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A BACKGROUND ON SYSTEM IDENTIFICATION IN STRUCTURAL ENGINEERING USING KALMAN FILTER

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ABSTRACT

System identification is an important step towards the aim of evaluating the existing condition, assessing the degree of damage and deterioration and predicting the response of structures. In this regard, this paper aims to provide the structural engineer a background on the application of system identification in the field of structural engineering using Kalman filter techniques. The basic concepts in system identification and parameter estimation are described and the linear discrete Kalman filter algorithm used to carry out the system identification is summarized. To illustrate the system identification by Kalman filter, a single degree-of-freedom system was analyzed. A survey of researches related to this field is also presented.

Keywords: Structural Engineering, Structural Dynamics, System Identification, Parameter Estimation, Kalman Filter

INTRODUCTION

The general subject of system identification started in the area of electrical engineering and had later extended to the fields of mechanical, control and aeronautical engineering. In the past decade, the problem of system identification had started to be recognized in the field of structural engineering and its importance has steadily increased in recent years in connection with the prediction of the response of structures due to various external loads and also with respect to the estimation of the existing condition for assessment of the degree of damage and deterioration of structures. Various system identification techniques applicable to structural engineering have been developed [1-2]. In recent years, system identification using filtering techniques such as Kalman filter has attracted many researchers.

This paper aims to provide the structural engineer a background on the application of system identification to structural engineering using Kalman filter. The general concepts in system identification and parameter estimation and the relation to filtering are discussed. The linear discrete Kalman filter algorithm used to carry out the system identification and parameter estimation is summarized. A weighted global iteration procedure used to achieve convergence in the estimation of the parameters is also described. An example illustrating the application of system identification using Kalman filter is presented by analyzing a single degree-of-freedom system. Researches related to the topic are also surveyed.

THE GENERAL PROBLEM OF SYSTEM IDENTIFICATION IN STRUCTURAL ENGINEERING

Civil engineering structures are designed and built to perform certain functions. After many years of service, structures will be damaged due to adverse environmental loadings. A building will be affected by strong winds and large earthquakes. Bridges will deteriorate as a result of repeated loading due to traffic loads causing fatigue. The problem of evaluating the existing condition of structures should be a concern of the structural engineer. This is important in connection with the maintenance, repair and rehabilitation of civil structures. In this regard, the field of system identification has a special significance.

The problem of system identification as applied to structural engineering can be described with the use of Figure 1.

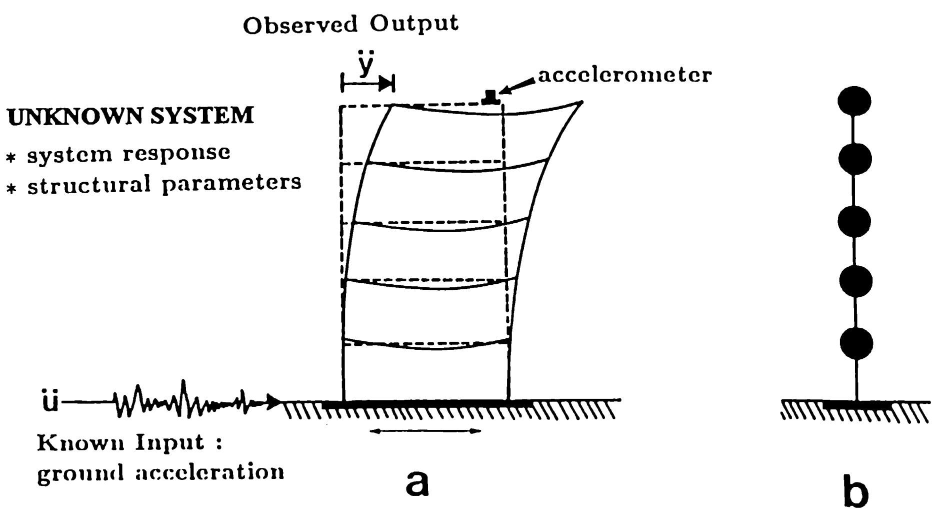

Figure 1. A System Identification Problem in Structural Engineering.
(a) multistorey building (b) lumped mass model

The behavior and characteristics of an unknown (e.g., multistorey building) is represented by the state (e.g., building response and structural parameters) of the system. The state of the system changes with time due to a known input (e.g., ground acceleration). In order to determine the state of the system, the engineer builds a measuring device (e.g., accelerometer) and takes measurements or observations of his system. The measurements are generally corrupted with noise caused by the electronic and mechanical components of the measuring device. To identify the state of the system, the engineer needs to represent his system by a mathematical or analytical model; the parameters of which are unknown. In structural dynamics, structures have been represented discretely by lumped mass or finite element models with stiffness and damping as parameters. The dynamics of the structures relating the input to the output is then formulated using the equation of motion. The general problem of system identification is then defined *as the process of using the observed input to a system and its observed output or response to derive an analytical model of the system which can be used to predict its response to future inputs* [4]. In structural engineering, the mathematical models of structures have been well developed. Hence, the problem of system identification usually reduces to that of identification of unknown structural parameters in the mathematical models. This problem is called parameter estimation.

THE FILTERING PROBLEM

Definition of Filtering

The filtering problem can be described graphically in Figure 2. The quantity or process, X_t, which represents the state of the system is observed through a noisy measurement Y_t. The system noise resulting from the input to the system is w_t and the measurement noise is ν_t. A filter is used to produce an estimate $\hat{X}_t$ of X_t from the observations. This problem of determining the state of a system from noisy measurement is called filtering.

