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Technical Note

A Straightforward Method for Determining the Pulse Widths of Sinusoidal PWM Waveforms

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ABSTRACT

A simple, speedy and straightforward method is given for calculating the widths of a sinusoidally modulated pulse train. The method employs the fact that a pulse has its average value proportional to its duty ratio. By setting this duty ratio proportional to the height of a sine wave, the width is thereby easily calculated. The method is shown for the generation of a single phase sinusoidal PWM.

INTRODUCTION

In inverters, sinusoidal output is usually attained by a modulation technique known as sinusoidal pulse width modulation. In this case, a high frequency triangular wave is compared with a sine wave as shown in Figure 1. The pulse is at level + V when the sine wave amplitude exceeds the triangular amplitude, otherwise it is at level -V, thus a pulse train of variable width results.

From the figure, the widths may be calculated by computing the intersections of the triangular wave with the sine wave. This will require satisfying two simultaneous equations, one linear, the other trigonometric and furthermore, the interval for positive and negative slopes has to be determined during the iteration procedure.

The method discussed here eliminates simultaneous equations hence no numerical technique is required. Also, adding an offset eliminates the need for the determination of positive and negative slopes within the interval. Straigthforward calculation is thus obtained based on the fact that a pulse train has its average value proportional to its duty ratio, τ /T, where τ is the proportion of time when it is at the positive level and T is the period of the triangular wave:

THE METHOD

Refer to Figure 2.

FIRST, let $y(\phi) = A \sin(\phi)$

Divide one cycle into P intervals

let $\theta = 2 * p / P$

then $dA = A \sin(\phi)\, d\phi$

Area of the nth interval = $\int_{\alpha}^{\beta} A \sin(\phi)\, d\phi$

where $\alpha = (n-1)\theta$
$\beta = n\theta$
A = peak value of sine wave
ϕ = angle in radians

The nth interval is represented by height hn whose area ϕ *hn equals the area of the nth interval. See Figure 3.

Thus,

$$An = \text{area of the nth interval} = \int_{(n-1)\theta}^{n\theta} A \sin(\phi)\, d\phi = hn*\theta f$$

Integrating:

$$-A \cos(\phi) \Big]_{(n-1)\theta}^{n\theta} = hn * \theta$$

Solving for hn :

$$hn = \frac{-A \cos(n*\theta) + A \cos([n-1]*\theta)}{\theta}$$

Thus hn represents the average value of the nth interval.

NEXT, we add an offset, B + A to hn

where B is an offset value, $B \geq 0$.

We then compute the duty ratio for the nth interval as

$$Dn = \frac{hn + A + B}{2(A + B)}$$

Refer to Figure 3.

A + B set the depth of modulation or width to height sensitivity. Thus, the duty ratio, D, is bounded between an upper bound (Du) and a lower bound (D1) given by:

$$\text{upper bound} = \frac{2A + B}{2(A + B)}$$

and

$$\text{lower bound} = \frac{B}{2(A + B)}$$

Finally, we compute the proportion of time at which the pulse train is at the positive level, tn, for the nth interval as

tn = Dn * Tc

where Tc = carrier period
= 1/Fc
= 1/ (P*Fo), where Fo = sine wave frequency

EXAMPLE :

Fo = 60 Hz, sine wave frequency
P = 16 pulses per cycle
Fc = 960 Hz, carrier frequency
B = 10
A = 100

then

$\theta = \pi / 16 = 0.1963495$ radians

INTERVAL	DUTY RATIO	TON(+ LEVEL)
1	0.5960	0.00062079
2	0.7733	0.00080549
3	0.9090	0.00094685
4	0.9824	0.00102336
5	0.9824	0.00102336
6	0.9090	0.00094685
7	0.7733	0.00080549
8	0.5960	0.00062079
9	0.4040	0.00042088
10	0.2267	0.00023618

11	0.0910	0.00009481
12	0.0176	0.00001831
13	0.0176	0.00001831
14	0.0910	0.00009841
15	0.2267	0.00023618
16	0.4040	0.00042088

Figure 4 shows the time domain and the power spectrum of the example.

