"Every physical factor of transient travelling-wave is affected by frequency-dependent parameters . . . easily considered in the solution."

Analytic Solution of Electromagnetic Transient in a Polyphase Transmission System

by

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ABSTRACT

For the first time, this paper gives analytic solutions in the time-domain and natural frequencies of nonsymmetrical exciting system after solving the matrix partial differential equations which describe electromagnetic behavior on polyphase lossy-transmission lines, the frequency-dependent parameters of which are considered, by using classical method strictly. It is beneficial to the analysis of principle of protections based on travelling-wave and to the proposition of new criteria for protection.

To verify the result, an example is taken which is offered by G. W. Swiftn [1] and solved by Bergeron's method [2]. In the paper, several concepts are given. At the appendix, the correctness of solutions are proved mathematically.

INTRODUCTION

People pay more attention to the research of analytic solution of electromagnetic transient in electric power system while its numerical procedure [2, 3] is tending to be perfected. It is significant for us to understand deeply the electromagnetic transient principle, particularly in the case of very-high-speed protection for UHV system.

For a distributed single-phase transmission-line, it is easy to get the analytic solution of electromagnetic transient in lossless case. The study of lossy-line is not ideal [4, 5]. In the Laplace-Transform method, it is also necessary for us to solve the complex transcendental equation.

For a nonsymmetrical exciting system, many documents [6, 7] make use of the decoupling method to solve coupled problems in the polyphase system, but do not get solution in time-domain because of the difficulty in the determination of analytic boundary conditions.

By using classical method strictly, first we obtain the analytic boundary condition of unsymmetrical exciting system in s-domain; then the solutions in s-domain. Finally, we obtain the solutions in time-domain after using the computer to obtain the natural frequencies of the system. In the expressions of transient travelling-wave group, all physical factors of travelling-wave with different frequencies are determined by transmission-line parameters and faulted distance. Therefore, it is beneficial to the understanding of physical concepts.

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LIST OF MAIN SYMBOLS

x, t	variable of space & time
A, B, C	phase A, B & C
l_r , l_s	distance from exciting source to receiving & sending end of line
$[u_e]$, $[i_e]$, $[U_e]$, $[I_e]$	exciting voltage & current vector in time-domain & s-domain
$[u]$, $[i]$, $[U]$, $[I]$	voltage & current vector in the time-domain & s-domain when distance is x
$[u_O]$, $[i_O]$, $[U_O]$, $[I_O]$	voltage & current vector in the time-domain & s-domain when x = O
$[u_r]$, $[i_r]$, $[U_r]$, $[I_r]$	voltage & current vector in the time-domain & s-domain when $x = l_r$
$[u_s]$, $[i_s]$, $[U_s]$, $[I_s]$	voltage & current vector in the time-domain & s-domain when $x = -l_s$
s, m	subscripts of self & mutual parameters
k, n	subscripts of kth. & nth. travelling-wave
$1/\alpha_n$	time constant of nth. travelling wave
β_n	eigenfrequency of nth.
w	frequency of exciting source
[L]	inductance matrix per unit length
[C]	capacitance matrix per unit length
[R]	resistance matrix per unit length
[G]	conductance matrix per unit length
[T]	eigenvector matrix
[r]	lumped resistance matrix
$[L_e]$	lumped inductance matrix
[Z]	$= [R] + s[L]$
[Y]	$= [G] + s[C]$
$[Z_e]$	$= [r] + s[L_e]$
$[Z_d]$	$= [T]\ [Z]\ [T]^{-1} = \mathrm{diag}\ [Z_{d0}\ Z_{d1}\ Z_{d2}]$
$[Z_{ed}]$	$= [T]\ [Z_e]\ [T]^{-1}$
	$= \mathrm{diag}\ [Z_{ed0}\ Z_{ed1}\ Z_{ed2}]$
	$= [T]\ [Z]\ [Y]\ [T]^{-1}$
	$= [T]\ [Y]\ [Z]\ [T]^{-1} \triangleq$
	$\triangleq [\tau]^2 = \mathrm{diag}\ [\tau_0^2\ \tau_1^2\ \tau_2^2]$
$[Z_w]$	$= [Z_d]\ [\tau]^{-1}$

MATHEMATICAL DESCRIPTION

According to the superposition theorem, an equivalent superposition system of power transmission-lines is shown in Figure 1.

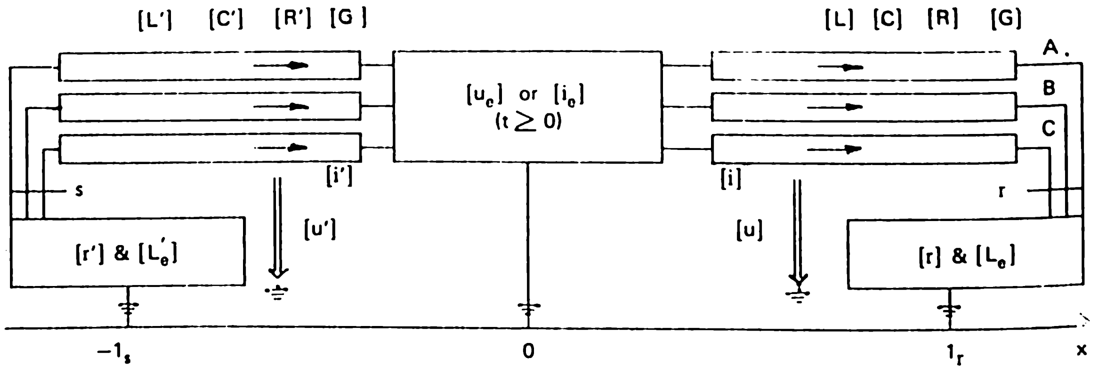

Figure 1. Equivalent Superposition System

$[u_e]$ and $[i_e]$ are the exciting voltage and current vectors respectively. For example, the exciting source of 'B, C' – earth solid fault is