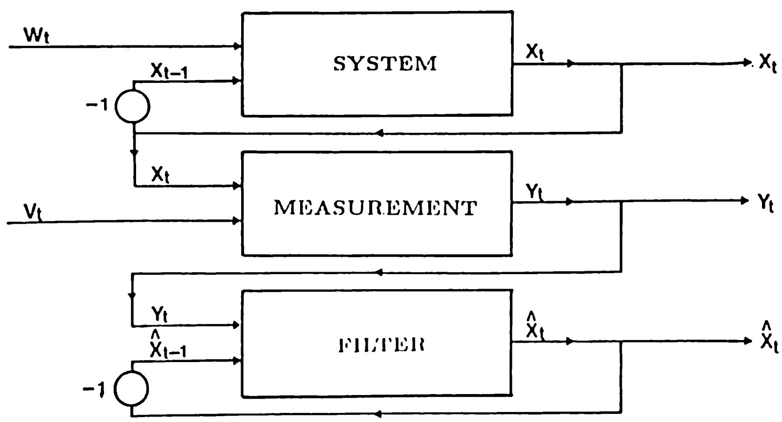

Figure 2. The Filtering Problem [5]

State Space Formulation

In order to know the state of system, one has to know the dynamics of the system, i.e. one has to know how one state is transformed into another as time passes. The dynamics of a system is described in terms of state transition. For continuous systems, the dynamic system is usually expressed as a difference equation. Since our interest on the state is usually centered on a discrete time, the discussion will focus only on discrete systems.

A discrete time process X_k (scalar or vector) called the state of the system, that describes the value at time, $t_k = k\Delta t$, of some property (or properties) of a system, can be generally described by a state equation as

$$X_{k+1} = \Phi(X_k, t_{k+1}, t_k) + \Gamma(X_k, t_k)\, w_{k+1}, \quad k = 0, 1\ldots \tag{3.1}$$

where $\Phi()$ is called the state transition function and w_{k+1} represents the system noise.

The state X_k may not be observed directly. Instead a vector of measurements, y_k may be observed at time t_k which is a function of X_k but is corrupted by a measurement noise v_k. The relationship between the measurements and the state is given by a measurement equation as

$$Y_k = h(X_k, t_k) + v_k, \quad k = 1, 2\ldots \tag{3.2}$$

Given now a sequence of observations $Y_1, Y_2, \ldots, Y_k$. The filtering problem involves computing an estimate of X_k based on these measurements by overcoming the presence of noise.

The common problem in system identification is when the dynamics and observations are linear. This means that the state transition function Φ and the measurement function h are linear in X_k. The dynamic system of a linear discrete system is then written as

$$X_{k+1} = \Phi(t_{k+1}, t_k)\, X_k + \Gamma(X_k, t_k)\, w_{k+1}, \tag{3.3}$$

$$Y_k = M(t_k)\, X_k + v_k. \tag{3.4}$$

Since nonlinear systems are solved by reducing them into equivalent linear systems, only discrete linear systems will be considered in the succeeding discussion.

The Discrete Linear Kalman Filter

The discrete linear Kalman filter will now be described briefly. The equations used in the Kalman filter algorithm will be given without proof. The reader can refer to the references for details [5-6]. In the following discussion on Kalman filter the notations A(k/k) and A(k + 1/k) are used for simplicity. A(k/k) refers to the value of the quantity A at time t_k given the observations up to t_k while A(k + 1/k) refers to the value of the quantity A at time t_{k+1} given the observations up to time t_k only.

Our dynamic model is represented by the state and measurement equations, respectively:

$$X_{k+1} = \Phi(t_{k+1}, t_k)\, X_k + \Gamma(X_k, t_k)\, w_{k+1}, \quad k = 0, 1, 2, ... \tag{3.5}$$

$$Y_k = M(t_k)\, X_k + \upsilon_k, \quad k = 1, 2, ... \tag{3.6}$$

The noise w_k and υ_k are white Gaussian independent random variables with zero means and covariance matrices, Q and R, respectively. The initial estimate of the state is $\hat{X}(0/0)$ and the error, $(X_o - \hat{X}(0/0)$ is also a Gaussian random variable with zero mean and error covariance P(0/0).

The Kalman filter is a recursive procedure. At the beginning ($k = 0$), the initial estimate $\hat{X}(0/0)$ of the state and its corresponding error covariance P(0/0) are assumed. The state at $k = 1$ is then predicted by

$$\hat{X}(1/0) = \Phi(1/0)\, \hat{X}(0/0). \tag{3.7}$$

When time at $k = 1$ arrives, the measurement Y_1 will be available. The estimate at $k = 1$ is then corrected or filtered by a weighted average of $\hat{X}(1/0)$ and Y_1 using the following expression

$$\hat{X}(1/1) = \hat{X}(1/0) + K(1)\, [Y1 - M(1)\, \hat{X}(1/0)] \tag{3.8}$$

where the quantity $\nu_1 = Y_1 - M(1)\, \hat{X}(1/0)$ is a correction term referred to as the filter innovation and K(1) is weight called the Kalman or filter gain. The Kalman gain is chosen such that the variance estimation error is minimum. The calculation of the Kalman gain involves the quantities, P(k/k) and P(k + 1/k) called error covariance matrices and they are defined as

$$P(k/k) = E\{[\{\hat{X}(k/k) - X_k]\ [\hat{X}(k/k) - \hat{X}_k]^T\}, \tag{3.9a}$$

$$P(k + 1/k) = E\{[\hat{X}(k + 1/k) - X_{k+1}]\, [\hat{X}(k + 1/k) - X_{k+1}]^T\}. \tag{3.9b}$$

The filtering is performed recursively to find the estimate $\hat{X}(k/k)$ at time t_k when Y_1, Y_2,...Y_k have been measured.

The equations involved in the Kalman algorithm will now be given without proof. These equations are used recursively starting with $k = 0$ up to $k = N$ as follows:

(1) Store the filter state $\hat{X}$ (k/k) and P(k/k);

(2) Compute the predicted state:

$$\hat{X}(k + 1/k) = \Phi\, (k + 1/k)\hat{X}(k/k); \tag{3.10}$$

(3) Compute the predicted error covariance matrix:

$$P(k + 1/k) = \Phi(k + 1/k)\, P(k + 1/k + 1)\Phi^T (k + 1/k) + \Gamma(k + 1)\, Q(k + 1)\Gamma^T (k + 1) \tag{3.11}$$

(4) Compute the Kalman gain matrix:

$$K(k+1) = P(k+1/k)M^T(k+1)[M(k+1)P(k+1/k)M^T(k+1) + R(k+1)]^{-1} \tag{3.12}$$

(5) Process the observation Y_{k+1}:

$$\hat{X}(k+1/k+1) = \hat{X}(k+1/k) + K(k+1)[Y_{k+1} - M(k+1)\hat{X}(k+1/k)] \tag{3.13}$$

(6) Compute the new (filtered) error covariance matrix:

$$P(k+1/k+1) = [I - K(k+1)M(k+1)]P(k+1/k) + [I - K(k+1)M(k+1)]^T + K(k+1)R(k+1)K^T(k+1). \tag{3.14}$$

(7) set $k = k + 1$ and return to step (1).

