Philippine Engineering Journal (1994) XV (1): 17-40

MEASURING PROCESS CAPABILITY

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ABSTRACT

Process capability indices are succinct unitless statistical metric which measures the amount of common cause variation present in a process. They indicate the ability of the process to meet engineering specifications or whether process centering poses a problem. First and second generation indices: C_p, C_{pL}, C_{pu}, C_{pk}, k, C_{pm} and Pearson process capability indices are presented, examined and compared. Single and confidence interval estimates of these indices are described. Finally, applications, drawbacks and uses of these indices are discussed.

Keywords: Process Capability Index, C_p, C_{pL}, C_{pu}, k, C_{pk}, C_{pm}, Pearson Capability Index

INTRODUCTION

Uniformity of a process is the current prevailing thinking about manufacturing quality. Quality engineering in particular deals primarily with the systematic understanding and reduction of process variability (or increasing process capability). Superior quality is now equated with how small process variation is or how far the process, is from a pre-specified target. Today, it is considered unacceptable to speak of quality as simply meeting the specification limits. The traditional "goal post syndrome" of meeting the specifications has been replaced with the Taguchi-based loss function or variation-based deviation from a target. Quality has been redefined as minimum variation from a pre-defined target.

Universally accepted as a metric for measuring process uniformity is process capability. While there are many definitions associated with process capability, it is generally defined as the inherent or natural variation present in a process. Process capability can either be instantaneous (short term) or can be viewed over time (long term). It can also be taken as quantifying the spread over which the outputs of a process can vary, Kane, (1986) [15]. In short, process capability determines the amount and behavior of the common cause variation found in a process.

Table 1.
C_p Value and Associated Process Fall Outs
for a Normally Distributed Process
(with Process Cénter at the Nominal)

Cp	Process Fall Out (defective ppm)
0.25	453,225
0.50	133,614
0.60	71,861
0.70	35,729
0.80	16,395
0.90	6,934
1.00	2,700
1.10	967
1.20	318
1.30	96
1.40	27
1.50	7
1.60	2
1.70	0.34
1.80	0.06
2.00	0.0018

Source: Montgomery (1991) p. 372 [17]

First and second generation measures of process capability exist. These measures are popularly known as process capability indices. These indices are succinct statistical measures which relate the process location **u** and the spread σ to engineering specifications. Process capability indices serve these major purposes: They measure the spread and deviation from a predetermined value. They measure the amount of nonconforming products of a process and they are unitless comparative measures between processes with different quality characteristics over time. Process capability indices exist for both single-sided and double-sided tolerances with or without target values. In this paper only those process capability indices involving variable quality characteristics are discussed.

The objective of this paper is to examine, compare and evaluate the capability indices C_p, C_{pL}, C_{pu}, C_{pk}, k, C_{pm}, and Pearson process capability indices. The underlying assumptions, sampling and estimation procedures of these indices are reviewed and described. Also discussed are the limitations, selection and applications of these indices. Finally, areas for further investigation are presented.

PROCESS POTENTIAL INDEX, C_p

A customary measure of process capability is to determine the natural tolerance six sigma (6 σ) or simply (σ) and relate it to engineering specifications. One popular alternative measure is the process potential index C_p expressed as the ratio of the allowable process spread and the actual process spread. Mathematically,

$$C_p = \frac{\text{Allowable Process Spread}}{\text{Actual Process Spread}} \qquad [1]$$

$$= \frac{USL - LSL}{6\sigma}$$

where USL = upper specification limit
LSL = lower specification limit
σ = process standard deviation
6σ = natural tolerance
= actual spread

C_p assumes that the underlying process is approximately normally distributed and has achieved a state of statistical stability or control. As indicated in Equation 1, C_p values are meaningful only for two-sided tolerances. Under conditions of normality and statistical control, a capable process theoretically will result in 2,700 nonconforming parts per million (nppm) beyond the specifications limits. One important use of C_p is to make various types of process comparisons. Kane (1986) [15] presented different C_p indices for varying widths of the process distribution. It can be seen easily that a $C_p = 1$ will allow production of 2,700 nppm; a $C_p =$ 1.33 will allow 64 nppm; a $C_p = 1.66$ will allow 0.6 nppm while a $C_p = 2$ will allow about zero nppm. Equivalently, a $C_p = 2$ can be taken as consuming 50% of specifications while a $C_p = 1$ consumes exactly 100% of specifications. Shown in Table 1 are C_p values and associated process fall outs for a normally distributed process with target value centered at the nominal. {Montgomery (1991), p. 372} [17].