$$[u_e] = [u_{fb} \; u_{fc}]^T$$

and 'A' – break exciting source is

$$[i_e] = i_{ka}$$

It is well known that the electromagnetic behavior of polyphase transmission-lines can be described by

$$-\frac{\partial [u]}{\partial x} = [L] \frac{\partial [i]}{\partial t} + [R] \; [i] \tag{1a}$$

$$-\frac{\partial [i]}{\partial x} = [C] \frac{\partial [u]}{\partial t} + [G] \; [u] \tag{1b}$$

The boundary and initial conditions are

$$[u_0] = [u_{a0} \; u_{b0} \; u_{c0}]^T \tag{2a}$$

$$[i_0] = [i_{a0} \; i_{b0} \; i_{c0}]^T \tag{2b}$$

$$[u_s] = -[L'_e] \frac{d\,[i_s]}{dt} - [r'] \; [i_s] \tag{2c}$$

$$[u_r] = [L_e] \frac{d\,[i_r]}{dt} + [r] \; [i_r] \tag{2d}$$

$$[u]\Big|_{t=0} = [i]\Big|_{t=0} = [0] \tag{2e}$$

$$\frac{\partial [u]}{\partial t}\Big|_{t=0} = \frac{\partial [i]}{\partial t}\Big|_{t=0} = [0] \tag{2f}$$

MATHEMATICAL SOLUTIONS

Solutions in S-Domain

For convenience, we assume perfect conductor transposition in the three-phase power transmission line for which frequency dependent parameters will be considered. Initially we will consider the case of "A" – earth solid fault.

From Eq. (1) and (2e), (2f), we obtain the equivalent form in S-domain

$$\frac{d^2[U]}{dx^2} = [Z]\,[Y]\,[U] \tag{3a}$$

$$\frac{d^2[I]}{dx^2} = [Y]\,[Z]\,[I] \tag{3b}$$

Combining Eq. (3) with Eq. (2a), (2d), a deterministic problem is formed and its solutions are

$$[U] = [T]^{-1}\,\mathrm{diag}\,[W_0\ \ W_1\ \ W_2]\ \ [T]\ [U_0] \tag{4a}$$

$$[I] = [T]^{-1}\,\mathrm{diag}\,[\widehat{W}_0\ \widehat{W}_1\ \widehat{W}_2]\ \ [T]\ [U_0] \tag{4b}$$

$$0 \leq x \leq 1_r$$

Where

$$[T] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{bmatrix}, \quad [T]^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$W_k = (Q_k \exp(\tau_k x)) + K_k \exp(-\tau_k x))\,/\,(Q_k + K_k)$$

$$\widehat{W}_k = Z_{wk}^{-1}\ (-Q_k \exp(\tau_k x) + K_k \exp(-\tau_k x)\ )\,/\,(Q_k + K_k)$$

$$Q_k = (Z_{edk}\, Z_{wk}^{-1} + 1)\ \exp\,(-\tau_k\, 1_r)$$

$$K_k = (Z_{edk}\ \ Z_{wk}^{-1} + 1)\ \ \exp\,(\tau_k\, l_r)$$

$$k=0, 1, 2$$

. Another deterministic problem, obtained by combining Eq. (3) with Eq. (2a), (2c), has the answers as

$$[U'] = [T]^{-1}\,\mathrm{diag}\ [W_0'\, W_1'\, W_2'\,]\ \ [T]\ \ [U_0] \tag{5a}$$

$$[I'] = [T]^{-1}\,\mathrm{diag}\ [\widehat{W}_0'\ \widehat{W}_1'\ \widehat{W}_2']\ \ [T]\ \ [U_0] \tag{5b}$$

$$-1_s \leq x \leq 0$$

Where

$$W_k' = (Q_k' \exp(\tau_k'\ x) + K_k'\ \exp(-\tau_k'\ x)\,)\ /\ (Q_k' + K_k')$$

$$\widehat{W}_k' = (Z_{wk}')^{-1}(-Q_k' \exp(\tau_k'\ x) + K_k'\ \exp(-\tau'_k\, x)\ )\ )\ /\ (Q_k' + K_k')$$

$$Q_k' = (-Z_{edk}'\ (Z_{wk}')^{-1} - 1)\ \exp\,(\tau_k'\ 1_s)$$

$$K_k' = (-Z'_{edk}\ (Z_{wk}')^{-1} + 1)\ \exp\,(-\tau_k'\ 1_s)$$

$$k=0, 1, 2$$

Because of the assumption of perfect conductor transposition, we have

$$W_1 = W_2,\ \widehat{W}_1 = \widehat{W}_2,\ W_1' = W_2',\ \widehat{W}_1', = \widehat{W}_2'.$$

In Eq. (4) and (5),

$$[U_0] = [U_{fa}\ U_{b0}\ U_{c0}]^T\ ,$$

U_{b0}, U_{c0} are unknown.

