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DEVELOPMENT OF EQUIVALENT CIRCUIT MODELS FOR NON-LINEAR RESISTORS

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ABSTRACT

The modeling of non-linear devices confront both users and developers of computer-aided design programs. This paper shows how large-signal models for voltage and current controlled resistors can be developed based on the piece-wise-linear characteristics of the device. For both voltage controlled and current controlled non-linear resistors, two models are developed. One model is Foster-like in the sense that the sub-circuits are connected either in series or parallel to form the model. The other model is Cauer-like in the sense that the topology is of ladder type. Methods for developing the models are presented.

INTRODUCTION

Non-linear circuit analysis problems often confront the designers and users of Computer-Aided Analysis and Design packages. Electronic devices are basically non-linear in nature and thus require special analysis techniques. Some devices already have representative models in CAAD libraries. Models for certain devices have to be constructed using models for other devices found in library. For example, the thyristor elements are usually replaced by two-transistor models.

There are some problems which arise in developing and applying CAAD systems such as bias analysis when Direct Current (DC) analysis has to be made. The DC analysis problem centers on finding the operating point or points (terminal currents and voltages) in the non-linear resistive devices in the network. It is usually done as an initial step to determine stable states which could exist within the network. finding the DC operating voltages or currents in non-linear networks may be complicated by the fact that the network problems which are presented may have no solution or a multiplicity of solutions. There is also a problem of finding all of the solutions for the operating points systematically when several could exist.

In this paper methods for constructing large signal models in the form of equivalent circuits for non-linear resistive elements are discussed. The models developed may be used in CAAD programs to model two-terminal devices, even those with negative resistance

model. The model which will be developed will be applicable only to both voltage-controlled or current-controlled resistors which are what most non-linear resistors really are. The only type of non-linear element which will be used in the model will be the ideal diode.

The use of negative resistances was avoided in constructing the model. The main reasons for imposing the non-negative resistance and the ideal diode conditions are to make the model usuable in most CAAD programs since most of these programs already recognize these elements. Such restrictions will also simplify the creation of a specialized CAAD program since a linear circuit analysis program can be used with modified cirucits as the ideal diodes change state. A program for circuit analysis using the model has been developed and analog simulations of some devices with the use of transistors have been conducted.

THE RESISTANCE PARAMETER

The resistance parameter of a two terminal device characterizes the relation between the current (i) flowing through the device and the voltage (v) across it. The most familiar resistor, which is the linear resistor, has its v-i relationship described by Ohm's Law, $v = Ri$ where R is a constant called resistance. A resistor is linear if its current-voltage (i-v) characteristics can be described by a straight line passing through the origin of the i-v plane, otherwise, it is non-linear. Graphically, the relationship between the current and the voltage can be demonstrated using i-v plots (current vs. voltage) and v-i plots (voltage vs. current). The p-n junction diode may be considered a non-linear resistor with the i-v relationship described by the equation $i = I_o(e^{V/nV_T} - 1)$, where I_o, n and V_T are constants. For some other resistor types, the relationship between i and v cannot be expressed in the form of a closed equation but in graphical form. Examples of these are the characteristic curves of various semiconductor diodes. Figure 1 gives examples of characteristics of various non-linear resistors.

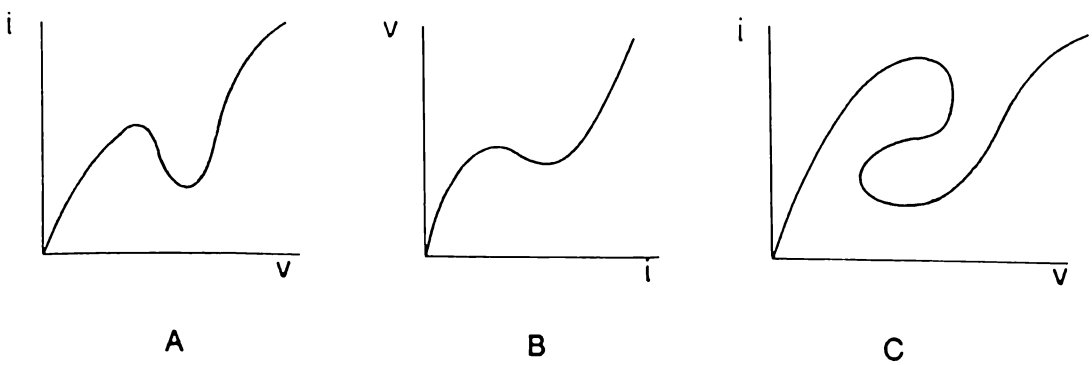


Figure 1

A voltage - controlled resistor is one in which the current i may be expressed as a function of the voltage v, that is, for each voltage v, there is a corresponding value of i. In

equation form, this may be written i = I(v). An example of the i-v characteristics of a non-linear voltage - controlled resistor is seen in Figure 1A.

A current-controlled resistance is one in which the voltage v can be expressed as a function of the current i, that is, for every value of i, there is a corresponding value of v. In equation form, this can be written v = v(i). An example of the v-i characteristics of a current - controlled non-linear resistor is shown in Figure 1B.

There are some resistors which are neither voltage-controlled or current-controlled. Figure 1C illustrates the i-v characteristics of such a device. The model development procedures to be presented in this paper will apply only to voltage - or current-controlled resistors.

PIECEWISE LINEAR (PWL) CHARACTERISTICS

For purposes of simplifying network analysis, the often-difficult-to-represent, characteristics of semi-conductor devices are replaced with piecewise linear curves. It was hoped that piecewise linearization will result in simpler and more straightforward equations which can easily be solved using algorithms for linear equations, hence shortening both the computation and development times of the CAAD programs. The piecewise linearization process will be used here as part of the synthesis procedure. Examples of PWL characteristics which are based on the devices of Figure 1 are shown in Figure 2.

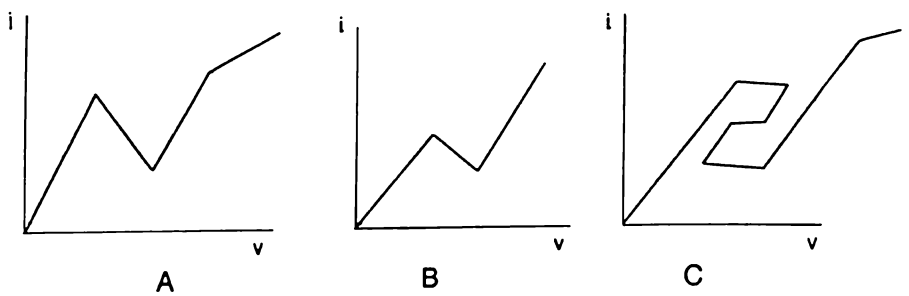


Figure 2

As seen by comparing both the original and PWL characteristics, the i-v characteristics of the voltage- controlled resistor and the v-i characteristics of the current-controlled resistor are modified so that portions of the characteristic curves are replaced by straight lines. Better approximations to the characteristics can be made by using more straight lines or segments of shorter length.