Since the actual process variation σ is not normally readily available, C_p is usually estimated by

$$\hat{C}_p = \frac{USL - LSL}{6S} \qquad [2]$$

where S = process sample standard deviation computed from a sample of size n

$$S = \left[\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} \right]^{1/2}$$

Using confidence interval estimation, the true value of C_p can also be estimated. Knowing that $(n - 1) (\hat{C}_p/C_p)$ follows a chi-square distribution with $(n - 1)$ degrees of freedom, a $100 (1 - \alpha) \%$ confidence interval of C_p can easily be procured. Montgomery (1991) [17], Kane [15] and Chou et.al. (1990) [5] presented confidence interval estimates of C_p.

A two-sided $100 (1-\alpha)\%$ confidence interval estimate of C_p may be obtained from

$$\frac{USL - LSL}{6S} \sqrt{\frac{X^2_{1-\alpha/2,\, n-1}}{n-1}} \leq C_p \leq \frac{USL - LSL}{6S} \sqrt{\frac{X^2_{\alpha/2,\, n-1}}{n-1}}$$

or

$$\hat{C}_p \sqrt{\frac{X^2_{1-\alpha/2,\, n-1}}{n-1}} \leq C_p \leq \hat{C}_p \sqrt{\frac{X^2_{\alpha/2,\, n-1}}{n-1}} \qquad (3)$$

where $X^2_{1-\alpha/2,\ n-1}$, and $X^2_{\alpha/2,\ n-1}$, are the lower $\alpha/2$ and upper $\alpha/2$ percentage points of the chi-square distribution with n-1 degrees of freedom respectively. A lower confidence estimate of C_p can be obtained from

$$\hat{C}_p \sqrt{\frac{X^2_{1-\alpha,\ n-1}}{n-1}} < C_p \qquad (4)$$

Chou et. al. (1990) [5] developed two tables which facilitate computing confidence interval estimates of C_p, 95% lower confidence limit C for C_p when n and C_p are given. The minimum values of C_p for which the process is.capable 95% of the time can be found in Chou et. al [5].

The process potential index C_p can be generalized to a case where the process has a target value, T. C_{pt}, when there is a target value is now computed as

$$C_{pt} = \text{Min}\left[\frac{T - LSL}{3\sigma},\ \frac{USL - T}{3\sigma}\right] \qquad (5)$$

where T = target value
σ = process standard deviation
USL = upper specification limit
LSL = lower specification limit

When the target value T is equal to the nominal or to the midpoint of the natural tolerance (USL-LSL), $C_p = C_{pt}$. The relationships between the target values T, the process capability C_p and associated costs are clearly shown in Kane (1986)[15].

Analogously as in Equation 2 C_{pt} can be estimated by substituting sigma σ with the sample standard deviation s in Equation 5.

Since C_p relates only the process spread to the specification limits, the centering of the process is not considered. Thus, C_p can only be used to measure the potential performance of the process. [1] [2] [10] [14] [15] [17] [21].

PROCESS PERFORMANCE INDICES

Process performance indices measure process capability by explicitly considering the magnitude of the process variation as well as the location of the process or the departures from the target value associated with the process. Among the most popularly used indices are C_{pu}, C_{pL}, C_{pk}, and k, Chan et.al. (1988) [3] and Spiring (1992) [20] proposed a new measure of process capability C_{pm} which takes into account both the proximity to the target value as well as the process variation when assessing process performance.

Upper Capability Index, CPu

The upper capability index, CPu is defined by

$$CPu = \frac{\text{Allowable Upper Spread}}{\text{Actual Upper Spread}}$$

$$= \frac{USL - u}{\frac{(USL - LSL)}{2}}$$

$$= \frac{USL - u}{3\sigma} \qquad (6)$$

where USL = upper specification limit
u = process mean
σ = process standard deviation

Kane (1986) [15] reported that CPu index was developed in Japan and is being utilized by a number of Japanese companies. It is specifically useful for processes where only an upper specification limit is given. To estimate CPu, we use the process average **x** and standard deviation s computed from a sample of size **n** in the formula for computing CPu. This estimated CPu is given:

$$CPu = \frac{USL - \bar{x}}{3S} \qquad (7)$$

CPu can also be estimated using confidence interval estimates. Chou et.al. (1990) [6] developed confidence interval tables which estimate CPu. The confidence interval tables indicate the minimum value of CPu for which the process is capable 95% of the time. They also show the 95% Lower Confidence Limit Cu for given values of **n** and CPu.

In like manner for the case where the process has a target value **T** given, CPu can be computed as [16]

$$CPu_t = \frac{USL - T}{3\sigma} \left[1 - \frac{|T-u|}{USL-T} \right] \qquad [8]$$

To estimate CP_{ut}, we simply replace σ with the process sample standard deviation s and process mean **u** with the process average **x**.