The variables involved in the algorithm are described as follows:

$\hat{X}(k/k)$: n x 1 filtered state estimate at t_k given $Y_1, \ldots, Y_k$
$P(k/k)$: n x n error covariance matrix in $\hat{X}(k/k)$
$\Phi(k+1/k)$: n x n state transition matrix
$\Gamma(k)$: n x r system noise coefficient matrix
$Q(k+1)$: r x r system noise covariance matrix
$\hat{X}(k+1/k)$: n x 1 state estimate at t_{k+1}
$P(k+1/k)$: n x n error covariance matrix in $\hat{X}(k+1/k)$
$M(k+1)$: m x n measurement matrix
$R(k+1)$: m x m measurement noise covariance matrix
$K(k+1)$: n x m Kalman gain matrix
Y_k: m x 1 measurement (observation) at t_k

PARAMETER ESTIMATION AND THE WEIGHTED GLOBAL ITERATION

Estimation of the parameters of a system can be easily incorporated in the Kalman filter. If ξ represents the response of the system and θ denotes the system parameters, the state vector will then consist of state variables x and augmented state variable q defined as

$$X = \begin{Bmatrix} \xi \\ \theta \end{Bmatrix} \tag{3.15}$$

The dynamics of the system can be defined as a state equation using this state vector. Assuming that the parameters are constant with time then

$$\theta_{t+1} = \theta_t \tag{3.16}$$

Eq. (3.16) can be incorporated in the general state equation considering the total state vector and the Kalman filter can be carried out to estimate the parameters by first assuming initial values to them. The system model in the filter, may however be different from the actual system, due to approximations in the model, improper initial values for the system parameters, etc., so that the estimates of the parameters may converge to incorrect values or else diverge. Many remedial schemes were developed to achieve convergence. Recently, a weighted global iteration was proposed by Hoshiya and Saito [8] and successfully applied to identification of many structural dynamic systems.

In the weighted global iteration procedure, global iterations of the Kalman filter are carried out by overweighting the error covariance matrix at each global iteration to achieve faster convergence. One global iteration means performing the Kalman filter algorithm recursively using the total set of observed data from $k = 1$ to $k = N$. At first, filtering is performed with initial guesses $\hat{X}_{(1)}(0/0)$ and $P_{(1)}(0/0)$ to obtain $\hat{X}_{(1)}(N/N)$ and $P_{(1)}(N/N)$ where the subscript (1) denotes the first global iteration. Then the second iteration is carried out utilizing the estimates of $\hat{X}_{(1)}(N/N)$ and $P_{(1)}(N/N)$. The estimates of the parameters at $\hat{X}_{(1)}(N/N)$ are used as initial values in $\hat{X}_{(2)}(0/0)$. The diagonal elements of $P_{(1)}(N/N)$ corresponding to the parameters are multiplied by a weighted value and used as initial values in $P_{(2)}(0/0)$. The initial values for the state variables in $\hat{X}_{(2)}(0/0)$ and $P_{(2)}(0/0)$ will be the same as that in the first global iteration. Subsequent iterations are performed until convergence can be achieved in the parameters.

APPLICATION IN STRUCTURAL ENGINEERING

Single Degree of Freedom System

A sample problem will be given to illustrate the application of Kalman filter as a tool for system identification and parameter estimation. The dynamic characteristics of a one storey building and its reponse to external loads such as seismic or wind loads are desired. In order to evaluate the dynamic properties, the roof system is subjected to a harmonic load test and an accelerometer was installed at the roof to measure its horizontal acceleration. Using only the measured horizontal acceleration data, we are required to estimate the dynamic characteristics of the building.

To determine the dynamic characteristics of the building, first we have to represent our system by a mathematical model. Let us assume that the one storey building can be idealized as a single degree-of-freedom system as shown in Figure 3.

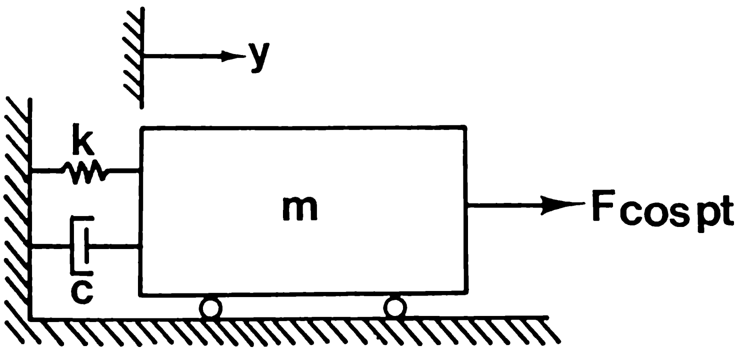

Figure 3. A Single Degree-of-Freedom System

The dynamics of our model can now be described mathematically. The differential equation of motion for a linear single degree-of-freedom system is given by the second order differential equation [7],

$$m\ddot{y} + c\dot{y} + ky = F\cos pt \qquad (4.1)$$

or

$$\ddot{y} + 2hwy + w^2y = f\cos pt \qquad (4.2)$$

where m, c and k are mass, damping coefficient and spring constant, respectively; h and w are respectively the damping ratio and natural frequency; p is the frequency of the force with peak amplitude F or $f = F/m$.

Our problem is now reduced to the identification of the parameters of the mathematical model using the measured acceleration data of the real structure. If we will use Eq. (4.2), we have to identify the following dynamic parameters: h, w, f and p of the single degree-of-freedom system.