EQUATIONS BASED ON PWL CURVES

Given the PWL characteristic curve of a device, as long as it is voltage controlled or current-controlled and continuous, a describing function can be written. This equation is of the form:

$$y(x) = a + bx + \sum_{k=1}^{n} c_k r(x - x_k) \qquad \text{Eg. 1}$$

where n is the number of segments - 1

In the equation, i and v have been replaced by y and x for generality. a, b, c_1, c_2 . . . c_n are constants and r(x) is the unit ramp function defined as:

$$r(x) = \begin{cases} x, & x > 0 \\ 0 & x < 0 \end{cases} \qquad \text{Eq.2}$$

$r(x - x_k)$ is the unit ramp function shifted by x_k, and it can be defined as:

$$r(x - x_k) = \begin{cases} x - x_k, & x > x_k \\ 0 \quad , & x < x_k. \end{cases} \qquad \text{Eq.3}$$

Figures 3A and 3B show the plots of the ramp function and shifted ramp functions.

Figure 3

An example will be used to illustrate how the constants of Equation 1 may be derived from the piecewise linear curves. For purposes of this discussion, a segment in the PWL characteristic curve, is a section of the PWL curve between two successive breaks. Breaks will be defined as points in which a new line is used to represent the characteristics. From these definitions, there is exactly one straight line in a segment. For voltage-controlled resistors, breaks occur at some voltages and segments can be found between two

successive break voltages. For current-controlled resistors, breaks occur at some currents and segments can be found between two successive break currents.

In Figure 4 the PWL characteristics will be used to show how the analytical expression relating y and x may be written. A PWL curve is shown with four segments. The form of Equation 1 will be used to describe the characteristics of the resistor with the characteristics shown in Figure 4. It is simply necessary to find the constants a, b, c_1, c_2 . . . c_n. The following discussion will show how these constants may be derived from Figure 4. Equation 1 may be expanded and written as:

$$y(x) = a + bx + c_1r(x - x_1) + c_2r(x - x_2) + \ldots + c_nr(x - x_n). \qquad \text{Eq.4}$$

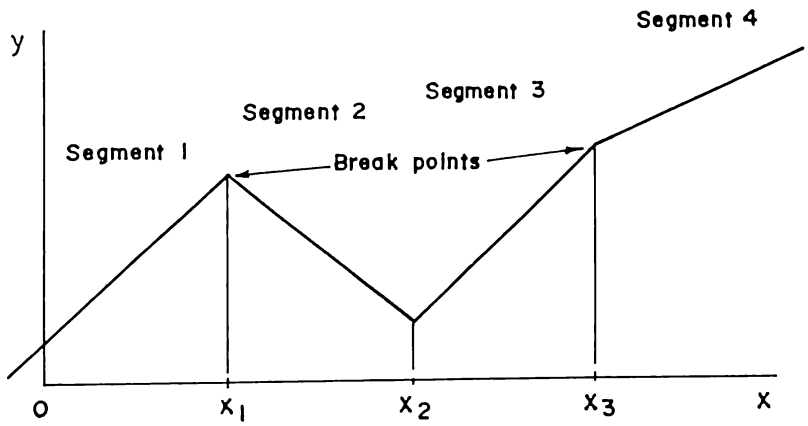


Figure 4

It will be noted that if y(x) is plotted vs. x, each time a term $c_kr(x - x_k)$ is introduced, a break in the PWL characteristics at x_k will be expected, hence the number of terms of this form in the expression will depend on the number of breaks. For the example curve (Figure 4) shown, there are three breaks so that the trial expression for y(x) will become:

$$y(x) = a + bx + c_1r(x - x_1) + c_2r(x - x_2) + c_3r(x - x_3). \qquad \text{Eq. 5}$$

It can also be observed that if y(x) is to correspond to the graph of Figure 4, the values of c_1, c_2, . . . c_n may be obtained by computing the differences in the slopes between two successive segments, that is, c_1 is the difference in the slopes of segments 2 and 1; c_2, the difference in slopes of segments 3 and 2 and c_3, the difference in the slopes of segments 4 and 3. The values of c_1, c_2 and c_3 can be computed accordingly.

It is also observed that the value of $c_1r(x - x_1) + c_2r(x - x_2) + \ldots + c_nr(x - x_n)$ is zero before the first break point, thus $y_1(x) = a + bx$ is the equation of the line in the first

(leftmost) segment. The constants a and b can thus be found by finding the equation of the first segment.

A synthesis procedure which parallels Foster's Method of passive network synthesis will be presented based on the equation just derived.

SYNTHESIS OF VOLTAGE-CONTROLLED RESISTORS

The v-i characteristics of a voltage-controlled resistor can be written as:

$$i(v) = I_o + gv + \sum_{k=1}^{n} i_k r(v - v_k) \qquad \text{Eq. 6}$$

$$i(v) = I_o + gv + i_1 r(v - v_1) + i_2 r(v - v_2) + \ldots + i_n r(v - v_n). \qquad \text{Eq.7}$$

The resistor to be modeled draws a current i(v) for the given value of v. An equivalent circuit for such a resistor may be developed by connecting in parallel, devices which draw currents equal to each term being summed up in Equation 7. Figure 5 shows this stage of the development of the model.

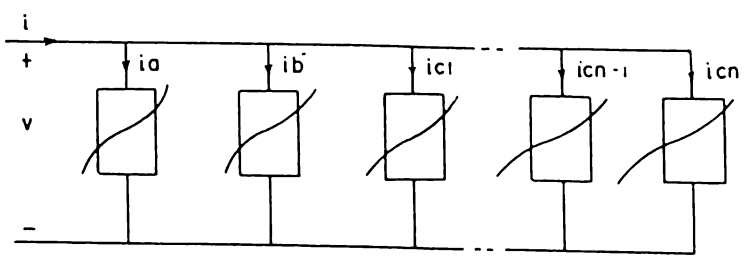


Figure 5

$$i_a = I_o \qquad \text{Eq. 8}$$

$$i_b = gv$$

$$i_{c1} = i_1 r(v - v_1)$$

$$i_{c2} = i_2 r(v - v_2)$$

$$i_{cn} = i_n r(v - v_n).$$

Element A can be modeled with a constant current source I_o. Element B can be modeled by a linear resistor of conductance g. However, since the use of negative resistances is avoided in the model to be developed, the circuit shown in Figure 6 is used instead for more generality. Note that if g is positive, the circuit of Figure 6A reduces to the circuit of Figure 6B which has a positive resistance.

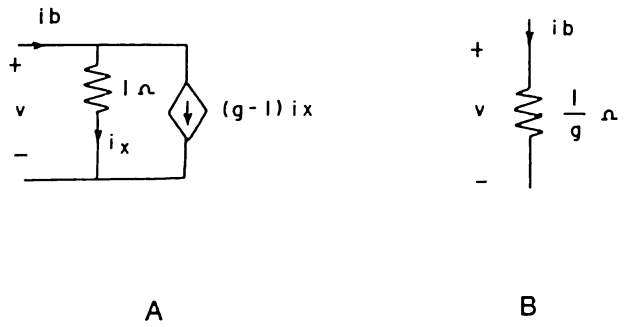


Figure 6

Elements C_1, C_2 ... C_n may be modeled using the building block shown in Figure 7. For positive values of i_k, the network shown in Figure 7A can be reduced to the structure shown in Figure 7B. The diode is ideal. The building block has the i-v characteristics shown in Figure 7C.