Lower Capability Index, C_{pL}

The lower capability index is defined by:

$$C_{pL} = \frac{\text{Allowable Lower Spread}}{\text{Actual Lower Spread}}$$

$$= \frac{u - LSL}{3\sigma} \qquad [9]$$

where u = process mean
LSL = lower specification limit
σ = process standard deviation

C_{pL} is specifically useful for processes where only a lower specification limit is desired. To estimate C_{pL}, we use the process average x and standard deviation s computed from a sample of size **n** in Equation 9. This estimate is given by:

$$C_{pL} = \frac{\bar{x} - LSL}{3S} \qquad [10]$$

As in the upper capability index CPu, C_{pL} can also be estimated using lower confidence interval estimates. Chou et.al. (1990) [6] developed confidence interval tables which estimate C_{pL}.

In the case where there is a given target value T, C_{pL} can be generalized into

$$CpL = \frac{T - LSL}{3\sigma}\left[1 - \frac{|T-u|}{T-LSL}\right] \qquad [11]$$

where T = target value
LSL = lower specification limit
σ = process standard deviation
u = process mean

To estimate C_{pLt}, as in Equation 8, we simply replace σ with the process standard deviation s and the process mean u with the process average x. That is

$$C_{pLt} = \frac{USL - T}{3S} \left[1 - \frac{|T-u|}{USL-T} \right]$$

Process Performance Index, C_{pk}

The process performance index C_{pk} is defined as

$$C_{pk} = \min \{ Cpu, C_{pL} \} \qquad (12)$$

C_{pk} index relates the scaled distance between the process mean and the closest specification limit (Kane (1980)) [15]. Another way of defining C_{pk} is in relation to C_p,

$$C_{pk} = (1-k)\ C_p \qquad \text{where} \qquad (13)$$

$$k = \frac{2\ |m - u|}{(USL - LSL)}$$

where m = midpoint of $\frac{(USL + LSL)}{2}$

USL = upper specification limit
LSL = lower specification limit
u = process mean

Equations 11 and 12 are algebraically equivalent for $0 < k < 1$. Chan (1989) {3}, Kane (1986) [15]. Note that the value k describes the amount that the process mean is off-center.

C_{pk} can be used for both bilateral and unilateral tolerances. C_{pk} is normally estimated as

$$C_{pk} = \min \{ \hat{Cpu}, \hat{C}_{pL} \} \qquad (14)$$

$$\hat{C}_{pk} = (1-k)\ C_p \quad \text{where} \tag{15}$$

$$k = \frac{2\ |\ m-\bar{x}\ |}{USL - LSL}$$

C_{pk} can also be estimated using confidence interval estimates. Chou et. al. (1990) [6] constructed confidence interval tables which estimate C_{pk}. The confidence interval tables show the 95% lower confidence limit for C_{pk} values given **n** and C_{pk} and the minimum values of C_{pk} for which the process is considered capable 95% of the time.

In like manner for the case where the process has a target value T given. C_{pk} can be computed as

$$C_{pkt} = \min \{ CPu_t, C_{pLt} \} \tag{15}$$

where

$$C_{pLt} = \frac{T - LSL}{3\ \sigma} \left[1 - \frac{|\ T - u\ |}{T - LSL} \right]$$

$$C_{put} = \frac{USL - T}{3\ \sigma} \left[1 - \frac{|\ T - u\ |}{USL - T} \right]$$

To estimate C_{pk}, we simply replace **u** and σ with their estimators **x** and **s** respectively.

C_{pk} values can take on positive or negative values. When the value of $C_{pk} < 0$, the process lies outside the specification limits. Specifically, if $C_{pk} < -1$, the entire process lies outside the specification limits {Montgomery (1991)} [17]. Gunter (1989) [12] [13] defines

$$C_{pk} = \frac{|\ u - \text{nearer specification limit}\ |}{3\sigma} \tag{16}$$

where u = process mean
σ = process standard deviation

He claims that C_{pk} considers how close the process mean is to the target and how spread the distribution is compared to the specification limits but ignores what the shape of the distribution is. Mc Coy (1991) [16] argues that Equation 16, is an inappropriate definition of C_{pk}. He points out that as long as the calculated mean is positioned so that the estimated 3-sigma is within the nearest spec limit, the process is satisfactory. As a measure of process centering, C_{pk} alone is still an inadequate measure. (Montgomery (1991), p. 378 [17]. Montgomery further states that C_{pk} depends inversely on σ and becomes large as σ approaches zero for any fixed value of **u** within the specification limits. A large value of C_{pk} does not indicate anything about the location of the process mean in the (LSL-USL) interval. To characterize process centering satisfactorily, Montgomery (p.378) recommends that C_{pk} must be compared to C_p to provide both a measure of the location and dispersion effects of the process.