After considering the connected condition

$$[I_{b0}\ I_{c0}]^T = [I_{b0}'\ I_{c0}']^T$$

we obtain

$$[U_0] = [D_{g1}] \ [U_e]$$

Where

$$[U_e] = [\, U_{fa}$$

$$[D_{g1}] = [1 \ (P_0 - P_1)/(2P_1 + P_0) \ (P_0 - P_1)/(2P_1 + P_0)\,]^T$$

$$P_0 = Z_{w0} \, Z'_{w0} \, (K_1 Q'_1 - K'_1 Q_1) \, (K_0 + Q_0) \, (K'_0 + Q'_0) \, (Z_{w1} + Z'_{w1})$$

$$P_1 = Z_{w1} \, Z'_{w1} \, (K_0 Q'_0 - K'_0 Q_0) \, (K_1 + Q_1) \, (K'_1 + Q'_1) \, (Z_{w0} + Z'_{w0})$$

Final solutions are:

$$[U] = [T]^{-1} \operatorname{diag} [W_0 \ W_1 \ W_1] \ [T] \ [D_{g1}] \ [U_e] \tag{6a}$$

$$[I] = [T]^{-1} \operatorname{diag} [\widehat{W}_0 \ \widehat{W}_1 \ \widehat{W}_1] \ [T] \ [D_{g1}] \ [U_e] \tag{6b}$$

$$0 \le x \le 1_r$$

Transfer Matrix Functions

In terms of electrical network theory, we can define the transfer matrix functions of Eq. (6)

$$[H_{g1}] = [T]^{-1} \operatorname{diag} [W_0 \ W_1 \ W_1] \ [T] \ [D_{g1}] \tag{7a}$$

$$[\hat{H}_{g1}] = [T]^{-1} \operatorname{diag} [\widehat{W}_0 \ \widehat{W}_1 \ \widehat{W}_1] \ [T] \ [D_{g1}] \tag{7b}$$

$$0 \le x \le 1_r$$

Similarly, when the exciting source is a current source, and

$$[I_e] = [I_{kb} \ I_{kc}]^T \quad .$$

the solutions are

$$[I] = [T]^{-1} \operatorname{diag} [V_0 \ V_1 \ V_1] \ [T] \ [D_{b2}] \ [I_e] \tag{8a}$$

$$[U] = [T]^{-1} \operatorname{diag} [\hat{V}_0 \ \hat{V}_1 \ \hat{V}_1] \ [T] \ [D_{b2}] \ [I_e] \tag{8b}$$

$$0 \le x \le 1_r$$

We define the transfer matrix functions as

$$[H_{b2}] = [T]^{-1} \operatorname{diag} [V_0 \ V_1 \ V_1] \ [T] \ [D_{b2}] \tag{9a}$$

$$[\hat{H}_{b2}] = [T]^{-1} \operatorname{diag} [\hat{V}_0 \ \hat{V}_1 \ \hat{V}_1] \ [T] \ [D_{b2}] \tag{9b}$$

$$0 \le x \le 1_r$$

Where

$$[D_{b2}] = \begin{bmatrix} (J_0 - J_1)/(2J_0 + J_1) & (J_0 - J_1)/(2J_0 + J_1) \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$J_0 = (K_1 Q'_1 - K'_1 Q_1) \, (K_0 + Q_0) \, (K'_0 + Q'_0) \, (Z_{w1} + Z'_{w1})$$

$$J_1 = (K_0 Q'_0 - K'_0 Q_0) \, (K_1 + Q_1) \, (K'_1 + Q'_1) \, (Z_{w0} + Z'_{w0})$$

$$V_k = (Q_k \exp(\tau_k x) + K_k \exp(-\tau_k x)) / (Q_k + K_k)$$

$$\hat{V}_k = Z_{wk} \, (-Q_k \exp(\tau_k x) + K_k \exp(-\tau_k x)) / (Q_k + K_k)$$

$$Q_k = (Z_{edk}^{-1} \, Z_{wk} - 1) \exp(-\tau_k 1_r)$$

$$K_k = (Z_{edk}^{-1} Z_{wk} + 1) \exp(\tau_k l_r)$$
$$Q'_k = (-(Z'_{edk})^{-1} Z'_{wk} - 1) \exp(\tau'_k l_s)$$
$$K'_k = (-(Z'_{edk})^{-1} Z'_{wk} + 1) \exp(-\tau'_k l_s)$$
$$k=0, 1$$

In the same way, we can also define other transfer matrix functions when other $[U_e]$ and $[I_e]$ are connected.