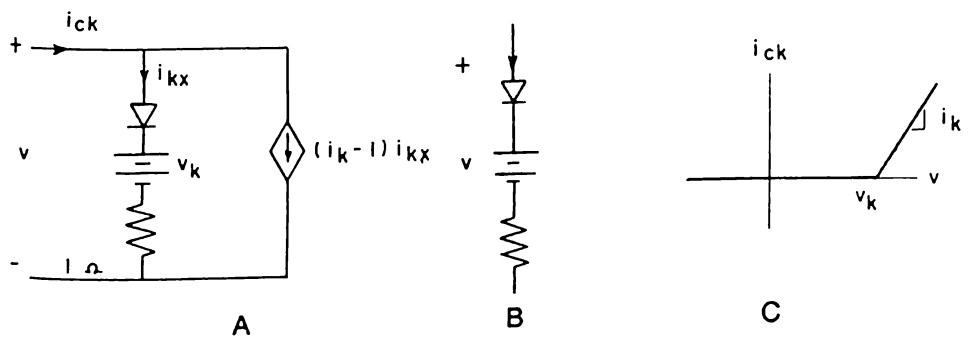


Figure 7

Consider the circuit shown in Figure 7A. If v V_k, the diode will be off. The current through the controlled source will be zero and the current i_{ck} will also be zero. If v V_k, then the ideal diode will conduct. The current i_{kx} will be equal to (v - V_k)/1 ohm and ik will be

equal to $(v - V_k)(1 + (i_k - 1))$ or $i_k (v - V_k)$. The v-i characteristic curve of this resistor is shown in Figure 7C.

FOSTER SYNTHESIS OF CURRENT-CONTROLLED RESISTORS

The expression for the characteristics of the current-controlled resistor has the same form as that of the voltage-controlled resistor. However, in the expression for current-controlled resistors, v is replace by i and vice versa.

$$v(i) = V_o + r_i + \sum_{k=1}^{n} v_k r(i - i_k) \qquad \text{Eq. 9}$$

$$v(i) = V_o + ri + v_1 r(i - i_1) + v_2 r(i - i_2) + \ldots + v_n r(i - i_n). \qquad \text{Eq.10}$$

The resistor to be modeled drops a voltage equal to v if the current passing through it is i. In a manner analogous to that of the voltage-controlled resistor, a model will be developed. This time, however, the voltage v will be treated as the sum of voltage drops across elements connected in series. It can be recalled that the current i was treated as the sum of the currents through different elements connected in parallel in the model for the voltage-controlled resistor. This is shown in Figure 8.

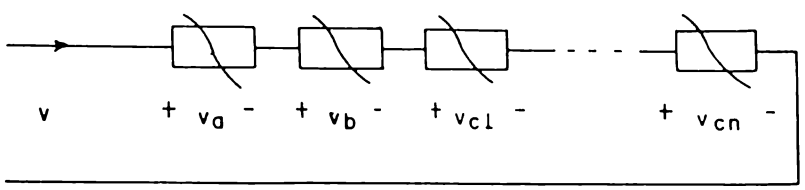


Figure 8

$$v_a = V_o \qquad \text{Eq. 11}$$

$$v_b = ri$$

$$v_{c1} = v_1 r(i - i_1)$$

$$v_{c2} = v_2 r(i - i_2)$$

$$v_{cn} = v_n r(i - i_n).$$

Element A may be modeled by a DC voltage source V_o. Element B will simply be a resistor of resistance b. However, since the use of negative resistances is to be avoided in developing the model, the circuit of Figure 9A is proposed. Again, for positive values of r, the circuit reduces to that shown in Figure 9B.

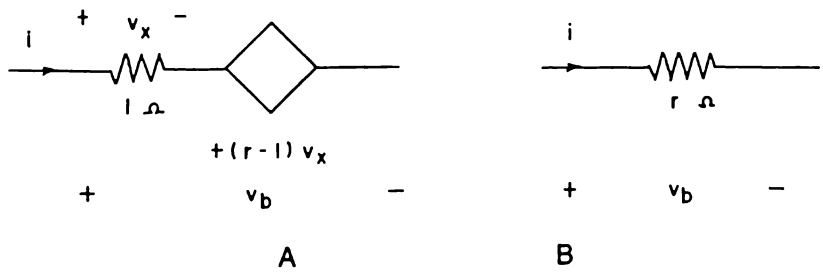


Figure 9

Elements C_1, C_2 ... C_n may be modeled using the building block shown in Figure 10A. For positive values of c_k the network shown in Figure 10A can be reduced to the structure shown in Figure 10B. Figure 10C shows the v-i characteristics of this building block.

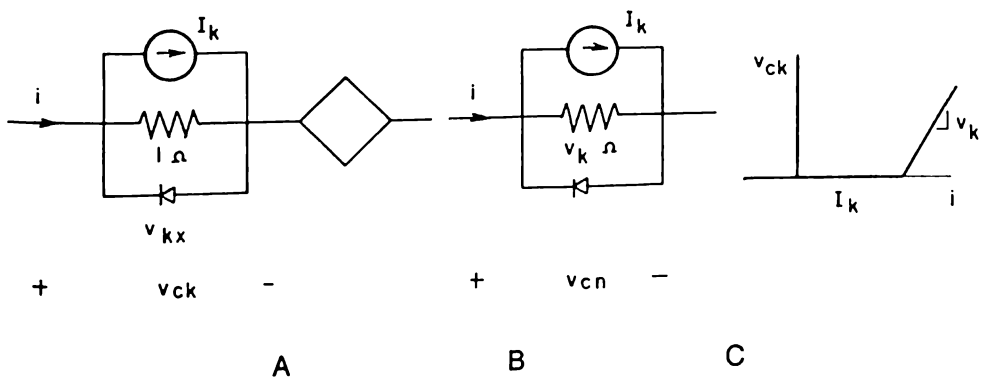


Figure 10

For the circuit of Figure 10A, if i I_k, then i - I_k < 0. The diode conducts. The voltage v becomes zero, hence v_{ck} also becomes zero. If i > I_k, the diode will not conduct. v will be equal to (i - I_k)x1 ohm and v_{ck} will be equal to v_k(i - I_k). The v-i characteristics of this resistor are shown in Figure 10C.

LADDER SYNTHESIS OF RESISTIVE NON-LINEAR NETWORKS

The methods previously discussed resulted in equivalent circuits which are composed of parallel combinations of elements for the voltage-controlled resistors and parallel combinations of elements for the current-controlled resistors. These methods, in a way, parallel the Foster I and II synthesis methods for passive RC, RL or LC networks. If there is a method that parallels the Foster methods, can there be one which parallels the Cauer methods? That is, can there be a method for developing equivalent circuits for non-linear resistors with the ladder topology? The discussion to follow will show how ladder models can be developed for voltage-or current-controlled networks.