Process Capability Index, C_{pm}

A new measure which explicitly accounts for process variation and centering was proposed by Chan et. al. (1988) [3] and Spiring (1991) [20]. The new measure is called C_{pm}. The C_{pm} index is defined as

$$C_{pm} = \frac{USL - LSL}{6\sigma'} \quad [17]$$

where USL = upper specification limit
LSL = lower specification limit

$$\sigma' = \sqrt{\sum_{i=1}^{n} (x_1 - T)^2}$$

Since $\sigma'^2 = E(x-u)^2 + (u-T)^2$ and $\sigma^2 = E(x-u)^2$

Equation 17 can be rewritten as

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (u - T)^2}} \quad [18]$$

To estimate C_{pm}, we use the general estimator

$$C_{pm} = \frac{USL - LSL}{6\sigma'} \quad [19]$$

where

$$\sigma' = \sqrt{\frac{\sum_{i=1}^{n} (x_1 - T)^2}{n - 1}}$$

C_{pm} possesses the necessary properties for assessing process capability. If the process variation increases the C_{pm} decreases. If the process drifts from its target value, C_{pm} declines. If both the process variation and the process mean drifts from the target, C_{pm} reflects these changes as well.

C_{pm} is related to C_p by the equation

$$C_{pm} = \frac{C_p}{\sqrt{1 + \frac{(u - T)^2}{\sigma^2}}} \qquad [20]$$

For fixed values C_p, C_{pm}, and C_{pk} have a one-to-one relationship with each other. (Spring) [20]. Table 2 shows a comparison among C_p, C_{pL}, C_{pu}, C_{pk}, and C_{pm}.

When the target T is not the midpoint of the specification limits ((USL-T) not equal to (T-LSL)), C_{pm} can be generalized into

$$C^*_{pm} = \frac{\min \{ USL-T,\ T-LSL \}}{3\sigma'} \qquad (21)$$

where

$$\sigma' = \sqrt{\sum_{i=1}^{n} (x_1 - T)^2}$$

$$= \sqrt{\sigma^2 + (u - T)^2}$$

An estimate of C_{pm} can readily be obtained from

$$\hat{C}_{pm} = \frac{\min \{ USL-T,\ T-LSL \}}{3s'} \qquad (22)$$

where

$$s' = \sqrt{\frac{\sum_{i=1}^{n} (x_1 - T)^2}{n = 1}}$$

C^*_{pm} can be used for analyzing two-sided and one-sided non-symmetric specification limits. For the case where only one specification limit is of interest, set the other limit to infinity (α) and compute the value of C^*_{pm} using Equation 22.

C_{pm} does not have a unique relationship to the amount of nonconforming products resulting from a process, similar to C_{pk}. However, due to its relationship to $C_{p,}$ unique upper limits on the number of nonconforming units can be set. (Spring (1991)) [20].

When the process mean u equals the target **T**, $C_{pm} = C_p = C_{pk}$. Both C_{pm} and C_{pk} decrease as u moves away from the target **T**. A definite analysis of the C_{pm} including its usefulness in measuring process centering was conducted by Boyles (1989) as reported by Montgomery (1991) [17].

For details on the C_{pm} ratio, its various estimators and their sampling properties, the interested reader is referred to Chan et.al. (1988) [3], Cheng et.al. [4], and Spiring (1991) [20].

Pearson Capability Indices

When the underlying distribution is non-normal, the indices C_p, C_{pk}, C_{pu}, C_{pL}, C_{pm} become misleading and inadequate. Statements about expected process fall out might not only be misleading but erroneous. One popular approach to deal with this situation is to transform the data so that the new transformed data will have a normal distribution appearance. Various graphical and analytical approaches exist for selecting a transformation (Montgomery (1991), p. 378) [17].

Another approach for calculating process capability indices for any shape of distribution was proposed by Clements (1989) [22] using the Pearson family of curves. These indices have the following advantages:

(1) When the distribution is normal, the indices are exactly the same as those given by C_p, C_{pL}, C_{pk}, C_{pu}, and C_{pm};

(2) Do not require mathematical transformation of data;

(3) Easy to visualize graphically;

(4) Relatively easy to calculate manually:

(5) System of Pearson curves on which the indices are based provides estimates of percentage out of specification for a wide variety of distributions and;

(6) They can be applied for other families of probability curves for which the median and tails can be tabulated. Shown in Figure 1 are the Pearson process capability indices

based on 99.73% confidence interval (or + 3-sigma). The median is used to measure location to ensure that C_{pu} and C_{pL} measure the relationships between the upper and lower halves of the data and the upper and lower tolerances. (Clements (1989)) [22]. For normally distributed characteristics, Figure 2 depicts the Pearson process capability indices which are shown to exactly the same as the conventional C_p, C_{pL}, C_{pu}, and C_{pk} indices. Table 3 summarizes the true and estimation equations for the Pearson process capability indices.