Solutions In Time-Domain

From Eq. (6), the voltage and current of exciting phase are expressed as

$$U_a = U_{fa} \ (P_0 W_0 + 2P_1 W_1) \ / \ (2P_1 + P_0) \tag{10a}$$

$$I_a = U_{fa} \ (P_0 \widehat{W}_0 + 2P_1 \widehat{W}_1) \ / \ (2P_1 + P_0) \tag{10b}$$

Where

$U_{fa} = (w\cos\phi + s \sin\phi) \ / \ (s^2 + w^2)$

w ________ working frequency

ϕ ________ phase angle

For short,

$P(s) = 2P_1 + P_0$

$E(s) = \frac{d}{ds} \ ((s^2 + w^2) \ P(s))$

$A_k(s) = (w \cos\phi + s \sin\phi) \ Q_k \ P_k \ (k+1)/(Q_k + K_k)$

$B_k(s) = A_k(s) K_k / Q_k \qquad k=0, 1$

On the basis of residue theorem, the inverse-transform forms of Eq. (10) are (see Appendix):

$$u_a(x, t) = \sum_{k=0}^{1} (u_k^+(x, t) + u_k^-(x, t)) + \sum_{k=0}^{1} \sum_{n=0}^{\infty} (u_{kn}^+(x, t) + u_{kn}^-(x, t))$$
$$= u_{as}(x, t) + u_{at}(x, t) \tag{11a}$$

$$i_a(x, t) = \sum_{k=0}^{1} (i_k^+(x, t) - i_k^-(x, t)) + \sum_{k=0}^{1} \sum_{n=0}^{\infty} (i_{kn}^+(x, t) - i_{kn}^-(x, t))$$
$$= i_{as}(x, t) + i_{at}(x, t) \tag{11b}$$
$$0 \le x \le l_r$$

Where

$u_k^+(x, t) = v_k^+ \exp(-a_k x) \cos(wt - b_k x + p_k)$

$u_k^-(x, t) = v_k^- \exp(+a_k x) \cos(wt + b_k x + q_k)$

$i^{+}_{k}(x,t) = (v^{+}_{k}/z_k)\exp(-a_k x)\cos(wt-b_k x+p_k-d_k)$

$i^{-}_{k}(x,t) = (v^{-}_{k}/z_k)\exp(+a_k x)\cos(wt+b_k x+q_k-d_k)$

$u^{+}_{kn}(x,t) = v^{+}_{kn}\exp(-a_{kn}x)\exp(-\alpha_n t)\cos(\beta_n t-b_{kn}x+p_{kn})$

$u^{-}_{kn}(x,t) = v^{-}_{kn}\exp(+a_{kn}x)\exp(-\alpha_n t)\cos(\beta_n t+b_{kn}x+q_{kn})$

$i^{+}_{kn}(x,t) = (v^{+}_{kn}/z_{kn})\exp(-a_{kn}x)\exp(-\alpha_n t)\cos(\beta_n t-b_{kn}x+p_{kn}-d_{kn})$

$i^{-}_{kn}(x,t) = (v^{-}_{kn}/z_{kn})\exp(+a_{kn}x)\exp(-\alpha_n t)\cos(\beta_n t+b_{kn}x+q_{kn}-d_{kn})$

$u_{as}(x,t)$, $i_{as}(x,t)$ ——— steady state voltage and current of phase A.

$u_{at}(x,t)$, $i_{at}(x,t)$ ——— transient voltage and current of phase A.

All the above physical factors are satisfied with the following complex-number equations.

$$P(-\alpha_n \pm j\beta_n) = 0$$

$$v^{+}_{k}\exp(jp_k) = A_k(jw)/E(jw)$$

$$v^{-}_{k}\exp(jq_k) = B_k(jw)/E(jw)$$

$$v^{+}_{kn}\exp(jp_{kn}) = A_{kn}(-\alpha_n+j\beta_n)/E(-\alpha_n+j\beta_n)$$

$$v^{-}_{kn}\exp(jq_{kn}) = B_{kn}(-\alpha_n+j\beta_n)/E(-\alpha_n+j\beta_n)$$

$$a_k + jb_k = ((R_k+jwL_k)(G_k+jwC_k))^{1/2}$$

$$a_{kn}+jb_{kn} = ((R_k+(-\alpha_n+j\beta_n)L_k)(G_k+(-\alpha_n+j\beta_n)C_k))^{1/2}$$

$$z_k\exp(jd_k) = ((R_k+jwL_k)/(G_k+jwC_k))^{1/2}$$

$$z_{kn}\exp(jd_{kn}) = ((R_k+(-\alpha_n+j\beta_n)L_k)/(G_k+(-\alpha_n+j\beta_n)C_k))^{1/2}$$

EXAMPLE

The results obtained with the use of the solution in this paper compare favorably with those obtained by G. W. Swiftn [1] for a case in which H. W. Dommel's EMTP program [2] was used.

Using our system of units and coordinates (see Figure 1) the original parameters given by Swiftn are rewritten as follows:

$u_{fa} = 500\ 2^{1/2}/3^{1/2}\sin(377t+90^{o})$ Kv,

$l_r = 386.225$ Km, $1_s = 32.185$ Km, $L_1 = 8.7*10^{-7}$ H/m,

$R_1 = 2.5*10^{-5}\,\Omega$/m, $C_1 = 1.296*10^{-11}$ F/m, $G_1 = 0$ ℧/m,

$L_0 = 3.33*10^{-6}$ H/m, $R_0 = 2.79*10^{-4}\,\Omega$/m, $C_0 = 7.0*10^{-12}$ F/m,

$G_0 = 0$ ℧/m, $r_1 = r_2 = 0\Omega$, $L_{e1} = 0.5L_{e2} = 0.531$ H.

and the distance from observation point to exciting source is 0.8 l_r.