TRANSFORMATIONAL PROPERTIES OF SOME TWO-PORT NETWORKS

Circuit 1

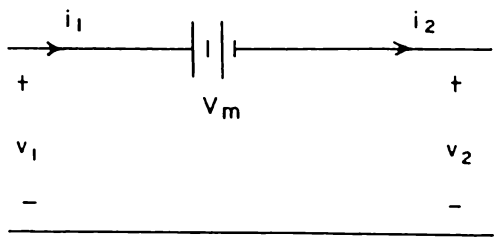


Figure 11

If a voltage-controlled resistor with characteristics $i_2 = i_2(v_2)$ is connected across port 2 of Circuit 1 as shown in Figure 11, the i-v characteristics seen at port 1 will be $i_1 = i_2(v_1 - V_m)$. Similarly if we wish the characteristics seen at port 1 to be $i_1 = i_1(v_1)$, then the resistor which should be connected at port 2 should have the i-v characteristics if $i2 = i1(v2 + Vm)$. This would mean that the i_1-v_1 and the i_2-v_2 curves are of the same general shapes; however, one is shifted either to the left or to the right of the other.

If a current-controlled resistor with v-i characteristics $v_2 = v_2(i_2)$ is connected across port 2 of Circuit 1 as shown in Figure 11, the v-i characteristics seen at port 1 will be $v_1 = v_2(i_1) + Vm$. If we wish that the v-i characteristics seen at port 1 be $v_1 = v_1(i_1)$, a resistor with v-i characteristics $v_2 = v_1(i_2) - V_m$. This would then mean that the v_1-i_1 and v_2-i_2 plots are similar in shape; however, one is shifted to be higher or lower than the other on the v-i plane.

Circuit 2

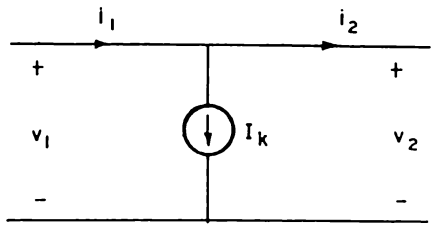


Figure 12

If a current-controlled resistor with v-i characteristics $v_2 = v_2(i_2)$ is connected across port 2 of Circuit 2 as shown in Figure 12, the v-i characteristics seen at port 1 will be $v_1 = v_2(i_1 - I_k)$. If we wish that the v-i characteristics seen at port 1 be $v_1 = v_1(i_1)$, a resistor with v-i characteristics $v_2 = v_1(i_2 + I_k)$. This would then mean that the v_1-i_1 and v_2-i_2 plots are similar in shape; however, one is shifted to a position on the v-i curve higher or lower relative to the other.

If a voltage-controlled resistor with i-v characteristics $i_2 = i_2(v_2)$ is connected across port 2 of Circuit 2 as shown in Figure 12, the i-v characteristics seen at port 1 will be $i_1 = i_2(v_1) - I_k$. Similarly, if we wish the i-v characteristics seen at port 1 to be $i_1 = f_1(v_1)$, then the resistor which should be connected at port 2 should have i-v characteristics if $i_2 = i_1(v_2) + I_k$. This would mean that the i_1-v_1 and the i_2-v_2 curves are of the same general shape. However, on the i-v plane one characteristic curve is shifted either to the left or to the right relative to the other.

Circuit 3

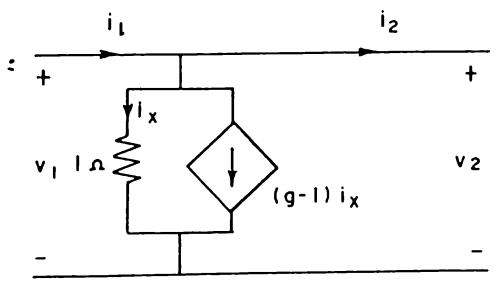


Figure 13

If a resistor with i-v characteristics $i_2 = i_2(v_2)$ is connected at port 2 of Circuit 3 as shown in Figure 13, then at port 1, the i-v characteristics seen will be $i_1 = i_2(v_1) + gv_1$. If it is required that the i-v characteristics at port 1 be equal to $i_1 = i_1(v_1)$, then a resistance with i-v characteristics $i_2 = i_1(v_2) - gv_2$ can be connected at port 2.

Circuit 4

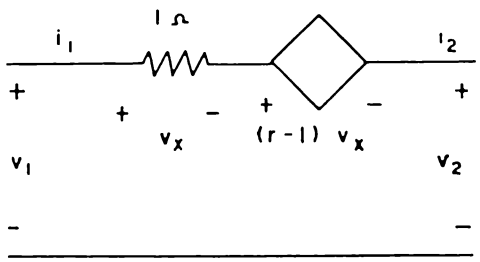


Figure 14

If a resistor with v-i characteristics $v_2 = v_2(i_2)$ is connected at port 2 of Circuit 4 as shown in Figure 14, then at port 1 the v-i characteristics of the resistor seen will be $v_1 = v_2(i_1) + ri_1$. If it is required that the v-i characteristics at port 1 be equal to $v_1 = v_1(i_1)$, then a resistance with v-i characteristics $v_2 = v_1(i_2) - ri_2$ can be connected at port 2.

Circuit 5

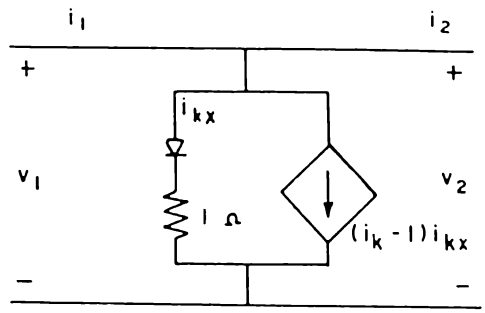


Figure 15

If a voltage-controlled resistor with i-v characteristics $i_2 = i_2(v_2)$ is connected across port 2 of Circuit 5 (Figure 15), the i-v characteristics seen at port 1 will be $i_1 = i_2(v_1) + c_k r(v_1)$. Similarly, if we wish the i-v characteristics at port 1 be $i_1 = i_1(v_1)$, then the i-v characteristics of the resistor to be connected at port 2 should be $i_2 = i_1(v_2) - c_k r(v_2)$.

Circuit 6

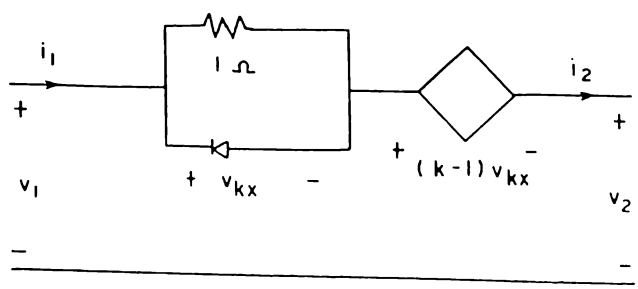


Figure 16

If a current-controlled resistor with v-i characteristics $v_2 = v_2(i_2)$ is connected across port 2 of Circuit 6 as shown in Figure 16, then the v-i characteristics seen at port 1 will be $v_1 =$

$v_2(i_1) - c_k r(i_1)$. Similarly, if a resistance with v-i characteristics $i_1 = v_1(v_1)$ at port 1 is needed, then the resistance which must be connected at port 2 should have the v-i characteristics $v_2(i_2) = v_1(i_2) + c_k r(i_2)$.

