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MATERIAL CHARACTERIZATION AND MODELING OF SILICONE PUTTY USING FRACTIONAL DERIVATIVES

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ABSTRACT

Material characterization and modeling of materials being used in energy-dissipating devices is a prerequisite in the development of accurate models which can predict the behavior of these devices. One such material in silicone putty used in the Shock Transmission Unit (STU). This paper discusses the material characterization of silicone putty using a flat plate rheometer and its modeling using a fractional derivative maxwell model under low frequencies of loading. The resulting constitutive equation is approximated using the L1-Algorithm for fractional derivatives.

I. Introduction

Energy-dissipating devices of various forms have been developed making use of different materials to dissipate energy, such as friction materials, viscoelastic polymers, viscous fluids, ductile metals and others. These devices are mechanical dampers which dissipate significant amount of energy, augmenting the energy-dissipating capability of structural systems. Other benefits provided by adding energy-dissipating devices (especially in bridges) are effective distribution of forces and induced load-sharing among the substructure components. One such device is the Shock Transmission Unit (STU) which makes use of silicone putty [Pritchard, 1989].

Silicone putty is classified as a reverse thixotropic material. A thixotropic material behaves like a solid when unstressed but will flow like a liquid when pressure is applied to it. Thus, silicone putty flows slowly when constant pressure is slowly applied to it, but will exhibit solid material behavior when it experiences an impact (high velocity) load [Brown, 1996].

At low frequencies of loading the silicone putty behaves viscoelastically but at high frequencies of loading it stiffens. It is during the low frequencies of loading that the STU dissipates energy which can be seen in the hysteresis of the device. However, at high frequencies of loading the silicone putty behaves like a solid effecting load sharing among substructure components without energy-dissipation [Tanzo, 1996]. It is during low frequencies of loading that is of interest in this paper, i.e. the characterization and modeling of the behavior of silicone putty at low frequencies only.

II. Material Characterization

The physical properties of silicone putty prevent it from being characterized by standard tests such as simple tension/compression, equibiaxial tension, simple shear and pure shear. However, high precision rheometers are now being routinely used in laboratories which can test materials like silicone putty using oscillation tests.