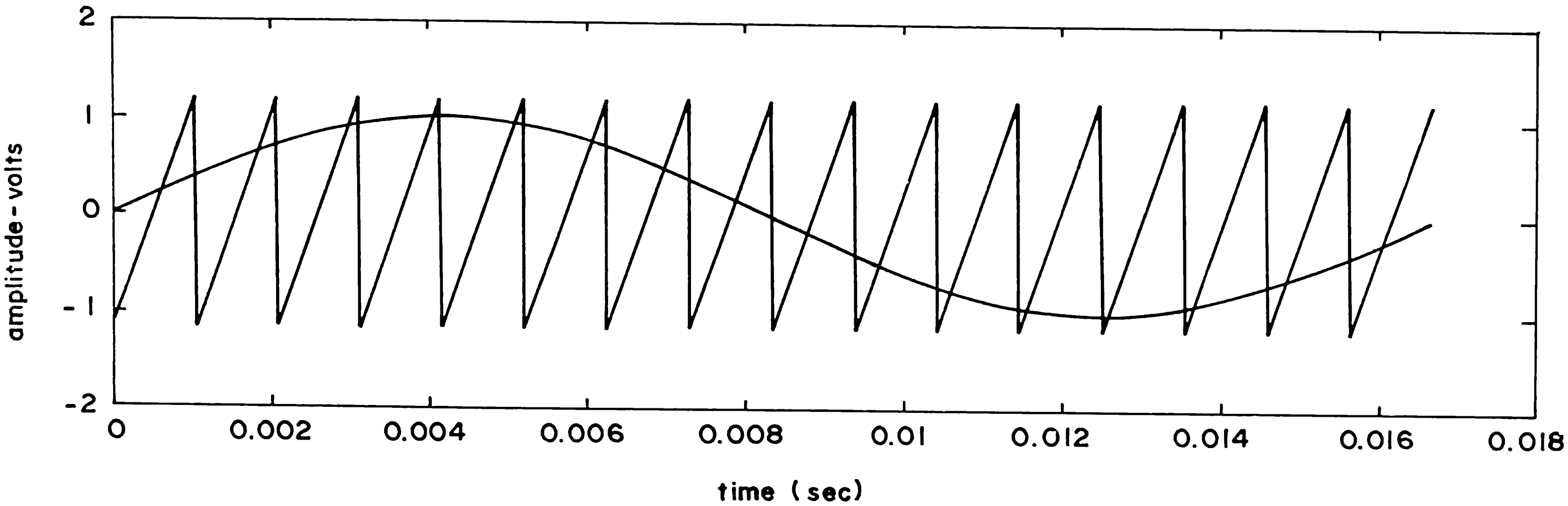


Fig. 1 Sine Waveform and Sawtooth Carrier