EXTRACTION OF THE ELEMENTS OF THE EQUIVALENT CIRCUIT

Given a resistor rs with characteristics (i-v or v-i) designated by f_s, extraction is defined as the process of finding an equivalent circuit for r_s using one of the six two-port networks shown above and a resistor r_m with characteristics designated by f_m. r_s is seen at port 1 of the two-port network while r_m is connected at port 2. r_m will be called the remainder resistor of the extraction. Figure 17 illustrates the extraction process. A sequence of extractions can be used to synthesize the equivalent circuit for the given non-linear resistor r_s and the resulting circuit is expected to have a ladder topology. The synthesis process stops when a zero resistance (short circuit) or zero admittance (open circuit) condition is reached. Let us define a Type n extraction to be one which uses two-port Circuit n as labeled in the past six figures.

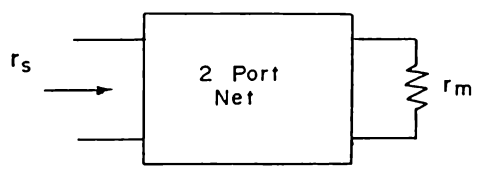


Figure 17

LADDER SYNTHESIS OF A VOLTAGE-CONTROLLED RESISTOR

Given f_s as the i-v characteristics of the resistor r_s which is to be replaced by an equivalent circuit, the following steps may be used to realize an equivalent network for r_s of ladder topology. In the following procedure, the r_m of an extraction step automatically becomes the r_s of the succeeding instruction step. Consequently the f_m of a step is also the f_s of succeeding step.

Step 1

Apply only a Type 1 Extraction or a Type 1 Extraction followed by a Type 2 Extraction to r_s, so that the i-v characteristics of the remainder resistor r_m which is f_m will intersect the origin of the i-v plane.

Step 2

Apply a Type 3 Extraction to r_s so the slope of f_m of the resulting r_m will be zero. This means using Circuit 3 with g set equal to the slope of the first segment of f_s.

Step 3

Apply a Type 1 Extraction to r_s so that the first breakpoint of f_m will be at the origin of the i-v plane. This means using a value of V_k in Circuit 1 equal to the value of the first break point in r_s.

Step 4

Apply a Type 5 Extraction to rs so that the number of segments in the resulting f_m will be one less than that of f_s. A value of c_k in Circuit 5 equal to the slope of the first segment of f_s should be used.

Step 5

If f_s is a straight horizontal line, the process ends. The resulting equivalent circuit is constructed by connecting ll the two-port networks used in the extraction in cascade from left to right in the order that they were extracted. Port 2 of the last network is left open-circuited. If f_s is not a straight horizontal line, go to Step 3.

NUMERICAL EXAMPLES

I - Voltage Controlled Resistors

Suppose the resistor with the given characteristics shown is to be modeled.

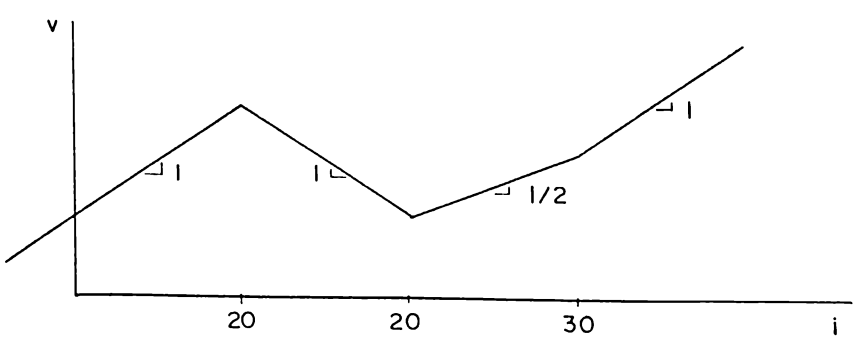


Figure 18

A.FOSTER METHOD

The first step is to find the equation describing the network characteristics.

Clearly $i_1 = -1 - 1 = -2 \quad V_1 = 10$
$i_2 = 1/2 - (-1) = 1.5 \quad V_2 = 20$
$i_3 = 1 - 1/2 = 1/2 \quad V_3 = 30.$

By the point-slope form, the equation of the line corresponding to the first segment can be found to be $i = 5 + Yv$. The equation for the characteristics then becomes:

$$i = 5 + Yv - 2r(v - 10) + 1.5r(v - 20) + 0.5r(v - 30)$$

The following circuit model is this developed for the resistor.

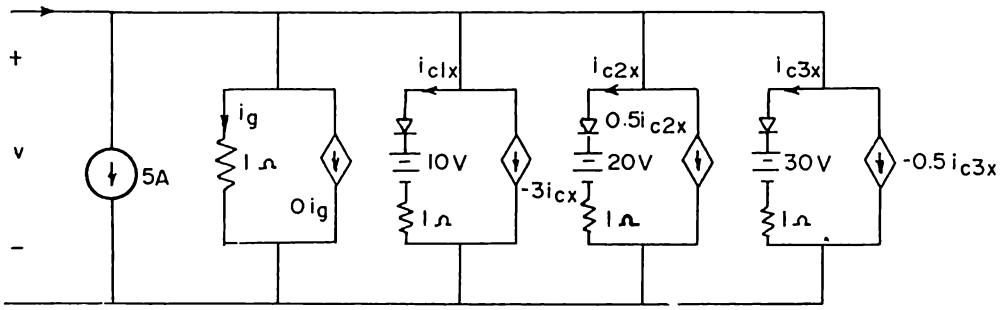


Figure 19

Note that some elements in the model may be combined and as a result, the simpler equivalent circuit shown can be synthesized.

