figure 20. Effect of compressible layer thickness and preconsolidation pressure on computational errors in settlement calculations for square footing on pre-loaded soil using finite sublayer method.

Figure 20 shows the numerical error in the settlement computed using the finite sublayer method in which the soil layer is divided into two parts: the upper layer which undergoes a combination of recompression and virgin compression, and the lower layer which undergoes recompression only. Each of these layers was further subdivided into a five sublayers. Results of the analyses show that for cases where the sublayers did not exceed a thickness of 2 meters, the predicted values were within 5% of the exact value.

V. SUMMARY AND CONCLUSIONS

In this paper, the numerical errors that arise when consolidation settlements are computed using the conventional one-point method were investigated. It was shown that consolidation settlements can be calculated accurately through the use of an appropriate numerical quadrature. Using this technique, exact solutions were obtained for purposes of estimating the numerical errors of the conventional one-point and finite sublayer methods. Studies show that these numerical errors are most significant in either normally consolidated or heavily consolidated soils, and are less pronounced in cases of lightly overconsolidated as well as underconsolidated soils. It was also shown that increasing the intensity of the load, surcharge and footing depth served to diminish these errors. In contrast, these numerical errors increase as the thickness of the compressible soil layer increases. It was also shown that these under-predictions in settlement are most pronounced for square and circular footings as compared to strip footings or extensive fills. Finally, for the finite sublayer method, it is recommended that thickness of each sublayer be restricted to no more than 2 meters in order to obtain values that are within 10% of the exact value.

REFERENCES

1. McPhail, J., P. Hellen, S. Britton, C. Colvin, T. Silvey, and J. Jones. *Evaluation of Consolidation Settlement Using the Sublayer Method.* Electronic Journal of Geotechnical Engineering, Vol. 5, 2000

2. Chapra, Steven C. and Raymond P. Canale. *Numerical Methods for Engineers, 3rd Ed.* McGraw Hill International. 1998, 924 pp.

3. Press, William H. *Numerical Recipes in FORTRAN 2nd Ed.* Cambridge University Press,. 1992, 963 pp.

4. Poulos, H. G., and E. H. Davis. *Elastic Solutions for Soil and Rock Mechanics.* John Wiley and Sons, 1974. pp 411.

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NUMERICAL ISSUES RELATED TO THE CALCULATION OF CONSOLIDATION SETTLEMENTS

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ABSTRACT

The evaluation of settlements due to consolidation is one of the most common computational procedures in geotechnical engineering. Recent research on this topic has indicated that current computational procedures underestimate consolidation settlements by as much as 70%. These errors are believed to result from the over-simplified manner by which the strains are numerically integrated. In this paper, the magnitude and nature of these numerical errors is investigated. A series of numerical experiments are performed to study the effects of load intensity and type, depth of foundation, thickness of soil layer, and preconsolidation pressure on these errors. Numerical errors are evaluated by comparing results obtained using the aforementioned procedures to either closed-form analytical solutions or numerical solutions using a high precision adaptive quadrature. Results of the numerical experiments indicate that such the underestimation of consolidation settlements is more pronounced in normally consolidated as well as heavily overconsolidated soils as compared to either lightly overconsolidated or underconsolidated soils. Recommendations are made regarding the proper use of the above-mentioned procedures in order to guarantee as sufficient degree of accuracy in the calculations.

I. INTRODUCTION

The evaluation of settlement due to consolidation is one of the most common computational procedures in geotechnical engineering. Traditionally, research on this topic has focused primarily on accurately estimating the time rate at which such settlements occur. However, recent study of this topic has indicated that present methods used for calculating consolidation settlements have the general tendency to underestimate such settlements by as much as 70%. Generally, the settlements due to surface loads are calculated by integrating the vertical strains resulting from such loads over the depth of the compressible soil layer. This process of integration is often approximate by dividing the entire soil layer into a finite number of sublayers, calculating the settlement in each layer based on the stress condition at the middle of the layer, and summing up the incremental settlements to obtain the settlement of the entire soil layer. This method shall be referred to in this paper as the finite sublayer method. This procedure is reduced to the conventional one-point method when one layer is used. While, the use of the one-point method and finite sublayer method for calculating consolidation settlements is described in most books on geotechnical engineering, none of these discussions provide clear guidelines for determining the number of sublayers required to achieve a prescribed level of accuracy. The limitations and inaccuracies of the conventional one-point method as well as the need for more rigorous methods