To identify the system parameters, we have to formulate the state and measurement equations. First, we have to define our state vector. Let the responses, y, $\dot{y}$ and $\ddot{y}$ be the state variables and the parameters, h, w, f and p be augmented state variables which are assumed constant with time. The state vector is then defined as

$$X = \{x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\}^T$$

$$= \{y\ \dot{y}\ \ddot{y}\ h\ w\ f\ p\}^T \qquad (4.3)$$

The dynamic system can be expressed in discrete form by using the linear acceleration method [7]. In the linear acceleration method, the following equations are used.

$$\dot{y}_{k+1} = \dot{y}_k + \ddot{y}_k \frac{\Delta t}{2} + \ddot{y}_{k+1} \frac{\Delta t}{2}, \qquad (4.4)$$

$$y_{k+1} = y_k + \dot{y}_k \Delta t + \ddot{y}_k \frac{\Delta t^2}{3} + \ddot{y}_{k+1} \frac{\Delta t^2}{6}. \tag{4.5}$$

Using the two linear acceleration equations and the equation of motion given by

$$\ddot{y}_{k+1} + 2hw\dot{y}_{k+1} + w^2 y_{k+1} = f\cos p t_{k+1}, \tag{4.6}$$

a state equation in difference form relating the state at t_{k+1} to the state at t_k can be derived as

$$X(k+1) = \begin{Bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ x_5(k+1) \\ x_6(k+1) \\ x_7(k+1) \end{Bmatrix} = \begin{Bmatrix} D_{11}x_1(k) + D_{12}x_2(k) + D_{13}x_3(k) + D_{14}x_6(k)\cos[x_7(k)t_{k+1}] \\ D_{21}x_1(k) + D_{22}x_2(k) + D_{23}x_3(k) + D_{24}x_6(k)\cos[x_7(k)t_{k+1}] \\ D_{31}x_1(k) + D_{32}x_2(k) + D_{33}x_3(k) + D_{34}x_6(k)\cos[x_7(k)t_{k+1}] \\ x_4(k) \\ x_5(k) \\ x_6(k) \\ x_7(k) \end{Bmatrix}$$

$$= g(X(k), t_k, t_{k+1}) + \Gamma(X(k), t_k)w_{k+1} \tag{4.7}$$

where,

$D_{11} = 1 + (\Delta t)^2 D_2/6,$ $D_{12} = (\Delta t)(1 + (\Delta t)D_3/6),$
$D_{13} = (\Delta t)^2(1 + D_4/2)/3,$ $D_{14} = -(\Delta t)^2 D_1/6,$
$D_{21} = (\Delta t)D_2/2,$ $D_{22} = 1 + (\Delta t)D_3/2,$
$D_{23} = (\Delta t)(1 + D_4)/2,$ $D_{24} = -(\Delta t)D_1/2,$
$D_{31} = D_2, D_{32} = D_3, D_{33} = D_4, D_{34} = -D_1,$

with,

$D_1 = -(1 + (\Delta t)x_4(k)x_5(k) + (\Delta t)^2 x_5^2(k)/6)^{-1}$
$D_2 = D_1 x_5^2(k)$
$D_3 = D_1(2x_4(k)x_5(k) + (\Delta t)x_5^2(k))$

and,

$D_4 = D_1(\Delta t)x_4(k)x_5(k) + (\Delta t)^2 x_5^2(k)/3)$

where w_k is the system noise vector with covariance matrix $Q(k)$ and Γ is the coefficient matrix of the system noise. It must be noted that the parameters x_4 - x_7 are constant parameters to be estimated.

The measurement equation relating the measurements to the state vector can also be derived. If the acceleration of the mass will be measured and used as observation, the measurement equation will be