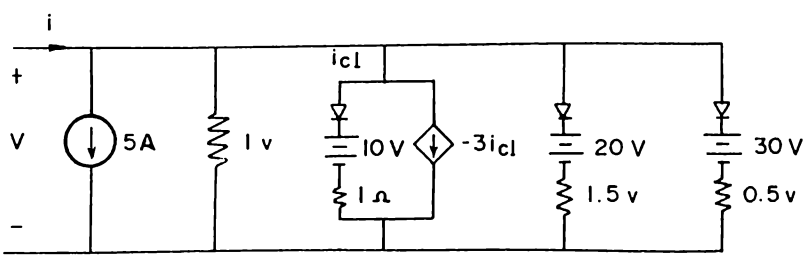
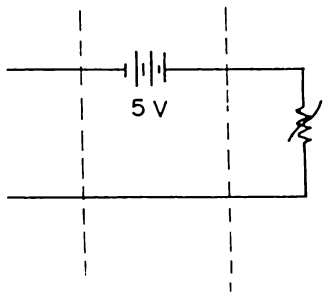


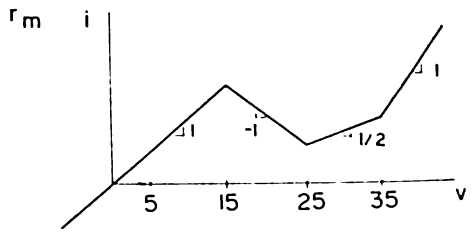
Figure 20

B. LADDER SYNTHESIS

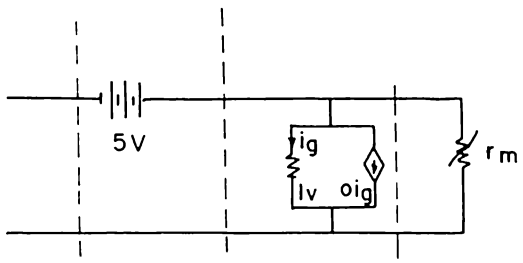
The following figure demonstrated the steps used in synthesizing the model. The characteristics shown on the right are the characteristics of the remainder resistor r_m which is f_m.

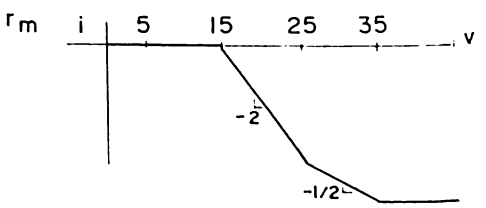
Type 1 Extraction



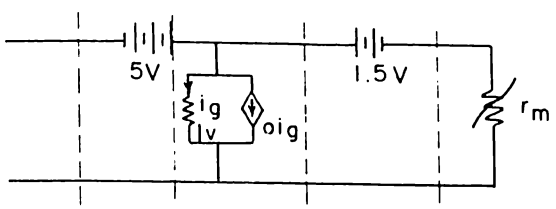


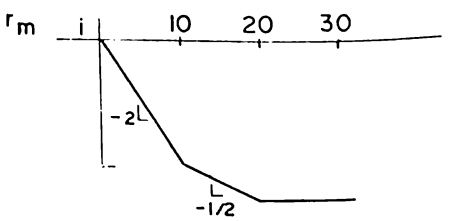
Type 3 Extraction



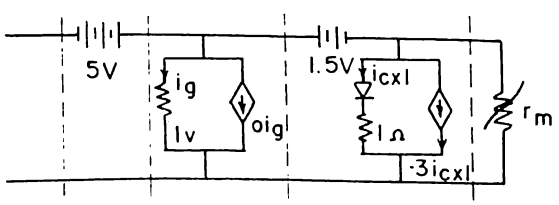


Type 1 Extraction





Type 5 Extraction



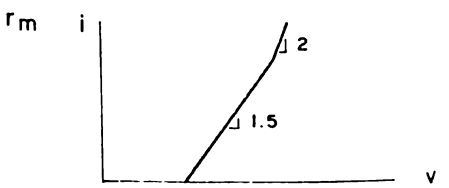
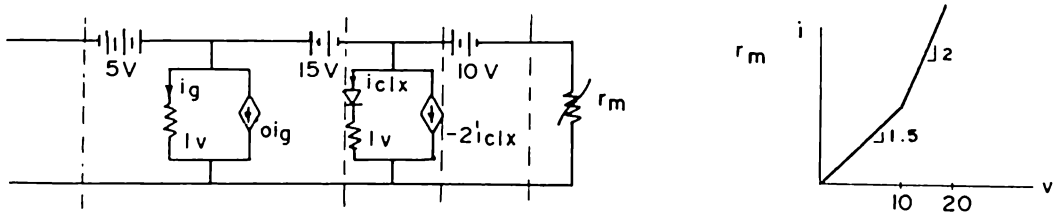
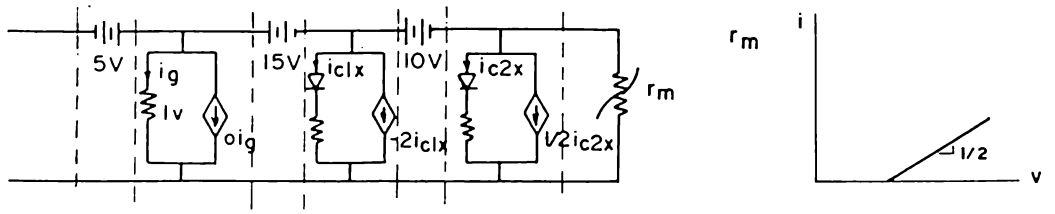


Figure 21

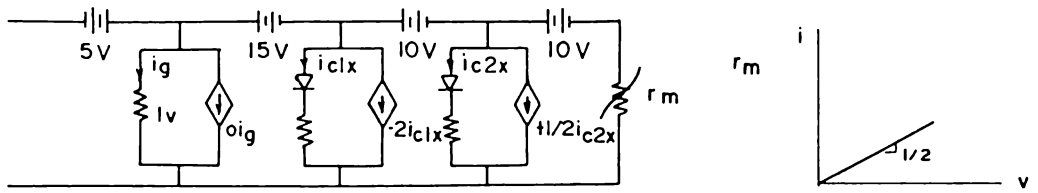
Type 1 Extraction



Type 5 Extraction



Type 1 Extraction



Type 5 Extraction

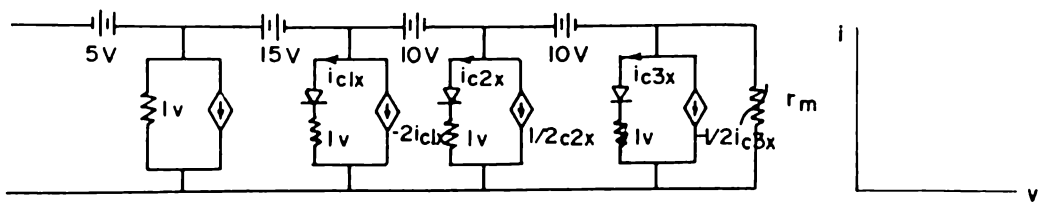


Figure 21 (cont'd)

Resulting Network

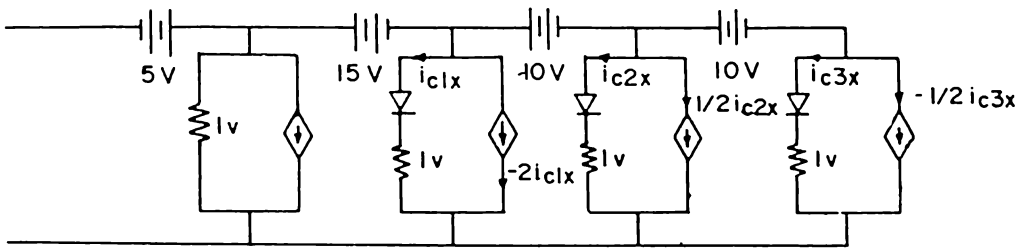


Figure 22

Some of the branches may be replaced by equivalent circuits of less elements.