To compute the Pearson capability indices, it is necessary to estimate the mean, standard deviation, skewness and kurtosis of the process when it is in a state of statistical control.

A simple worksheet for calculating Pearson capability Indices adapted from Clements (1989)[22] is given in Appendix 1.

A summary of the various process capability indices together with their true and estimation equations, assumptions and usage is shown in Table 4.

DRAWBACKS AND APPLICATIONS OF PROCESS CAPABILITY INDICES

Drawbacks

Kane (1986) [15] listed some of the drawbacks in using process capability indices stemming from inadequate understanding of statistical principles:

1. Stability of a Process

If the process is not in statistical control , the capability index is meaningless. When there are special causes of variation, the meaning of the capability index is not clear. It is therefore necessary to compute indices only when the process is in control.

2 Sampling Plan

The value of a capability index can be changed easily by merely changing the sampling plan. Spreading out samples within a subgroup increases process variation making stability easy to achieve but it decreases the value of the capability index. On the other hand, using consecutive piece sampling decreases variation (increases capability) but it hampers attaining stability. It is thus important to assess process capability and stability jointly.

3. Non-normality

C_p, C_{pL}, C_{pu}, C_{pk}, C_{pm} assume normally distributed processes. When there are significant departures from normality data transformations should be resorted to. Indices which do not assume normality like the Pearson capability indices should be used.

4. Tool Wear

Tool change frequently influences the value of the capability index.

5. Understanding and Computation of Index

Incomplete understanding of the meaning of the index and the lack of familiarity with mathematical formulas sometimes make computation difficult on the manufacturing floor. It is thus important that personnel charged with computations be adequately trained.

Applications and Uses

Some of the applications and uses of process capability indices are [Kane (1986) [15] and Montgomery (1992) [17]

1. Predict how well a process will hold tolerance

Process capability index indicates the percentage of the specifications consumed by the process.

2. Prevent Nonconforming Product

For machine and process qualification, a reasonable benchmark is $C_{pk} = 1.33$ which will make non-conforming units unlikely.

3. Measure Continuous Improvement

To measure improvement, it must be monitored through time. Capability indices can be used to indicate distributional shifts both in terms of process location and variation.

4. Measure Process Location or Variability

For each characteristics, C_p and C_{pk} must be compared. If C_p is close to C_{pk}, process location is not a problem; if C_{pk} is to low, C_p must be examined to determine if process variation is acceptable. [15].

5. Selection Criterion for Competing Supplies

Capability indices can be used to compare and rank competing suppliers. They can also be used to monitor supplier's quality improvement in a straightforward manner.

6. Prioritization of Process Improvement

Capability indices can be used to establish priority for process improvement. Unacceptable indices can aid in paretorizing improvement activities.

7. Specify Performance Requirements

Specify performance requirements for new equipment, materials and process. Capability indices can be used as criterion for acceptance of new equipment, materials or processes.

8. Audits

Comparison of in-process capabilities with capability indices obtained from audits can help establish problem areas.

9. Communication

Use of process capability index provides a common language for communicating potential and actual performance of production processes to all interest groups in the manufacturing floor.

SELECTION OF A PROCESS CAPABILITY INDEX

To select a process capability index, one must be guided by the requirements of the process. The current practice for using process capability indices is to compare the estimated index with the recommended minimum value. Table 5 provides recommended minimum values C_p for certain specific applications (Montgomery (1991) [17]'. If the estimated index is larger than or equal to the minimum value, then the process is considered to be capable. Otherwise, the process is sentenced to be incapable.

A simple guide for choosing a particular process capability index was developed by the author and is shown in Table 6.

CONCLUSIONS AND AREAS FOR FURTHER INVESTIGATION

First and second generation capability indices were presented, their properties and assumptions were examined. Single confidence interval estimates were described. Applications, drawbacks and selection were likewise discussed. It can be concluded that at this stage of the quality revolution, the indices C_p, C_{pu}, C_{pL}, C_{pk}, C_{pm}, and the Pearson process capability indices serve the purpose of providing an effective unitless comparative indicator of process performance. Nonetheless, other areas remain investigated. Some of these are:

1. What should be the standard metric for measuring process capability?

2. How robust are C_p, C_{pk}, C_{pu}, C_{pL}, and C_{pm}, under increasing departures from normality?

3. How effective are the various data transformation approaches in measuring the true capability index of the process?

4. How can we establish unique relationships between C_{pk}, and C_{pm} with the percentage of nonconforming units produced by a process?

5. What are the operational problems and barriers faced by manufacturing people in using these indices in the manufacturing floor?