$$Y_4 = [0, 0, 1, 0, 0, 0, 0]X(k) + v(k), \tag{4.8}$$

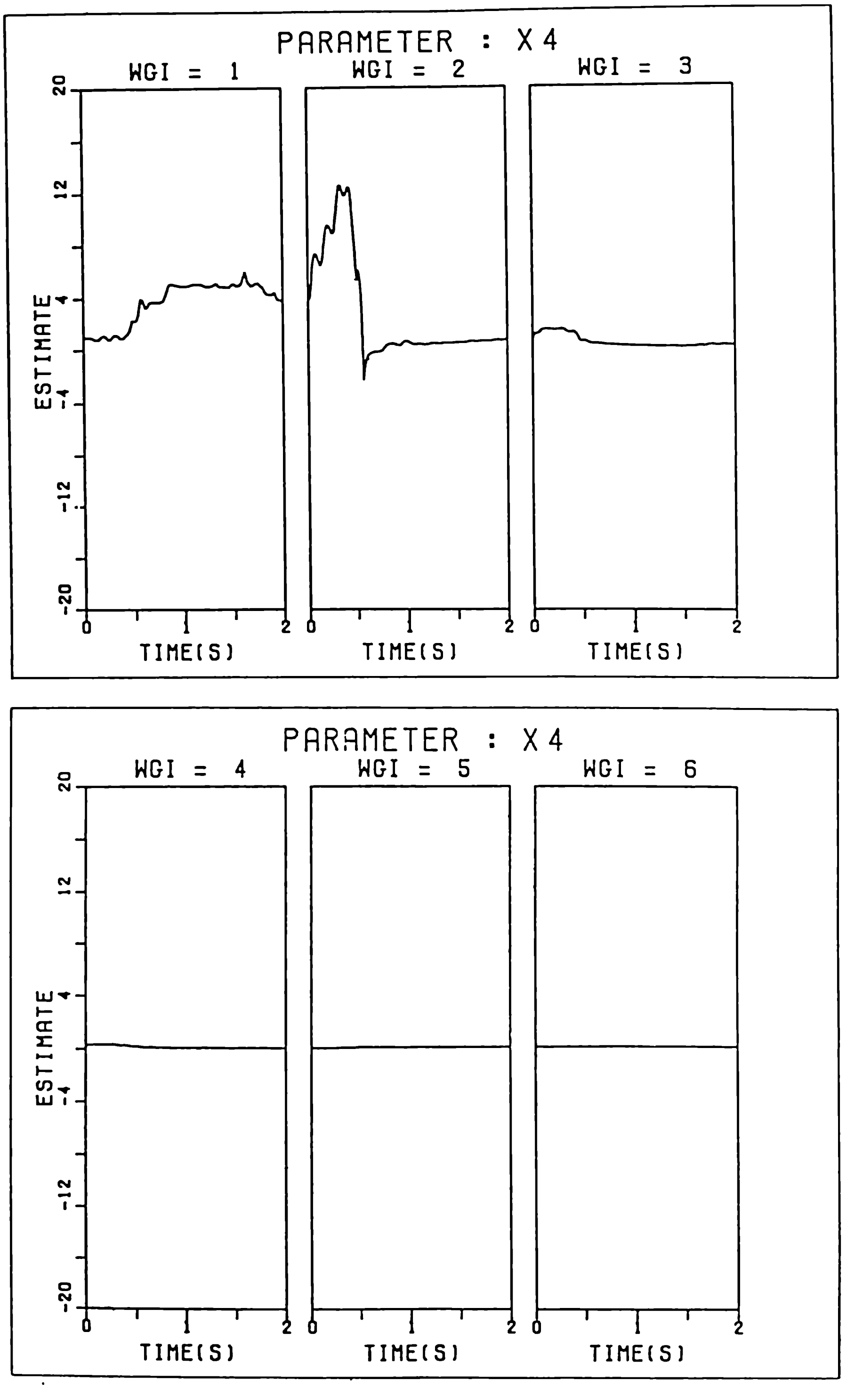

Figure 4. Numerical Convergence of Parameter $x_4 = h$
Using Weighted Global Iteration Procedure

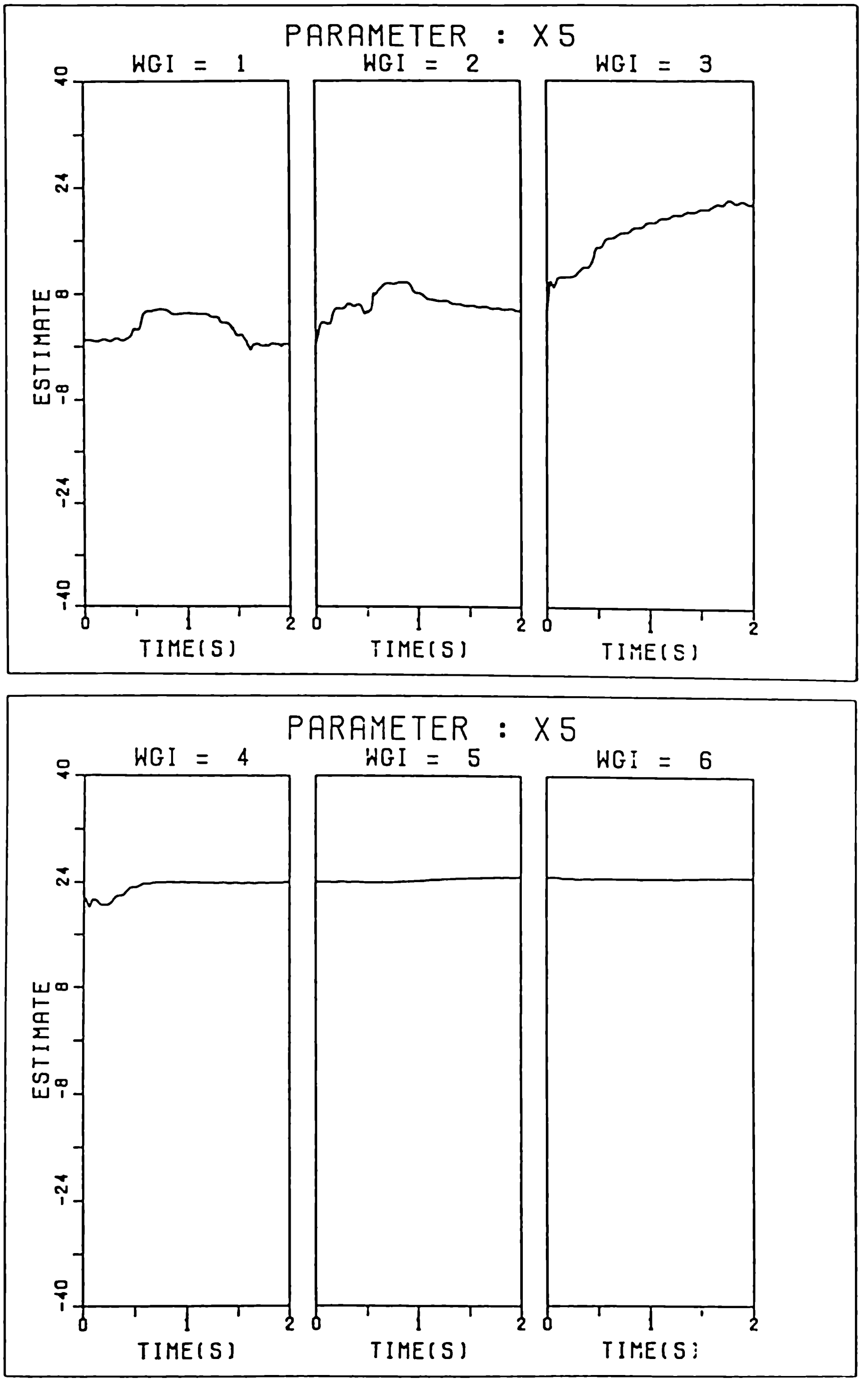

Figure 5. Numerical Convergence of Parameter $x_5 = w$
Using Weighted Global Iteration Procedure