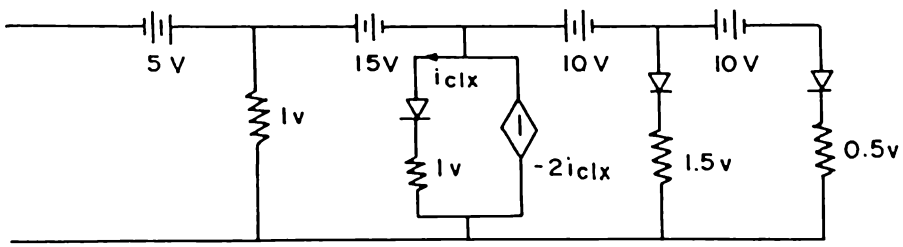


Figure 23

II.Current-Controlled Resistors

A. Foster Synthesis

The characteristics of the resistor to be synthesized are shown in the figure below.

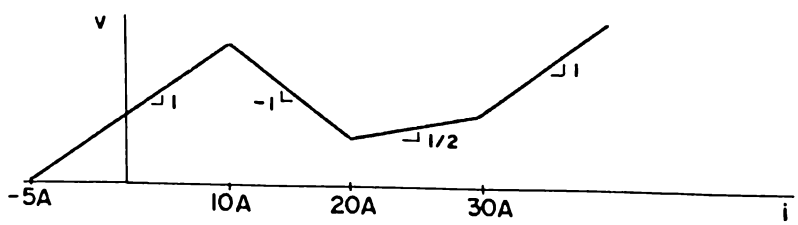


Figure 24

The equation describing the characteristics of this resistor is:

$$v = 5 + xi - 2r(i - 10) + 1.5r(i - 20) + 0.5r(i - 30)$$

The following circuit is synthesized as a result of following the procedure.

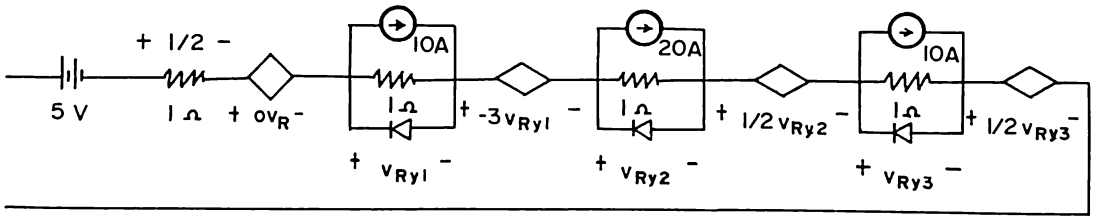


Figure 25

Some of the elements can be combined to result in a circuit with less elements.

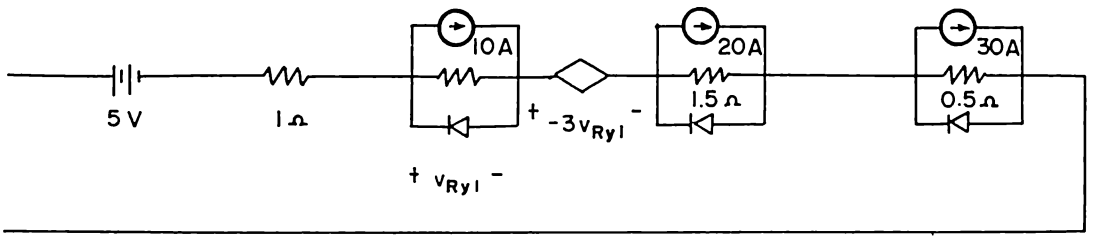
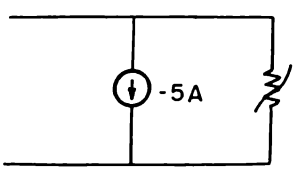


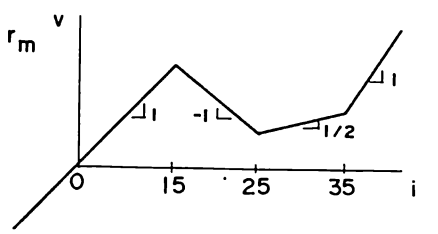
Figure 26

B. Ladder Synthesis

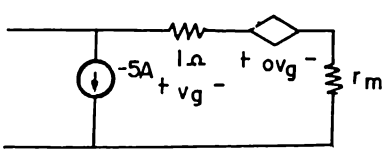
The following figure demonstrates the steps used in synthesizing the model. The characteristics shown on the right are the characteristics of the remainder resistor r_m which is f_m.

Type 2 Extraction





Type 4 Extraction



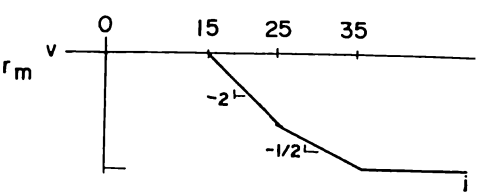
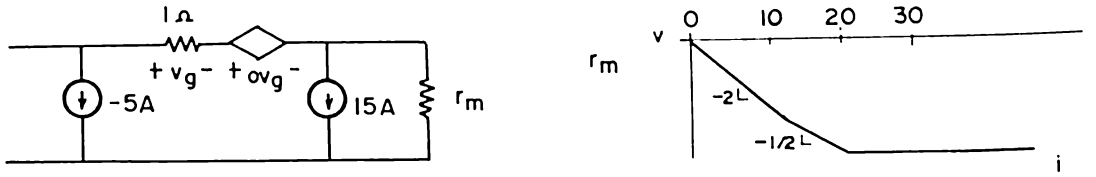
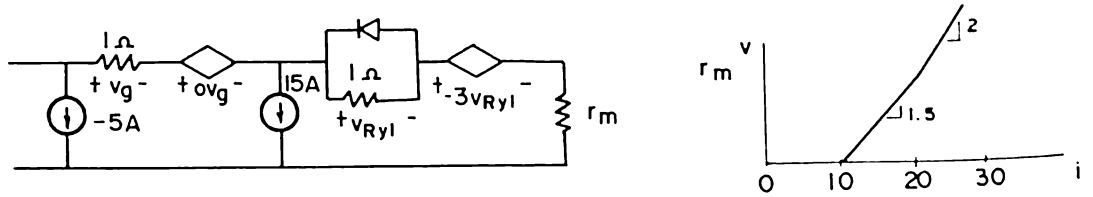


