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BEAMS

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ABSTRACT

An interactive computer program was developed for the universal solution of the differential equation of the elastic curve which was derived by the author in an earlier paper. This program determines the reactions at the supports and constructs the shear and bending moment diagrams of statically determinate as well as statically indeterminate beams. It computes the maximum bending and transverse shear stresses for eight beam cross-sections. It also makes a sketch of the elastic curve and determines the deflection at selected points as well as the maximum deflection. The beam supports may be at the same or at different levels.

MATHEMATICAL BASIS OF THE PROGRAM

In an earlier paper (Pacheco, E.S.,"The Integrated Equation of the Elastic Curve", U.P. Engineering Research Journal, July, 1972, pp. 24-28), the author derived the solution of the differential equation of the elastic curve for a beam that is acted on by any number of concentrated forces, any number of linearly varying distributed loads, and any number of concentrated couples. The beam can have any number of supports which may be located at the same or at different levels. The free body diagram of such a beam is shown in Fig. 1 below.

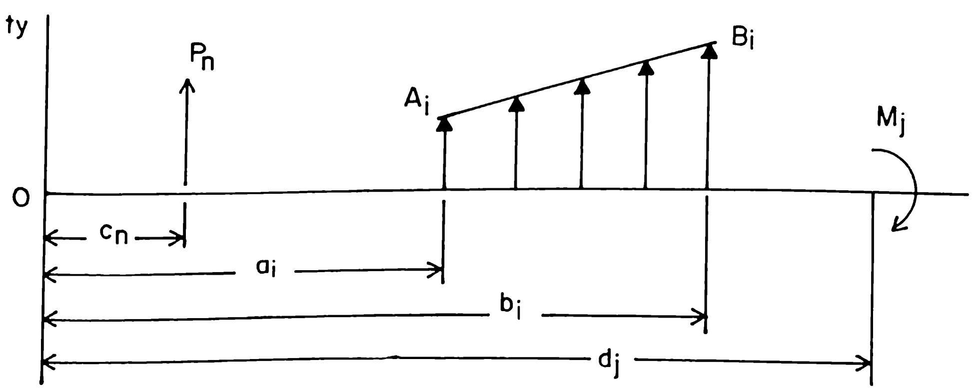

Fig. I

It was shown in the above mentioned paper that the integrated equation of the elastic curve of a homogeneous beam with uniform cross-section and loaded as in Figure 1 is

$$EIy(x) = EIy(0) + EIy'(0)x + \sum_{n} \frac{P_n}{6} (x - c_n)^3 \, u(x - c_n) +$$

$$\sum_{j} \frac{M_j}{2} (x - d_j)^2 \, u(x - d_j) +$$

$$\sum_{i} \left\{ \left[A_i + \frac{B_i - A_i}{b_i - a_i} \frac{(x - a_i)}{5}\right] \frac{(x - a_i)^4}{24} \, u(x - a_i) - \right.$$

$$\left. \left[B_i + \frac{B_i - A_i}{b_i - a_i} \frac{(x - b_i)}{5}\right] \frac{(x - b_i)^4}{24} \, u(x - b_i) \right\}$$

where n, j, and i range from one to the number of concentrated forces, concentrated couples, and distributed forces, respectively, acting on the beam. The unknown reactions are included in the P_n's and the M_j's in the free body diagram. E is the modulus of elasticity of the beam and I is the moment of inertia about the neutral axis of the beam cross-section.

In addition to the equations of equilibrium, as many equations as there are information concerning the deflections and slopes at different points in the beam are available. Thus, if the supports are rigid and all at the same level, equating the deflection y(x) at each support to zero yields as many independent equations as there are supports. A sufficient number of independent equations to determine all unknowns is always available when there is complete information about the position of the supports relative to the line of zero deflection (the x axis).

DESCRIPTION OF THE PROGRAM

Using the above integrated equation of the elastic curve, a computer program was developed which does the following:

1. Determines the reactions at the support for statically determinate as well as statically indeterminate beams.

2. Constructs the shear and bending moment diagrams.

3. Determines the location of the neutral axis, the moment of inertia of the beam cross-section about the neutral axis, and the first moment of parts of the cross-section about the neutral axis for the following beam cross-sections:

(a) Solid rectangle.

(b) Hollow rectangle.

(c) T-section.

(d) Inverted T.

(e) U-section.

(f) Inverted U.

(g) I-section (symmetrical).

(h) I-section (unsymmetrical).