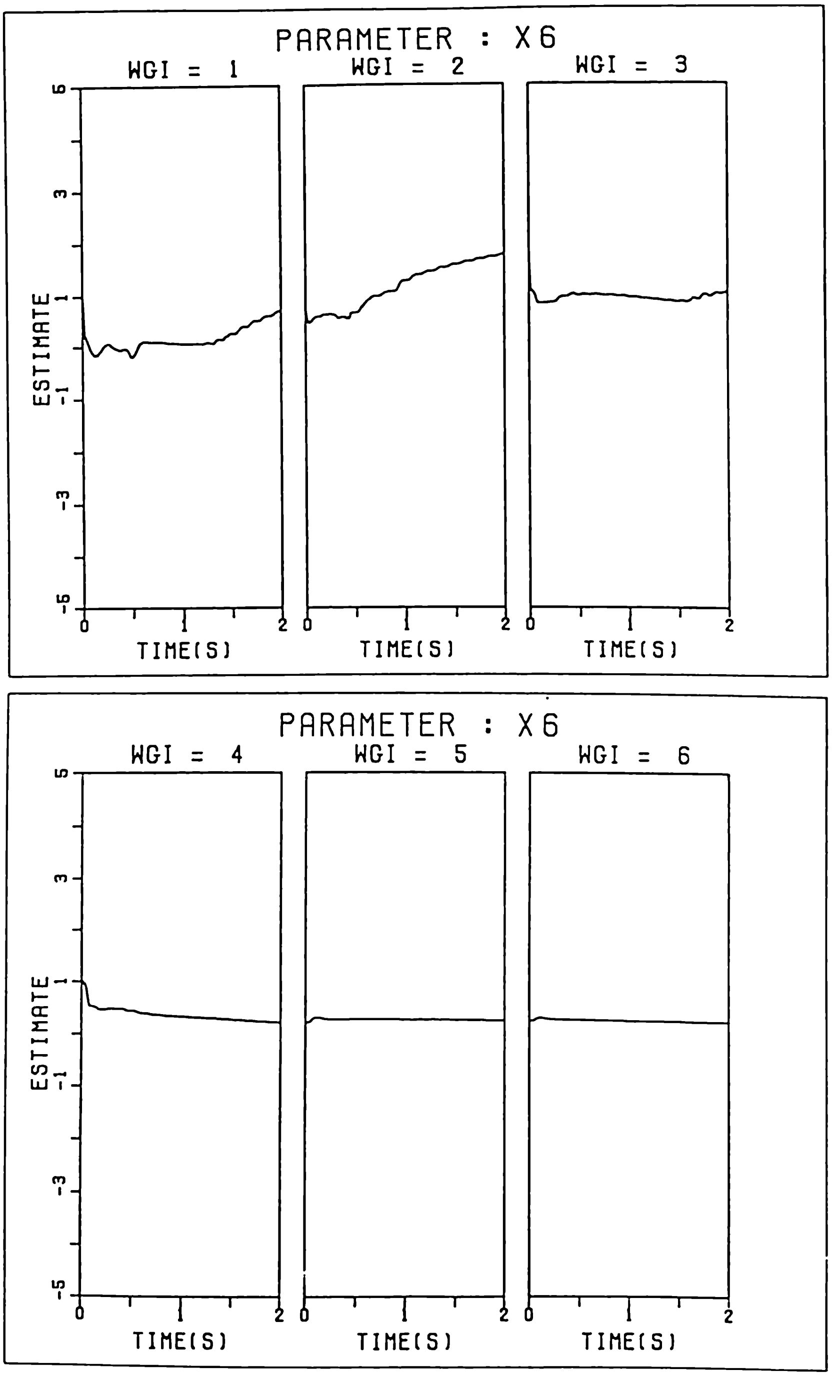

Figure 6. Numerical Convergence of Parameter $x_6 = f$
Using Weighted Global Iteration Procedure