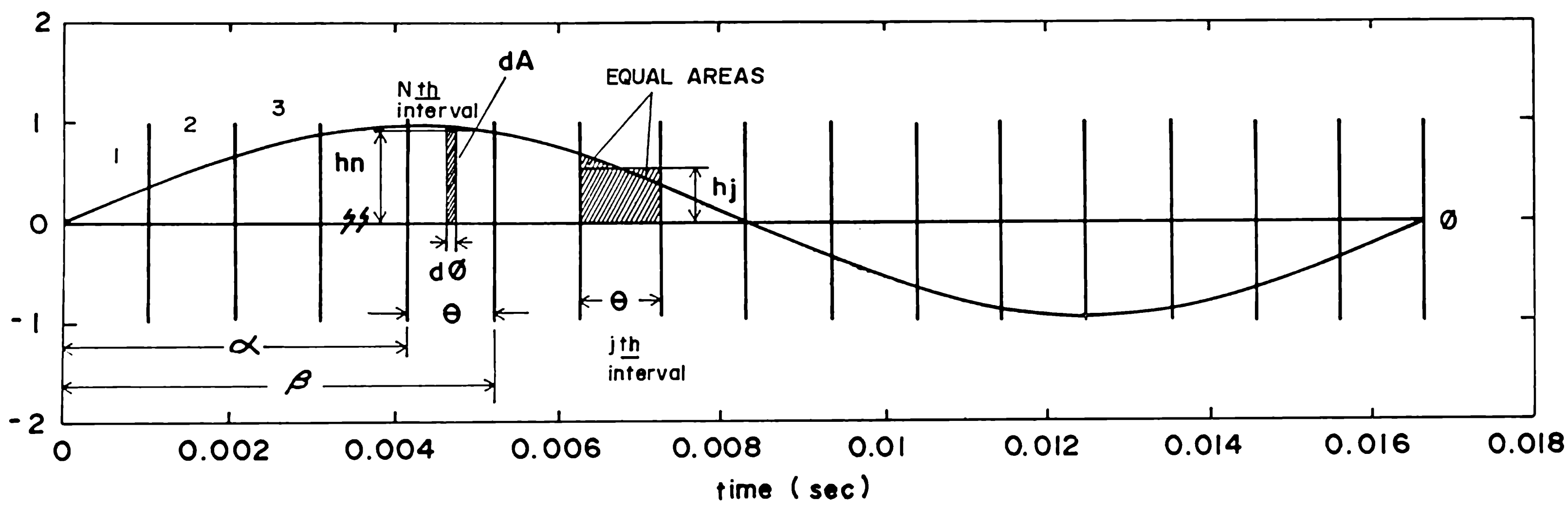
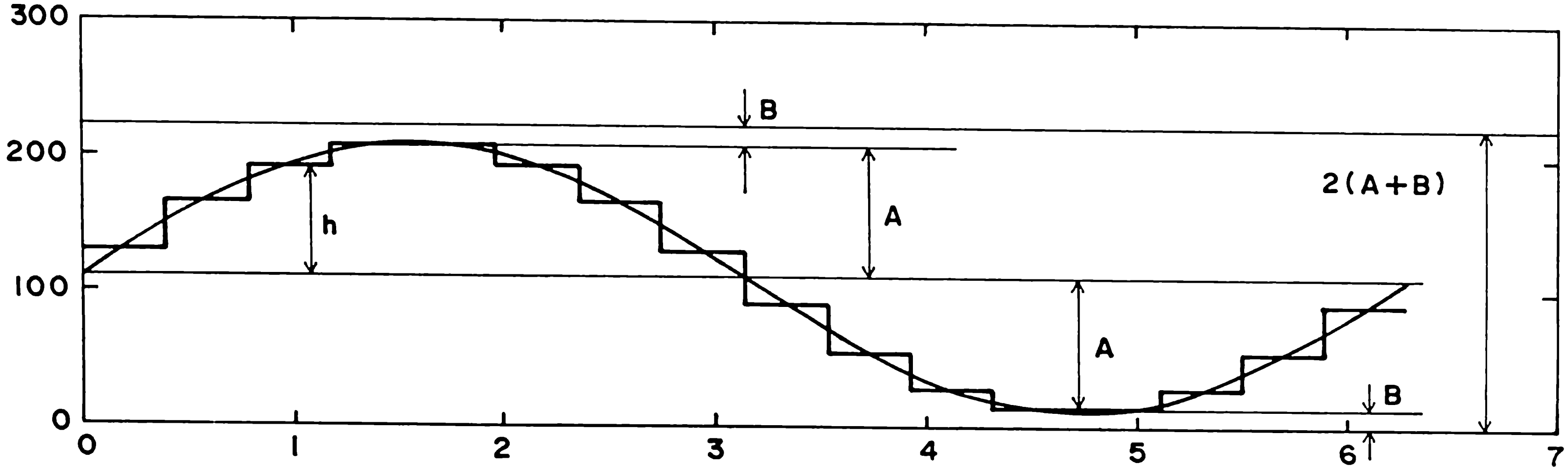