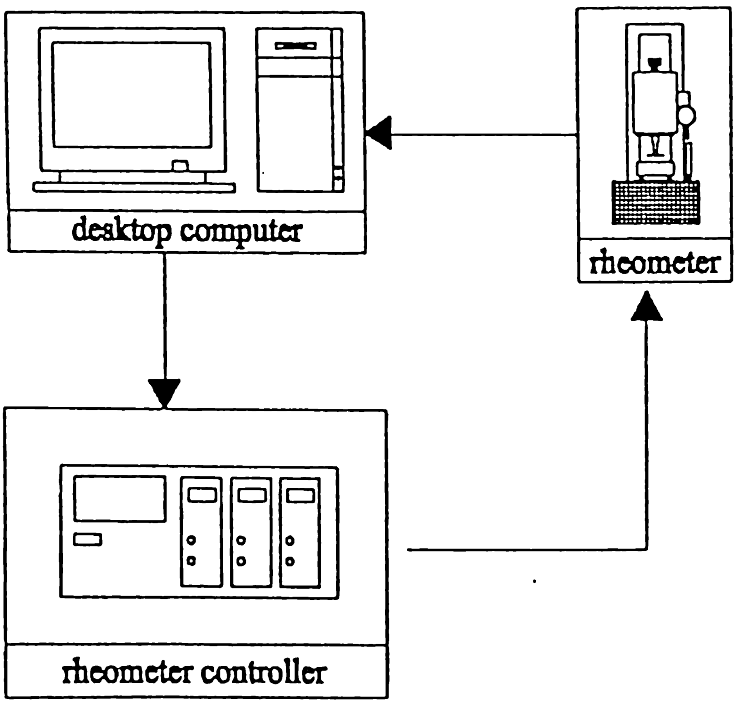

Figure 1. Flow Chart of the Experiment

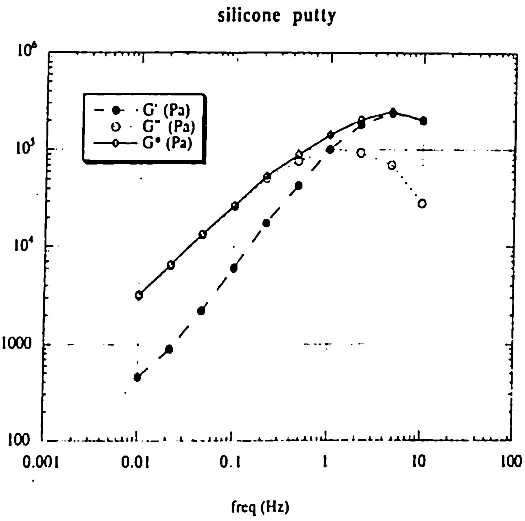

Figure 2. Storage *G'*, Loss *G"* and Complex *G** Modulus

The silicone putty was characterized using a flat plate rheometer at a constant oscillation stress of 3 Pa. It was subjected to different oscillation frequencies at a constant temperature of 20°C. The flow-chart of the experiment is shown in Figure 1. The material storage modulus G' and loss modulus G'' were determined. Using these two parameters a constitutive equation model for the silicone putty can be developed. Figure 2 shows the plot of G', G'' and the complex modulus $G*$.

III. Maxwell Fractional Derivative Model

Although a generalized standard solid model, of the form

$$\tau + \sum_{n=1}^{\infty} \alpha_n \frac{d^n \tau}{dt^n} = G\gamma + G\sum_{n=1}^{\infty} \beta_n \frac{d^n \gamma}{dt^n} \tag{1}$$

can be developed from the oscillation test on the silicone putty, a large number of terms and/or parameters must be used in order to come up with a suitable curve fit. However, with the use of fractional derivative modeling, the number of parameters needed are greatly reduced to as few as three parameters in modeling the material with high accuracy. Fractional derivative modeling takes the form,

$$\tau(t) + \alpha \frac{d^{\alpha}\tau(t)}{dt^{\alpha}} = G\gamma(t) + Gb\frac{d^{\beta}\gamma(t)}{dt^{\beta}} \tag{2}$$

where the generalized derivatives $\frac{d^{\alpha}\tau(t)}{dt^{\alpha}}$ and $\frac{d^{\beta}\gamma(t)}{dt^{\beta}}$ are defined as,

$$\frac{d^{\alpha}x(t)}{dt^{\alpha}} = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{x(\tau)}{(t-\tau)^{\alpha}}dt \tag{3}$$

with $0 < \alpha < 1$ and Γ is the gamma function.

A fractional derivative maxwell model was developed from the results of the oscillation tests,

$$\tau(t) + \lambda \frac{d^{q}\tau(t)}{dt^{q}} = \mu_1 \frac{d^{p}\gamma(t)}{dt^{p}} \tag{4}$$

with q assumed equal to p and less than unity. Taking the Fourier transform of both sides of the equation,

$$\tau^* + \lambda(i\omega)^q \tau^* = \mu_1(i\omega)^p \gamma^* \tag{5}$$

$$G^* = \frac{\tau^*}{\gamma^*}$$

$$= \frac{\mu_1(i\omega)^p}{1+\lambda(i\omega)^q} \tag{6}$$

Equation 6 can be manipulated to separate the real and imaginary parts which correspond to the storage and loss modulus, G' and G'', respectively. One arrives at the following expressions:

$$G'(\omega) = \frac{\mu_1 \omega^p \cos(\frac{p\pi}{2}) + \mu_1 \lambda \omega^{p+q} \cos\left[\frac{\pi}{2}(p-q)\right]}{1 + 2\lambda\omega^q \cos(\frac{q\pi}{2}) + \lambda^2 \omega^{2q}} \tag{7}$$

$$G''(\omega) = \frac{\mu_1 \omega^p \sin(\frac{p\pi}{2}) + \mu_1 \lambda \omega^{p+q} \sin\left[\frac{\pi}{2}(p-q)\right]}{1 + 2\lambda\omega^q \cos(\frac{q\pi}{2}) + \lambda^2 \omega^{2q}} \tag{8}$$

Knowing that $G^* = \sqrt{G'^2 + G''^2}$, the Levenberg-Marquardt algorithm is then used to apply a curve fit to the test data. The curve fit, shown in Figure 3, resulted in values of $\mu_l = 1.8981e+05$ Pa, $\lambda = 0.64133\ (sec)^{0.82234}$, and $q = p = 0.82234$.