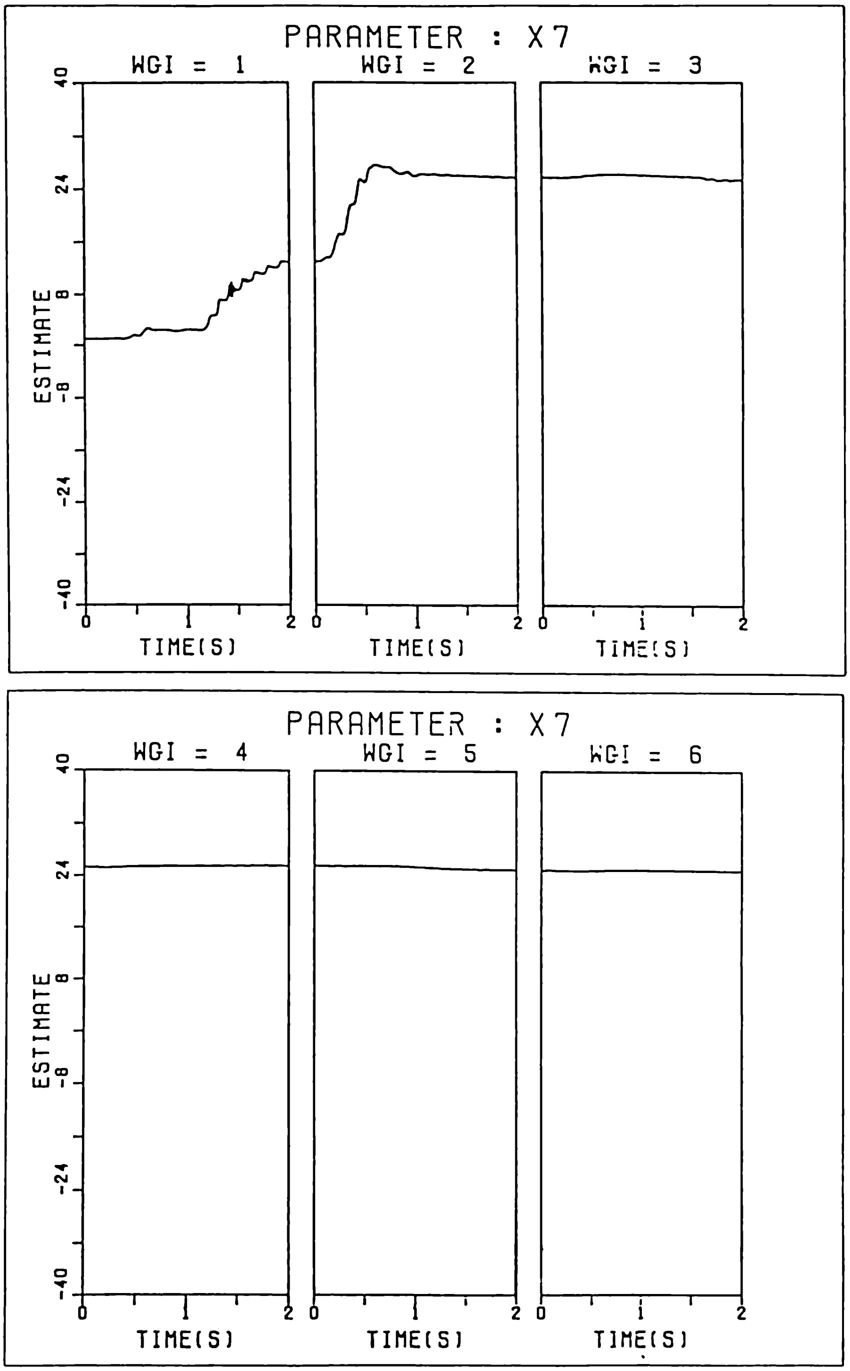

Figure 7. Numerical Convergence of Parameter $x_7 = p$
Using Weighted Global Iteration Procedure

where $v(k)$ is the observational noise vector with covariance matrix R(k).

It can be seen that the derived state equation is nonlinear and corresponds to Eq.(3.1). Hence, to apply the discrete linear Kalman filter algorithm, the state transition matrix in the algorithm must be obtained. The state transition matrix (see appendix) can be linearized with its elements given by

$$\phi_{ij}(k + 1/k) = \frac{\partial g_i(\hat{X}(k/k), t_k, t_{k+1})}{\partial x_j} \qquad (4.9)$$

Incorporating the state and measurement equations in the Kalman filter algorithm and using the acceleration data as observations, the parameters x_4 - x_7 can be estimated.

Numerical Example

To test the identification of the single degree-of-freedom system, numerical calculations were carried out. Let us assume that the SDOF system has the following properties: m = 20, c = 50, k = 12500, F = 5, h = 0.05, w = 25, p = 25 and f = 0.25 with initial values, y(0) = y(0) = 0.

Using standard techniques in solving linear differential equations, an analytical solution of the equation can be obtained, that is y(t), $\dot{y}(t)$ and $\ddot{y}(t)$ can be derived. The derived solution of $\ddot{y}(t)$ was used to generate the measurement data using a time interval of Δt = 0.01 sec. Zero initial values were used for the state variables x_1 - x_3. For the initial error covariance matrix, 0.001 was assumed for the diagonal elements corresponding to the state variables and 100 for the augmented variables (parameters). A value of R = 10.0 was assumed for the measurement noise. A sampling time of 2.0 sec. which corresponds to a data set from k = 1 to k = 200 was used. In the weighted global iteration, a weight of 100.0 was applied. The Kalman filter algorithm was implemented using 6 global iterations; the results of which are shown in Table 1. It can be seen that the estimates at the sixth global iteration are relatively in good agreement with the true values.

Table 1. True and Estimated Values of Parameters

Parameter	h	w	f	p
True Values	0.05	25.0	0.25	25.0
Initial Values	1.0	1.0	1.0	1.0
Estimated Values	0.05454	24.95	0.2646	25.05

Figures 4-7 show the convergence behavior of the parameters during the identification procedure using the weighted global iteration. It is shown in the figures that the first global iteration does not produce the optimal estimates of the parameters. Only after a number of global iterations that the parameters converged to the optimal values. At the sixth global iteration, convergence of all the parameters was achieved. The numerical results show that the weighted global iteration is necessary to achieve convergence of the parameters and it works well even with poor initial guesses of the parameters. It must be noted that convergence of the parameters is affected by many factors such as the time interval Δt, total sampling time, assumed initial values of state variables and parameters and initial error covariance matrix. Hence, to verify the correctness of the identification, these factors must be given consideration.

Survey of Related Researches

Researches on the application of system identification in structural engineering using Kalman filter are numerous. Identification of damping and stiffness parameters of multidegree-of-freedom (MDOF) systems of the shear type (Figure 8) was investigated by many researchers [8-9]. Identification of modal parameters of MDOF systems was also studied [10].

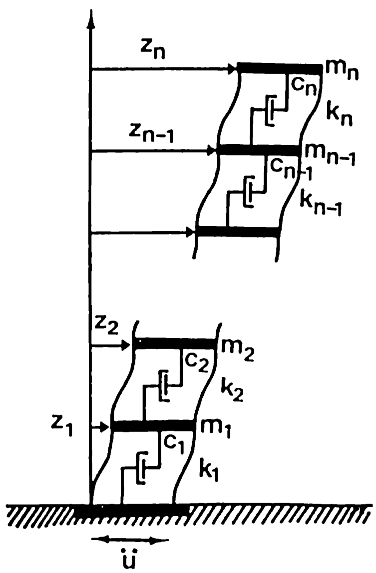

Figure 8. MDOF system of shear type

Applications to bridge structures have also been considered. Hoshiya and Maruyama [11] identified the dynamic parameters such as velocity, weight, natural frequency and damping coefficient of a running load on a beam (Figure 9) including the natural and damping coefficient of the beam. Yun and Shinozuka [2] identified the structural parameters of a damaged bridge structure. Modal parameter identification of an in-situ bridge deck using field data was also investigated [13-14].