Fig. 2 Sine Waveform and Area Calculation



Case for A=100, B=10, 16 pulses per cycle, X-axis in radians

Fig. 3 Sine Waveform and Offest

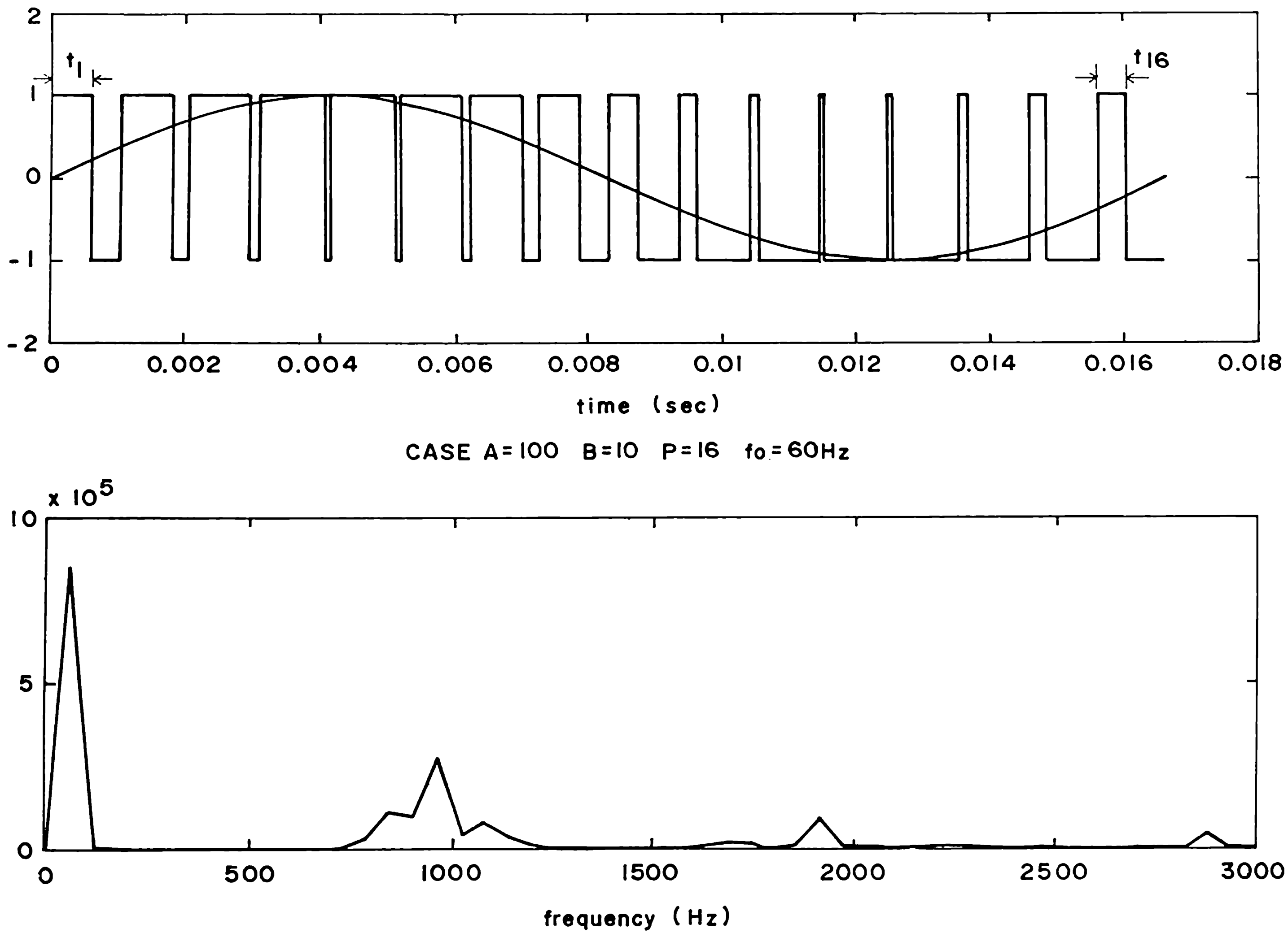


Fig. 4 PMW Waveform and Power Spectrum