Figure 27

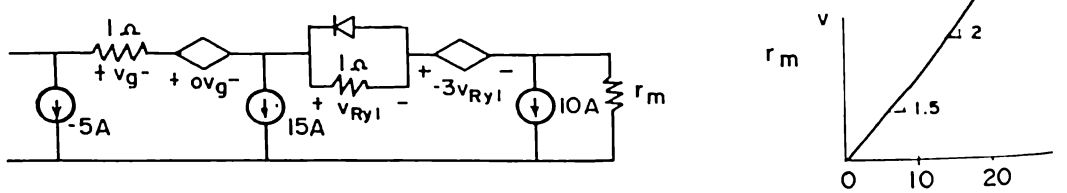
Type 2 Extraction



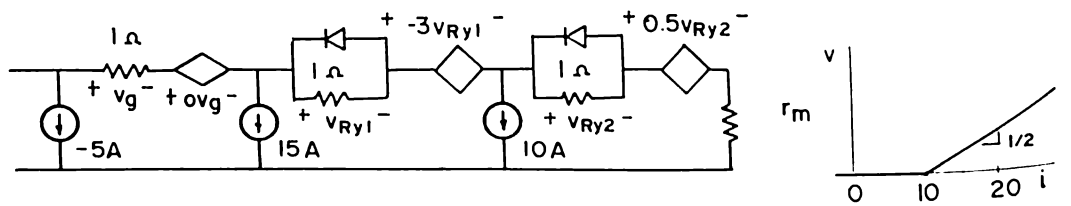
Type 6 Extraction



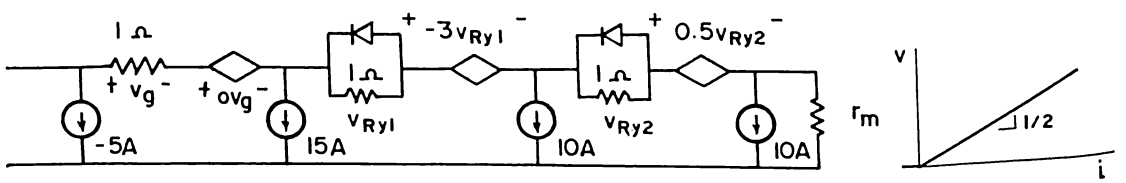
Type 2 Extraction



Type 6 Extraction



Type 2 Extraction



Type 6 Extraction

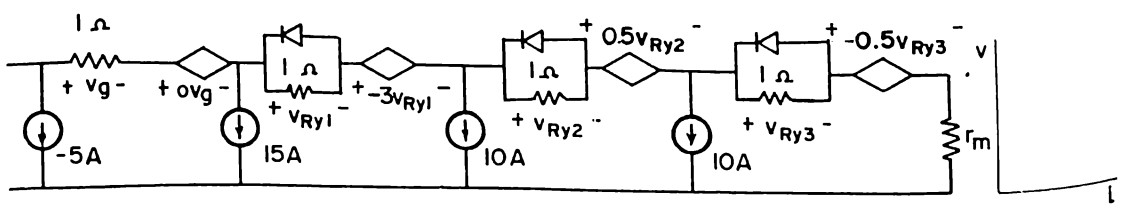


Figure 27 (cont'd)

Resulting Network

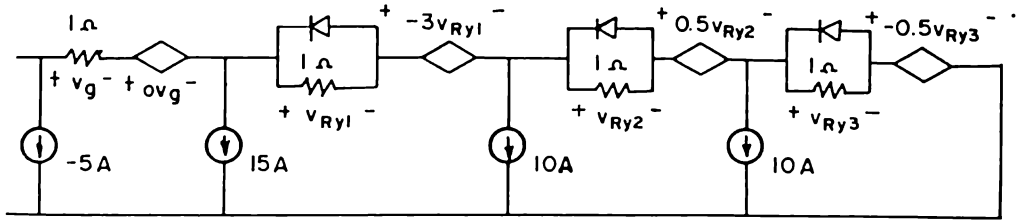


Figure 28

Some of the branches may be replaced by equivalent circuits of less elements.

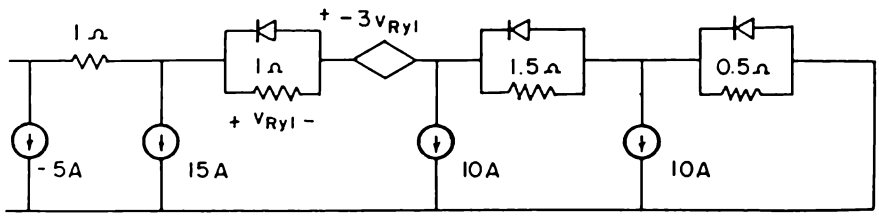


Figure 29

APPLICATIONS OF THE MODEL

I - Finding the DC Operating Point of Interconnected Non-linear Resistors.

One of the problems in non-linear network analysis is to find the DC operating point of several devices in a network. Difficulties often arise in network problems because of non-linearities. A non-linear resistive device may be replaced by one of the equivalent circuits just developed. The new circuit containing only diodes as the non-linear elements can be analyzed more easily. Since the diodes are ideal, they can only be switched on or off, equivalent either to an open circuit or a short circuit. A computer program can be developed to analyze such circuits.

The analysis of such a circuit can be made by assuming some state for the diodes and verifying whether or not the states are valid. For example in a calculation, a certain diode D can be assumed to be ON, thus equivalent to a short circuit. After calculations based on this assumption are made, it should be tested whether the current flowing from the anode to the cathode is positive. If the current turns out to be negative, the starting assumption is already wrong. If D is the only diode in the network, then it is sufficient to assume that D is OFF. If D is only one of the diodes in the network, then it is possible that the assumption for D being ON is still correct while the assumptions for the other diodes may be wrong.

One difficulty in non-linear network analysis is the possibility that there are many solutions or that no solutions exist. One way to determine all possible solutions is to consider all combinations of states of the diodes in the given network and make a linear analysis of the network for every combination of states. SInce a diode had two states, a network with n diodes will require 2n separate analysis. When using the models just developed, the number of combinations may be decreased. On Figure ( ) it can be seen that D2 is OFF as long as D1 is OFF and D3 is OFF as long as D3 is OFF. The allowable states for this network become 4 instead of 8. It can be shown that if one model has n diodes, then the number of possible states in the model is just 2n instead of 2^n.

Programs to analyze such circuits can be developed using algorithms based on linear circuit analysis. A main program can be used to manage diode state combinations. A linear circuit analysis algorithm is then called to analyze the networks with the diodes already replaced by open and short circuits. The solution is then checked for consistency. For all diodes assumed ON, a positive current should flow from the anode to the cathode while for diodes assumed OFF, a negative voltage should be measure between anode and cathode.

If the network contains no solutions, that is, no allowable DC operating point, then for all diode state combinations, no consistent solution will be found. If there is an infinite number of solutions, usually the determinant of the circuit matrix used in the linear circuit calculation part will be zero. This could cause the program to fail or give appropriate warnings.

The programs developed based on this model will definitely not yield very accurate results, however, results may be obtained faster due to the simplicity of the linear equations being computed.

II - Simulation of Non-linear Resistance Devices

The model could be used as starting point in developing circuits with prescribed driving point resistive characteristics. The controlled current and voltage sources may be developed using transistors, operational amplifiers and other active devices when the characteristics are not monotonic. The resulting devices could thus be used in actual circuits.

CONCLUSION

Methods to develop equivalent circuits for non-linear resistors have been presented. These methods are based on the piecewise-linear characteristics and may result in either. Foster type of Cauer (ladder) type networks. Non-linear analysis using these models can be made by considering possible combinations of diode states. Construction of actual circuits based on these models can be done using active devices.

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