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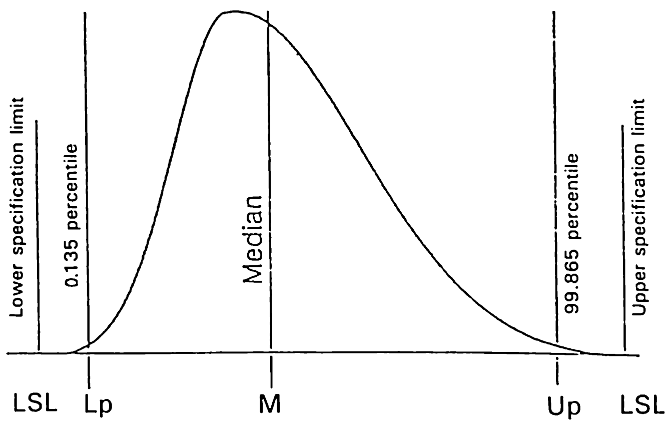


$$Cp = \frac{USL - LSL}{Up - Lp}$$

$$CpL = \frac{M - LSL}{M - Lp}$$

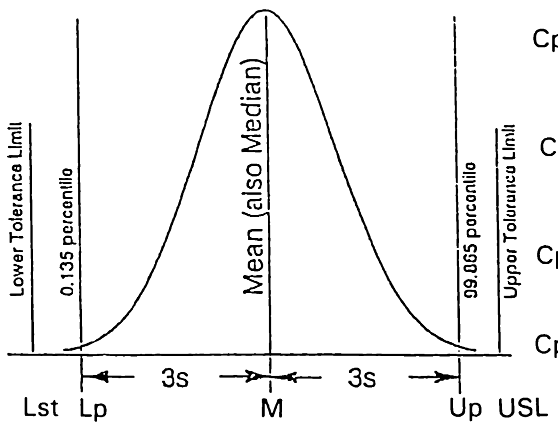
$$Cpu = \frac{USL - M}{Up - M}$$

$$Cpk = Min(Cpu, CpL)$$

M = Median (50 percentile)
Lt = Lower tolerance limit
Lp = 0.135 percentile
Ut = Upper tolerance limit
Up = 99.865 percentile

Figure 1
PROCESS CAPABILITY INDICES PEARSON DISTRIBUTION CURVES

Adapted from Clements, Quality Progress vol. 22, no. 9, pp. 95-97



$$Cp = \frac{USL - LSL}{Up - Lp} = \frac{USL - LSL}{6s}$$

$$CpL = \frac{M - LSL}{M - Lp} = \frac{M - LSL}{3s}$$

$$Cpu = \frac{USL - M}{Up - M} = \frac{USL - M}{3s}$$

$$Cpk = Min(Cpu, CpL)$$

M = Process mean (& median)
LSL = Lower specification limit
Lp = o.135 percentile
s = Process standard deviation
USL = Upper specification limit
Up = 99.865 percentile

Figure 2
PROCESS CAPABILITY INDICES NORMAL DISTRIBUTION
Special Case Pearson Curve

Adapted from Clements, Quality Progress vol. 22, no. 9, pp. 95-97

Table 2
Comparison Among Cp, Cpu, CpL, Cpk and Cpu

Given:

Lower Specification Limit, (LSL) = 10
Upper Specification Limit, (USL) = 20
Target Value, (T) = 15
Process Spread, (σ) = 1

Process Mean μ	**Process Potential Capability Index Cp**	**Upper Capability Index Cpu**	**Lower Capability Index CpL**	**Process Performance Index Cpk**	**Process Capability Index Cpu**
10	1.67	3.33	0.00	0.00	0.33
11	1.67	3.0	0.33	0.33	0.40
12	1.67	2.67	0.67	0.67	0.53
13	1.67	2.33	1.00	1.00	0.75
14	1.67	2.0	1.33	1.33	1.18
15	1.67	1.67	1.67	1.67	1.67
16	1.67	1.33	2.00	1.33	1.18
17	1.67	1.00	2.33	1.00	0.75
18	1.67	0.67	2.67	0.67	0.53
19	1.67	0.33	3.00	0.33	0.40
20	1.67	0.00	3.33	0.00	0.33

Table 3
Process Capability Indices for Non-Normal Distribution

Index	True Equation	Estimation	Assumption
Cp	$Cp = \frac{USL - LSL}{Up - Lp}$	$Cp = \frac{USL - LSL}{Up - Lp}$	• Stable Process • Non-normal distribution
Cpu	$Cpu = \frac{USL - M}{Up - M}$	$Cpu = \frac{USL - M}{Up - M}$	• Stable Process • Non-normal distribution
CpL	$CpL = \frac{M - LSL}{M - Lp}$	$CpL = \frac{M - LSL}{M - Lp}$	• Stable Process • Non-normal distribution
Cpk	Cpk = min {Cpu , CpL}	Cpk = min {Cpu , CpL}	• Stable Process • Non-normal distribution

where USL = Upper specification limit
LSL = Lower specification limit
M = Median (50th percentile)