4. Determines the maximum tensile bending stress and the maximum compressive bending stress in a beam whose cross-section is one of those listed in 3. above

5. Determines the maximum transverse shear stress for the beam sections listed above.

6. Determines the maximum deflection.

7. Makes a sketch of the elastic curve and gives the value of the maximum deflection.

The program is menu driven and the user provides information about position and magnitude of loads, location of supports, dimensions of beam cross-section, etc., by responding to questions that appear on the screen. A picture of the beam with the prescribed loads and supports appears on the screen after the user completes inputting the data. Another picture subsequently appears showing the same beam with the supports removed and replaced by the reactions at the supports with their correct direction, sense, and magnitude.

If the user wants to determine the maximum bending and transverse shear stresses, the program constructs the shear diagram and the bending moment diagram to scale with values at significant sections indicated. Where there is a possibility that the maximum transverse shear stress may occur either at the neutral axis or at some other location, the program automatically computes the transverse shear stress at the two locations and picks out the larger stress. If the maximum deflection is desired, the program sketches the elastic curve to scale and the value of the maximum deflection is indicated. In addition, the values of deflection at the ends of the beam and at points with horizontal tangents to the elastic curve are listed.

If the supports are not at the same level, the user must specify the amount by which each mis-aligned support is offset from the x axis. The program assumes that the supports are connected to the beam so that when there is initial misalignment of supports, the elastic curve of the beam before and after application of loads passes through the supports. If the misaligned support is such that it can only exert a force in one direction (e.g., a roller), the user must assume first that the beam touches the support after loading and then find out from this program if the reaction at that support is in the correct direction. If it is not, this means that the beam does not touch that support and this support is therefore unnecessary under the given loads. The program must then be re-run with the particular unnecessary support removed.

USES OF THE PROGRAM

The traditional course in Mechanics of Deformable Bodies (or Strength of Materials) extensively covers the subject of stresses and deformations of beams with symmetrical cross-sections. Roughly half the time allotted for the course is spent in the study of shear and bending moment diagrams, bending stresses, transverse shear stresses, deflections using various methods, and statically indeterminate beams.

As in any course involving the application of principles of physics and mathematics, the student must solve problems to be able to gain some insights on the phenomenon under consideration which do not get revealed by the statement of the principles alone. The conscientious student usually does this and he uses the list of answers to problems (if the book is provided with one) to verify if he did everything correctly.

Unfortunately, many problems about beams involve lengthy computations and even a trivial error made earlier will produce a result that has no resemblance to the correct answer. A search for the source of the error can be difficult and time-consuming. and if the student is unable to find it, he is left wondering if he misunderstood the problem, misapplied the principles, made a trivial computational error, or was misled by a wrong answer given by the author.

In a situation like this, what the student should do is consult his teacher and ask for his assistance in finding out what he did wrong, if he did anything wrong. However, consulting ones teacher is an activity that is shunned by many of our students. The few who do consult are often those who are doing well in the course while those who are struggling seem afflicted with teacher-phobia. And so, when an attempt is made to solve a problem and the correct answer does not come out, it can be speculated that some students just give up in frustration.

It will be nice if there is a textbook that does not just give the final answer to a problem but also gives intermediate results. For example, if the problem involves the determination of the maximum bending stress in a statically determinate beam, it will be most helpful to the student if he is informed what should come out as the correct values of the reactions at the supports, what the shear diagram and bending moment diagram should look like and what are the values of shear and bending moment at critical sections, where the neutral axis is located, what is the moment of inertia about the neutral axis, and finally, what is the value of the maximum bending stress. This way, the student will have less difficulty in finding out where he went wrong since the location of the error is almost pinpointed by the intermediate answers provided. Furthermore, the student is spared the waste of time and effort in continuing a solution that is based on wrong values of intermediate results. However, no such textbook exists.

It can be argued that the student will really be better off if only the final answer is provided since this will give his mental faculties useful exercise in debugging solutions. That argument is fine if the student really avails of that opportunity to search for an error that is located anywhere within one or two or more pages of solution, regardless of how much time it takes to trace the error. But if the effect on the student is discouragement and frustration, then solving problems shall have failed to become a rewarding experience for him.

This computer program is intended for students and teachers who believe that a step-by-step feedback to the solution of problems in mechanics is in fact a useful guide to learning. A prospective author intending to write a book in strength of materials can use the program for creating beam problems and obtaining their solutions. This program can solve most numerical problems on symmetrical bending of beams.

Anyone interested may get a copy of this program from the author.