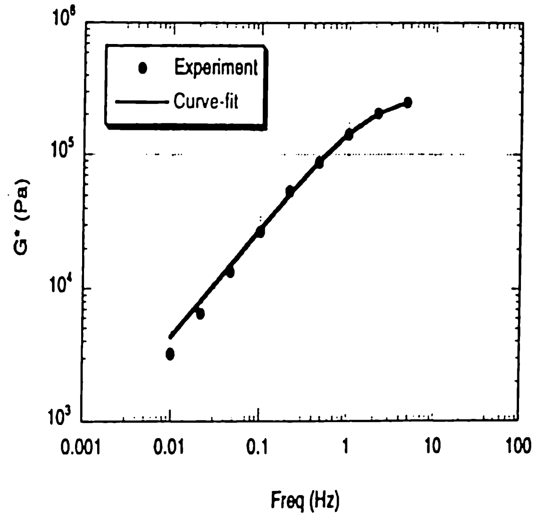

Figure 3. Fractional Derivative Maxwell Model Curve-Fit

IV. Linear L1 - Algorithm

A closed form solution of an equation involving fractional derivatives is very difficult to derive except for the order of the derivative equal to one-half. One can then take the Laplace transform of the equation and arrive at a solution. However, for the general case, numerical approximations are normally used in order to solve equations having fractional derivatives of arbitrary order. One such approximation is the linear L1-Algorithm which assumes that the function whose fractional derivative is being taken is piecewise linear in every subinterval.

The fractional derivative definition,

$$\frac{d^q \tau(t)}{dt^q} = \frac{1}{\Gamma(1-q)} \frac{d}{dt} \int_0^t \frac{\tau(\xi)}{(t-\xi)^q} d\xi \tag{9}$$

can be expressed in the following form,

$$\frac{d^q \tau(t)}{dt^q} = \frac{1}{\Gamma(1-q)} \left[\frac{\tau(0)}{t^q} + \int_0^t \frac{\tau(t-\xi)}{\xi^q} d\xi \right] \tag{10}$$

Applying the assumption that $\tau(t)$ is piecewise linear over each subinterval $[jh, (j+1)h]$, we obtain the L1-Algorithm. Following the formulation of Koh [1985], the derivative of the shear stress in the subinterval is approximated by

$$\tau(t-\xi) \cong \frac{\tau_{n-j} - \tau_{n-j-1}}{h} \tag{11}$$

$jh \le \xi \le (j+1)h$, which is independent of ξ in the subinterval. This makes it possible to integrate and simplify equation 10, yielding the L1-Algorithm.

$$\frac{d^q \tau_n}{dt^q} = \frac{1}{h^q \Gamma(2-q)} \left[\frac{(1-q)\tau_0}{n^q} + \sum_{j=0}^{n-1} (\tau_{n-j} - \tau_{n-j-1})[(j+1)^{1-q} - j^{1-q}] \right] \tag{12}$$

This equation can be written in quadrature form as,

$$\frac{d^q \tau_n}{dt^q} = \frac{1}{h^q} \sum_{j=0}^{n} \omega_j \tau_j, \qquad 0 < q < 1 \tag{13}$$

where,

$$\omega_0 = \frac{1}{\Gamma(2-q)} \left[(n-1)^{1-q} - n^{1-q} + (1-q)n^{-q} \right] \tag{14}$$

$$\omega_n = \frac{1}{\Gamma(2-q)} \tag{15}$$

$$\omega_{n-j} = \frac{1}{\Gamma(2-q)}\left[(j+1)^{1-q} - 2j^{1-q} + (j-1)^{1-q}\right], \quad 1 \le j \le n-1 \tag{16}$$