Up = 99.865 percentile
Lp = 0.135 percentile

Adapted from Clements, Quality Progress vol. 22, no. 9, pp. 95-97

Table 4
SUMMARY OF PROCESS CAPABILITY INDICES

INDEX	TRUE EQUATION WITHOUT TARGET VALUE	TRUE EQUATION WITH TARGET VALUE	ESTIMATION EQUATION WITHOUT TARGET VALUE	ESTIMATION EQUATION WITH TARGET VALUE	ASSUMPTIONS	USAGE
C_p	$C_p = \frac{USL-LSL}{6\sigma}$	$C_{pt} = \min \frac{T-LSL}{3\sigma}, \frac{USL-T}{3\sigma}$	$C_p = \frac{USL-LSL}{6s}$	$C_{pt} = \min\left\{\frac{T-LSL}{3s}, \frac{USL-T}{3s}\right\}$	• Process in statistical control • Distribution is normal	• Process potential for two-sided spec. limits
C_{pu}	$C_{pu} = \frac{USL-u}{3\sigma}$	$C_{pu_t} = \frac{USL-T}{3\sigma}, 1-\frac{\|T-u\|}{USL-T}$	$C_{pu} = \frac{USL-\bar{x}}{3s}$	$C_{pu_t} = \frac{USL-T}{3s}\left\{1-\frac{\|T-u\|}{USL-T}\right\}$	• Process in statistical control • Distribution is normal	• Process performance relative to upper spec. limit
C_{pL}	$C_{pL} = \frac{u-USL}{3\sigma}$	$C_{pu_t} = \frac{T-LSL}{3\sigma}, 1-\frac{\|T-u\|}{T-LSL}$	$C_{pL} = \frac{\bar{x}-LSL}{3s}$	$C_{pu_t} = \frac{T-LSL}{3s}\left\{1-\frac{\|T-u\|}{T-LSL}\right\}$	• Process in statistical control • Distribution is normal	• Process performance relative to lower spec. limit
K	$K = \frac{\|m-u\|}{\frac{USL-LSL}{2}}$	$K = \frac{\|T-u\|}{\min\{T-LSL, USL-T\}}$	$K = \frac{\|m-\bar{x}\|}{\frac{USL-LSL}{2}}$	$K = \frac{\|T-u\|}{\min\{T-LSL, USL-T\}}$	• Process in statistical control • Distribution is normal	• Deviation of process mean from midpoint (m) of spec. limits
C_{pk}	$C_{pk} = \min\{C_{pu}, C_{pL}\}$ $= (c-k)\, C_p$		$C_{pk} = \min\{\hat{C}_{pl}, \hat{C}_{pu}\}$ $= (1-k)\, \hat{C}_p$		• Process in statistical control • Distribution is normal	• Process performance for two-sided spec. limits
C_{pm}	$C_{pm} = \frac{USL-LSL}{6\sqrt{\sigma^2+(u-T)^2}}$ $= \frac{C_p}{\sqrt{1+\frac{(u-T)^2}{\sigma^2}}}$		$C_{pm} = \frac{USL-LSL}{6\sqrt{s^2+(x-T)^2}}$ $= \frac{C_p}{\sqrt{1+\frac{(x-T)^2}{s^2}}}$		• Process in statistical control • Distribution is normal	• Process performance for two-sided and one-sided spec. limits
C_{pm_t}	$C_{pm} = \frac{\min\{USL-T, T-LSL\}}{3\sqrt{\sigma^2+(u-T)^2}}$		$C_{pm_t} = \frac{\min\{USL-T, T-LSL\}}{3\sqrt{s^2+(x-T)^2}}$		• Process in statistical control	• Process performance for two-sided and one-sided speck limits