The first twelve terms of Eq. (11a) are taken and the result is shown in Figure 2.

For this example of 'A' – earth solid fault, the two curves do not vary significantly.

PHYSICAL CONCEPTS

Natural Frequency

In order to arrive at the form of Eq. (10), it is necessary to solve the equation for the eigenvalues s_n:

$$P(s) = 0 \tag{12}$$

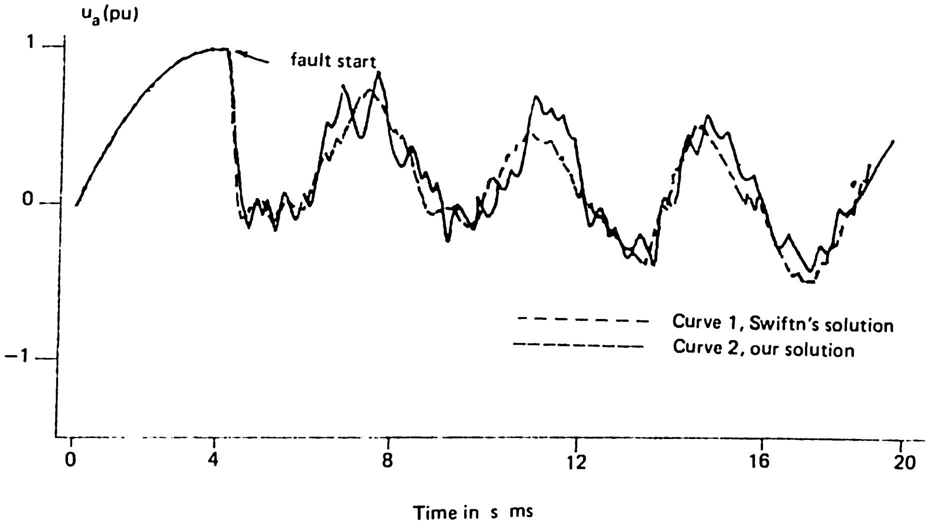

Figure 2. Comparison of Curves

By using an appropriate numerical procedure, we easily obtain the roots of Eq. (12):

$$s_n = -\alpha_n \pm j\beta_n$$
$$n = 0, 1, 2, 3 \ldots \infty$$

According to network theory, s_n is known as the natural frequency of the system. From Eq. (11), we know that the physical dimensions of α_n and β_n are all the inverse of time, that is, a frequency. The inverse of α_n is known as time-constant and β_n as eigenfrequency.

In the example above, the eigenfrequencies in rad./s. of the system are:

$\beta_0=0$ $\beta_1=1451.5$, $\beta_2=1921.3$, $\beta_3=2957.1$,
$\beta_4=4617.3$, $\beta_5=6539.6$, $\beta_6=7977.1$, $\beta_7=8852.7$,

The Transient Relationship Among Phases

1) *In the case of a symmetrical fault*

The exciting source is

$$[U_e] = [U_{fa}\ U_{fb}\ U_{fc}]^T$$

The solutions are

$$[U] = \text{diag}\ [W_1\ W_1\ W_1]\ [U_{fa}\ U_{fb}\ U_{fc}]^T \tag{13a}$$

$$[I] = \text{diag}\ [\hat{W}_1\ \hat{W}_1\ \hat{W}_1]\ [U_{fa}\ U_{fb}\ U_{fc}]^T \tag{13b}$$

The decoupled forms make the expression of every phase very simple. From this, we have mathematically proven that the transient research of symmetrical fault could be done by solving a single-phase system.

2) *In the case of a 'B, C' — solid fault*

The exciting source is

$$[U_e] = [U_{fb} \; U_{fc}]^T$$

The solutions are

$$\begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ W_1/2 & -W_1/2 \\ -W_1/2 & W_1/2 \end{bmatrix} \begin{bmatrix} U_{fb} \\ \\ U_{fc} \end{bmatrix} \qquad (14a)$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \hat{W}_1/2 & -\hat{W}_1/2 \\ -\hat{W}_1/2 & \hat{W}_1/2 \end{bmatrix} \begin{bmatrix} U_{fb} \\ \\ U_{fc} \end{bmatrix} \qquad (14b)$$

$$0 \leq x \leq 1_r$$

Obviously

$u_a\,(x, t) = i_a\,(x, t) = 0,$

$u_b\,(x, t) = -u_c\,(x, t) \;\; i_b\,(x, t) = -i_c\,(x, t)$

Comparing Eq. (13) to Eq. (14), we can see the simple relationship between symmetrical and two-phase solid fault.