Substituting back into the fractional derivative maxwell model, equation 4:

$$\tau_n + \frac{\lambda}{h^q}\sum_{j=0}^{n}\omega_j\tau_j = \frac{\mu_1}{h^p}\sum_{j=0}^{n}\upsilon_j\gamma_j \tag{17}$$

$$\tau_n(1+\frac{\lambda}{h^q}\omega_n) = \frac{\mu_1}{h^p}\sum_{j=0}^{n}\upsilon_j\gamma_j - \frac{\lambda}{h^q}\sum_{j=0}^{n}\omega_j\tau_j \tag{18}$$

if we let,

$$\varpi_n = 1 + \frac{\lambda}{h^q}\omega_n \tag{19}$$

$$\upsilon_j = \frac{\mu_1}{h^p}\upsilon_j, \qquad 0 \le j \le n \tag{20}$$

$$\varpi_j = \frac{\lambda}{h^q}\omega_j, \qquad 0 \le j \le n-1 \tag{21}$$

Then we can solve for τ_n as,

$$\tau_n = \frac{1}{\varpi_n}\left[\sum_{j=0}^{n}\upsilon_j\gamma_j - \sum_{j=0}^{n-1}\varpi_j\tau_j\right] \tag{22}$$

Similar to Koh's formulation, a shifted L1-algorithm was developed in order to lessen the number of terms which will come into the equation because $n = t/h$ becomes very large as t increases.

A fortran program was developed using the shifted L1-algorithm mentioned. For the purpose of verifying the developed program, a numerical example is provided. The L1-Algorithm is implemented for a shear strain, varying sinusoidally,

$$\gamma = \gamma_0 \sin \omega t \tag{23}$$

where ω is the frequency of the sinusoidal load. Taking the Laplace transform of the resulting fractional derivative maxwell model equation and solving for τ (s)

$$\tau(s) = \mu_1 \left(\frac{s^p}{1 + \lambda s^q} \right) \frac{\gamma_0 \omega}{s^2 + \omega^2} \tag{24}$$

taking $\lambda = \omega = \gamma_0 = 1.0,\ q = 0.5,\ p = 1$ and $\mu_1 = 1000$ in equation 24,

$$\tau(s) = 1000 \left(\frac{s}{1 + s^{0.5}} \right) \left(\frac{1}{s^2 + 1} \right) \tag{25}$$

The closed form solution of this equation was derived using Mathematics 3.0. One observes from the plot of the closed form solution and that obtained using the fortran program, shown in Figure 4, that the two are in very good agreement.

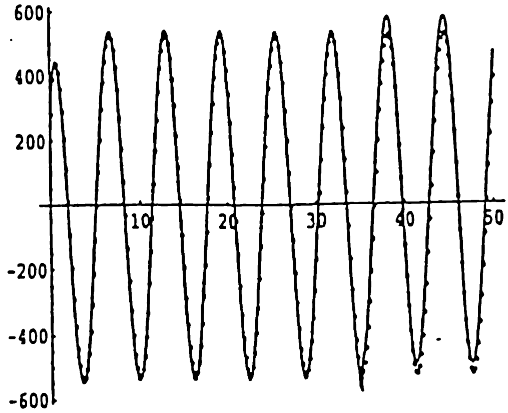

Figure 4. Comparison of Closed-Form Solution with L1-Algorithm Numerical Approximation $(\tau$ vs $t)$

V. Conclusions

Silicone putty subjected to low frequency sinusoidal loading was successfully modeled using a fractional derivative maxwell model. As a means of solving the resulting constitutive equation, the L1-Algorithm was implemented using a fortran program which was verified by solving a differential equation involving fractional derivatives which has a closed form solution. The material model can now be used in the modeling of energy-dissipating devices which uses silicone putty.

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