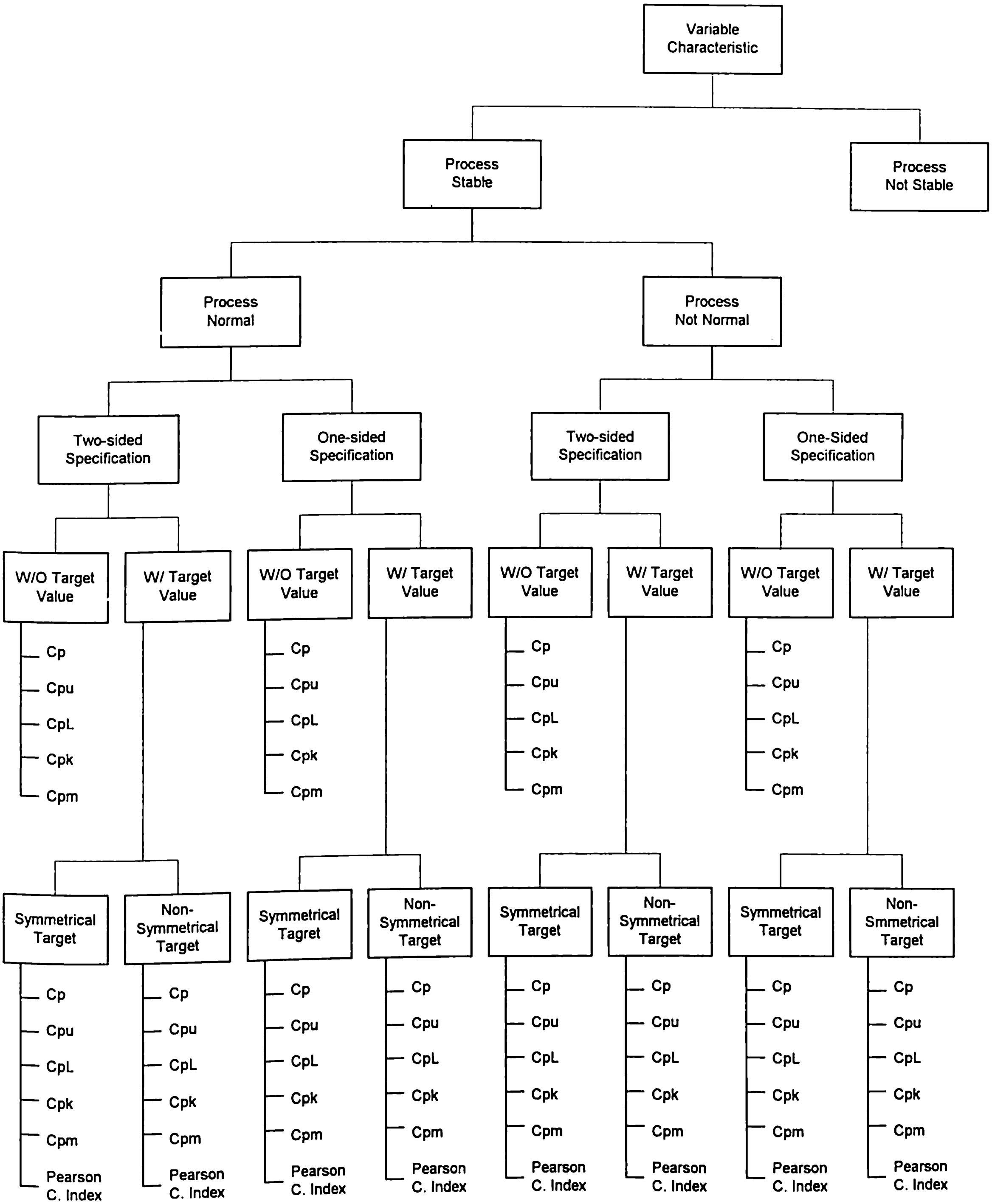
LEGEND:
USL = upper specification limit
LSL = lower specification limit
σ = process standard deviation
u = process mean
T = target value
x = process sample average
s = process standard deviation
m = midpoint of the upper and lower specification limits

Table 5
Recommended Minimum Values of C_p

	Two-Sided Spec.	One-Sided Spec.
Existing Process	1.33	1.25
New Process	1.50	1.45
Safety, Strength or Critical Parameter Existing Process	1.50	1.45
Safety, Strength or Critical Parameter New Process	1.67	1.60

Source: Montgomery (1991) p. 373

Table 6
Some Guidelines in the Selection of a Process Capability Index



Appendix 1
Worksheet for Calculating Pearson Capability Indices

Procedural Steps: **Value**

Step 1 Specify specification Limits
- Upper Specification Limit USL =
- Lower Specification Limit LSL =

Step 2 Compute process statistics
- Process average $\bar{X}$ =
- Sample std. Deviation s =
- Coefficient of skewness Sk =
- Coefficient of Kurtosis Ku =

Step 3 Obtain standardized percentile *
- standardized 0.135 percentile Lp' =
- standardized 99.865 percentile Up' =
- standardized median M' =

Step 4 Calculate estimated 0.135 percentile

$\bar{x}$ - (s x Lp') = Lp =

Step 5 Calculate estimated 99.865 percentile

$\bar{x}$ + (s x Up') = Up =

Step 6 Calculate estimated median

$\bar{x}$ + (s x M') = M =

Step 7 Calculate process capability indices

$$Cp = \frac{USL - LSL}{Up - Lp} =$$

$$CpL = \frac{M - LSL}{M - Lp} =$$

$$Cpu = \frac{USL - M}{Up - M} =$$

$$Cpk = Min\ (Cpu, CpL) =$$

* Refer to standardized values prepared by Clements [22]