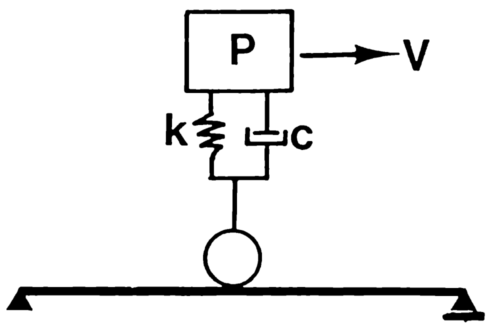

Figure 9. Running Load on Beam System

Offshore structures which are idealized as lumped mass systems subjected to wave forces have been investigated to identify the structural parameters and inertia and drag coefficients associated to wave forces [15-16]. Studies on nonlinear systems of hysteretic, non-degrading or degrading type have also been conducted [17-19]. More recently, identification of structures using a substructured approach was utilized with the objective of identifying efficiently the parameters of structures with many DOF [20] or to identify the parameters of only a small section of a structure [21].

The survey of researches on the field of system identification as applied to structural engineering shows that much work has to be done. Most of these researches used only numerically generated measurement data for the identification of parameters. Very few researches used field data. This means that research is still on its numerical testing stage. Presently, experimental studies are being conducted using laboratory models inorder to verify the validity of the identification techniques [10]. Upon completion of such verification, field experiments are expected to follow. Here, the system response and parameters will be identified using the measured field data of in-situ structures. Only at this stage that the usefulness and importance of system identification in structural engineering can be fully recognized.

CONCLUSION

A general picture of the problem of system identification using Kalman filter and its relation to the field of structural engineering was presented. The general concepts in system identification and parameter estimation and the relation to filtering were described. The linear discrete Kalman filter using a weighted global iteration was summarized. An illustrative example applying system identification and parameter estimation to a single degree-of-freedom system was also presented. To give the reader an idea of the extent of work being done on this field, researches related to the topic were presented. From the survey of the researches, it can be concluded that much work has still to be done particularly on the application of system identification to in-situ structures. It is hoped that through this paper the interested reader will gain some understanding of the basic problem and will refer to the references for details and ultimately will contribute to the development and advancement of this challenging and important field of system identification in structural engineering.

REFERENCES

1. F. Kozin and H.G. Natke: System Identification Techniques, *Structural Safety, 3(1986), pp.269-316.*

2. H.G. Natke and J.T.P. Yao: *Structural Safety Evaluation Based on System Identification Approaches*, Vieweg, Germany, 1988.

3. S.C. Liu and J.T.P. Yao: Structural Identification Concept, *Journal of Structural Division, Proceedings of the ASCE*, Vol. 104, No. ST12, Dec. 1978, pp. 1845-1858.

4. M.A.M. Toramani and H.E. Lee: Model Identification of an Arch Bridge for Dynamic Analysis, *Proc.* ICOSSAR 1989, pp. 1459-1465.

5. P.E. O'Connel (Ed.): Real Time Hydrological Forecasting and Control, *Proc. of First International Workshop*, July 1977

6. A.H. Jazwinski: *Stochastic Processes and Filtering Theory*, Academic Press, NY, 1970

7. M. Paz: *Structural Dynamics*, Van Nostrand, NY, 1980.

8. M. Hoshiya and E. Saito: Structural Identification by Extended Kalman Filter, *J. Engrg. Mech., ASCE,* Vol. 110, No. 12, Dec. 1984, pp. 1757-1770.

9. K. Toki, T. Sato and J. Kiyuno: Identification of Structural Parameters and Input Ground Motion from Response Time Histories, *Struct. Engrg./Earthquake Engrg., Proc. of the Japan Society of Civil Engineers*, Vol. 6, No. 2, Oct. 1989, pp. 413-421

10. O. Maruyama, C.B. Yun, M. Hosheya and M. Shinozuka: Program EXKAL2 for Identification of Structural Dynamic System, *NCEER Technical Report, No. NCEER-89-0014*, May 1989.

11. M. Hosheya and O. Maruyama: Identification of Running Load and Beam System, *J. Engrg. Mech., ASCE*, Vol. 113, No. 6, June 1987, pp. 813-824.

12. C.B. Yun, W.J. Kim and A.H.S. Ang: Damage Assessment of Bridge Structures by System Identification, Proc. ICOSSAR 1989, pp. 2179-2186.

13. A.W.C. Oreta and T. Tanabe: Application of Modal Parameter Identification to Reinforced Concrete Bridge, *Proc. Int. Symposium in Concrete Engineering*, Nanjing, China, 18-20 Sept. 1991, Vol. 3, pp. 1356-1361.

14. A. W. C. Oreta and T. Tanabe: Identification of Modal Parameters of a Reinforced Concrete Bridge by Kalman Filter Theory, *Proc. of the Japan Concrete Institute*, Vol. 13, No. 2, pp. 1051-1056.

15. A.M. Paliou and M. Shinozuka: Identification of Equivalent Linear Systems, *Structural Safety Evaluation Based on System Identification Approaches*, H.G. Natke and J.T.P. Yao (Eds.), Vieweg, 1988, pp. 161-181.

16. C.B. Yun and M. Shinozuka: Identification of Nonlinear Structural Dynamic Systems, *J. Struct. Mech.*, Vol. 8(2), 1980, pp. 187-203.

17. C.H. Loh and Y.H. Tsaur: Time Domain Estimation of Structural Parameters, *Eng. Struct.*, Vol. 10, April 1988, pp. 95-105.

18. M. Hoshiya, O. Maruyama and M. Shinozuka: Identification of Bilinear Severely Hysteretic Systems, *Proc. ICOSSAR 1989.*

19. M. Hoshiya, and O. Maruyama: Identification of Nonlinear Structural Systems, *Proc. 5th Int. Conf. on Applications of Statistics and Probability in Soil and Structural Engrg.*, Vancouver, Canada.

20. C.G. Koh, L.M. See and T. Balendra: Estimation of Structural Parameters in Time Domain: A Substructure Approach, *Earthquake Engineering and Structural Dynamics*, Vol. 20, Aug. 1991 pp. 787-801.

21. A.W.C. Oreta and T. Tanabe: Localized Identification of Structures by Kalman Filter, Submitted for possible publication to the Proc. of the Japan Society of Civil Engineers (Struct. Engrg./Earthquake Engrg), 1991.