3) *In the case of 'B, C' — earth solid fault*

The exciting source is

$$[U_e] = [U_{fb} \; U_{fc}]^T$$

The solutions are

$$\begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix} = (2P_0 + P_1)^{-1} \begin{bmatrix} P_0W_0 - P_1W_1 & P_0W_0 - P_1W_1 \\ P_0W_0 + P_1W_1 + P_0W_1 & P_0W_0 - P_0W_1 \\ P_0W_0 - P_0W_1 & P_0W_0 + P_1W_1 + P_0W_1 \end{bmatrix} \begin{bmatrix} U_{fb} \\ U_{fc} \end{bmatrix} \qquad (15a)$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = (2P_0 + P_1)^{-1} \begin{bmatrix} P_0\hat{W}_0 - P_1\hat{W}_1 & P_0\hat{W}_0 - P_1\hat{W}_1 \\ P_0 W_0 + P_1\hat{W}_1 + P_0\hat{W}_1 & P_0\hat{W}_0 - P_0\hat{W}_1 \\ P_0\hat{W}_0 - P_0\hat{W}_1 & P_0\hat{W}_0 + P_1\hat{W}_1 + P_0\hat{W}_1 \end{bmatrix} \begin{bmatrix} U_{fb} \\ U_{fc} \end{bmatrix} \qquad (15b)$$

$$0 \leq x \leq 1_r$$

After considering the constraints due to the characteristic equations:

$$2P_0 + P_1 = 0,$$

we obtain

$u_{bt}\,(x, t) = u_{ct}\,(x, t), i_{bt}\,(x, t) = i_{ct}\,(x, t).$

That is, transient components of the two faulted phases are equal.

4) *In the case of 'A' —earth solid fault*

From Eq. (6), we have

$$\begin{bmatrix} U_a \\ U_b \\ U_c \end{bmatrix} = (P_0 + 2P_1)^{-1} \begin{bmatrix} P_0W_0 + 2P_1W_1 \\ P_0W_0 - P_1W_1 \\ P_0W_0 - P_1W_1 \end{bmatrix} U_{fa} \tag{16a}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = (P_0 + 2P_1)^{-1} \begin{bmatrix} P_0\hat{W}_0 + 2P_1\hat{W}_1 \\ P_0\hat{W}_0 - P_1\hat{W}_1 \\ P_0\hat{W}_0 - P_1\hat{W}_1 \end{bmatrix} U_{fa} \tag{16b}$$

$$0 \leq x \leq 1_r$$

Obviously

$$u_b(x, t) = u_c(x, t) \quad i_b(x, t) = i_c(x, t)$$

Transients Effected By Lossy, Lossless and Frequency-Dependent Parameters

1) *Eigenfrequencies*

The eigenfrequencies of a lossy-line is lower than that of a lossless-line, and a lossy-line in which the frequency-dependent parameters are considered has the lowest eigenfrequencies.

2) *Attenuation*

In a lossless-line, every transient travelling-wave does not attenuate with time, but does so in a lossy-line. After considering frequency-dependent parameters, it attenuates faster.

3) *Speed*

The speed of every transient travelling-wave is defined as

$$v_{kn} = \beta_n / b_{kn}$$

The speed of every transient travelling-wave in a lossless-line is the same; in a lossy-line, the speed is frequency dependent. It is the slowest when the frequency-dependent parameters are considered in a lossy-line.

4) *Amplitude*

The amplitude of every transient travelling-wave in a lossy-line is higher than that of a lossless-line. It is the highest when we consider the frequency-dependent parameters.

CONCLUSIONS

The assumed conditions and the method of solution in this paper are strict. The mathematical answers describing transient travelling-wave group are satisfied with partial differential equations. The forms of answer for both current-source exciting system and voltage-source exciting system are similar, in which both voltage and current exhibit duality. We can accurately give the natural frequencies of a nonsymmetrical exciting system by using computer methods. Every physical factor of transient travelling-wave is affected by frequency-dependent parameters, which are easily considered in the solution.

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REFERENCES

1. SWIFTN, G. W., "The Spectra of Fault Induced Transient." *IEEE Transactions on Power Apparatus and System,* vol. PAS-98, May/June 1979, pp. 940-47.
2. DOMMEL, H. W., "Digital Computer Solution of Electromagnetic Transients in Single and Multiphase Networks." *IEEE Transactions on Power Apparatus and System,* vol. PAS-88, April 1969, pp. 388-99.
3. JOHNS, A. T., "Digital Simulation of Faulted E. H. V. Transmission-lines With Particular Reference to Very-High-Speed Protection." *Proc. IEEE,* vol. 123, No. 4, April 1976, pp. 353-59.
4. GETSINGER, W., "Analysis of Certain Transmission Line Networks in The Time-Domain." *IRE Trans. MTT-8,* May 1960, pp. 301-09.
5. GHAUSI, M. S., and HERSKOWITZ, G. J., "The Effect of Shaping and Loading on Distributed RC Lines." J. Franklin Institute, vol. 278, 1964, pp. 108-23.
6. WEDEPONL, L. W., "Application of Matrix Methods to the Solution of Travelling-Wave Phenomena in Polyphases Systems." *Proc. IEEE,* vol. 110, No. 12, Dec. 1963.
7. DOMMEL, H. W., "Computation of Electromagnetic Transients." *Proc. IEEE,* No. 7, 1974.

APPENDIX

Verification of the Correctness of the General Solutions

1. *S-Domain*

From Eq. (16), we have

$$-\frac{d[U]}{dx} = -\frac{U_{fa}}{P_0+2P_1}\begin{bmatrix} P_0 & P_1 & P_1 \\ P_0 & 0 & -P_1 \\ P_0 & -P_1 & 0 \end{bmatrix} \frac{d}{dx}\begin{bmatrix} W_0 \\ W_1 \\ W_1 \end{bmatrix} =$$

$$= \frac{-U_{fa}}{P_0+2P_1}\begin{bmatrix} P_0 & P_1 & P_1 \\ P_0 & 0 & -P_1 \\ P_0 & -P_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \tau_0 Q_0 \exp(\tau_0 x) & -\tau_0 K_0 \exp(-\tau_0 x) \\ \tau_1 Q_1 \exp(\tau_1 x) & -\tau_1 K_1 \exp(-\tau_1 x) \\ \tau_1 Q_1 \exp(\tau_1 x) & -\tau_1 K_1 \exp(-\tau_1 x) \end{bmatrix} =$$

$$= \frac{U_{fa}}{P_0+2P_1}\begin{bmatrix} Z_{d0} P_0 \hat{W}_0 + 2Z_{d1} P_1 \hat{W}_1 \\ Z_{d0} P_0 \hat{W}_0 - Z_{d1} P_1 \hat{W}_1 \\ Z_{d0} P_0 \hat{W}_0 - Z_{d1} P_1 \hat{W}_1 \end{bmatrix} =$$

$$= \begin{bmatrix} R_s + sL_s & R_m + sL_m & R_m + sL_m \\ R_m + sL_m & R_s + sL_s & R_m + sL_m \\ R_m + sL_m & R_m + sL_m & R_s + sL_s \end{bmatrix} \frac{U_{fa}}{P_0+2P_1}\begin{bmatrix} P_0\hat{W}_0 + 2P_1\hat{W}_1 \\ P_0\hat{W}_0 - P_1\hat{W}_1 \\ P_0\hat{W}_0 - P_1\hat{W}_1 \end{bmatrix} =$$

$$= [Z]\ [I]\ .$$

Similarly, we can also prove

$$-\frac{d[I]}{dx} = [Y]\ [U]$$

Combining the two results enables us to obtain Eq. (3).

2) *Time-Domain*

When the exciting sources are Symmetrical, the answers are decoupled, see Eq. (13). Therefore, the partial differential equation of each phase can be written as

$$-\frac{\partial u}{\partial x} = L_1 \frac{\partial i}{\partial t} + R_1 i \tag{17a}$$

$$-\frac{\partial i}{\partial x} = C_1 \frac{\partial u}{\partial t} + G_1 u \tag{17b}$$

From Eq. (13), we can obtain their answers, u (x, t) and i (x, t).

Substitute $\partial u / \partial x$, $\partial i / \partial x$ and i (x, t) into Eq. (17), comparing with the coefficients, we can show that

$$(-(R_1 - \alpha_n L_1) \sin d_n + \beta_n L_1 \cos d_n) / z_n = b_n \tag{18a}$$

$$(+(R_1 - \alpha_n L_1) \cos d_n + \beta_n L_1 \sin d_n) / z_n = a_n \tag{18b}$$

and

$$(-R_1 \sin d + wL_1 \cos d) / z = b \tag{19a}$$

$$(+R_1 \cos d + wL_1 \sin d) / z = a \tag{19b}$$

Since

$$a_n + jb_n = ((R_1 + (-\alpha_n + j\beta_n) L_1)(G_1 + (-\alpha_n + j\beta_n) C_1))^{1/2}$$

$$z_n \exp(jd_n) = (((R_1 - \alpha_n L_1) + j\beta_n L_1) / ((G_1 - \alpha_n C_1) + j\beta_n C_1))^{1/2}$$

we have

$$z_n = (((R_1 - \alpha_n L_1)^2 + (\beta_n L_1)^2) / ((G_1 - \alpha_n C_1)^2 + (\beta_n C_1)^2))^{1/4}$$

$$\sin d_n = (0.5 g^{1/2} - 0.5 (h+y))^{1/2} / g^{1/4}$$

$$a_n = (0.5 g^{1/2} + 0.5 (h-y))^{1/2}$$

$$\cos d_n = (0.5 g^{1/2} + 0.5 (h+y))^{1/2} / g^{1/4}$$

$$b_n = (0.5 g^{1/2} + - 0.5 (h-y))^{1/2}$$

Where

$$g = ((R_1 - \alpha_n L_1)^2 + (\beta_n L_1)^2)((G_1 - \alpha_n C_1)^2 + (\beta_n C_1)^2)$$

$$h = (R_1 - \alpha_n L_1)(G_1 - \alpha_n C_1)$$

$$y = \beta_n^2 L_1 C_1$$

Substitute them into Eq. (18), they are tenable.

By using the same way, Eq. (19) is also proven to be tenable.

Verification of the Correctness of the Particular Solutions

Correct particular solutions must satisfy the boundary conditions and initial conditions.

1) *Boundary Conditions*

Prove:

From Eq. (13a), we have

$$U_a = U_{fa} (Q_1 \exp(\tau_1 x) + K_1 \exp(-\tau_1 x)) / (Q_1 + K_1)$$

Let x = 0, obtain $U_a = U_{fa}$

That is, when x = 0, the answer is certainly the exciting source itself.

Similarly,

$$[U_r] = [Z_e]\ [I_r] \text{ and } [U_s] = -[Z'_e]\ [I_s]$$

are also tenable.

2) *Initial Condition*

For all situations, the answers in the time-domain satisfy the initial conditions.

An example is shown in Figure 3. The u and i are zero before 1.4 ms because the distance from an observer to exciting source is about 400 kilometers. This means the initial values are zero before travelling-waves arriving.

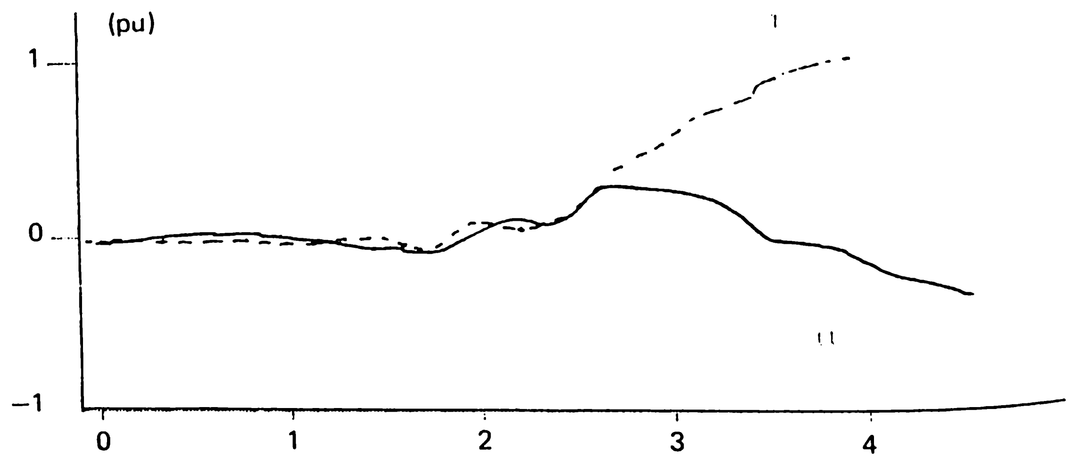

Figure 3. Zero-Initial Value

Derivation of Inverse S-Transform:

The Residue Theorem is defined as:

If $U(s) = N(s)/D(s)$

and the eigenvalues of its characteristics equation are

$$s_n^+ = -\alpha_n + j\beta_n$$
$$s_n^- = -\alpha_n - j\beta_n \qquad n = 0, 1, 2, 3, \ldots \infty$$

then,

$$u(t) = \sum_{n=0}^{\infty} \frac{N(S_n^+)}{D'(S_n^+)} \exp(S_n^+ t) + \sum_{n=0}^{\infty} \frac{N(S_n^-)}{D'(S_n^-)} \exp(S_n^- t).$$

where, $D'(^*)$ – The derivative of $D(^*)$.

For convenience, the simplest case, symmetrical fault on 3-phase, will be considered here.

In this case, voltage of phase A in S-domain is

$$U_a(x,s) = U_a(0,s)\,[Z_e(s)\,\tau(s) - \cosh(1-x)\,\tau(s) + Z(s)\,\sinh((1-x)\,\tau(s))] / [Z_e(s)\,\tau(s)\cosh(1\,\tau(s)) + Z(s)\,\sinh(1\,\tau(s))] =$$

$$= U_a(0,s)\,[N_1(s)\exp(x\,\tau(s)) + N_2(s)\exp(-x\,\tau(s))]/D(s).$$

Where,

$$N_1(s) = [Z_e(s)\,\tau(s) - Z(s)]\exp(-1\,\tau(s)),$$

$$N_2(s) = [Z_e(s)\,\tau(s) + Z(s)]\exp(1\,\tau(s)),$$

$$D(s) = [Z_e(s)\,\tau(s) + Z(s)]\exp(1\tau(s)) + [Z_e(s)\,\tau(s) - Z(s)]\exp(-1\,r(s)).$$

We obtain the eigenvalues from D (s) = 0, they are:

$$s_n^+ = -\alpha_n + j\beta_n$$

$$s_n^- = -\alpha_n - j\beta_n \qquad n=0,1,2,3\ldots\infty$$

Based on the Residue Theorem, one can directly write the transient voltage as follows:

$$u_{at}(x,t) = \sum_{n=0}^{\infty}[U_a(0,s_n^+)\,[N_1(s_n^+)\exp(x\,\tau(s_n^+)) + N_2(s_n^+)\exp(-x\,\tau(s_n^+))]/D'(s_n^+)]\exp(s_n^+ t) + \sum_{n=0}^{\infty}[U_a(0,s_n^-)\,[N_1(s_n^-)\exp(x\,\tau(s_n^-)) + N_2(s_n^-)\exp(-x\,\tau(s_n^-))]/D'(s_n^-)]\exp(s_n^- t).$$

We define that,

$$(V_n^+/2)\exp(jp_n) = [U_a(0,s_n^+)\,N_1(s_n^+)]/D'(s_n^+),$$

$$(V_n^-/2)\exp(-jp_n) = [U_a(0,s_n^-)\,N_1(s_n^-)]/D'(s_n^-),$$

$$(V_n^+/2)\exp(jq_n) = [U_a(0,s_n^+)\,N_2(s_n^+)]/D'(s_n^+),$$

$$(V_n^-/2)\exp(-jq_n) = [U_a(0,s_n^-)\,N_2(s_n^-)]/D'(s_n^-),$$

$$a_n \pm jb_n = \tau(-\alpha_n + j\beta_n).$$

This results in that,

$$u_{at}(x,t) = \sum_{n=0}^{\infty}[(V_n^+/2)\exp(jp_n)\exp(x(a_n+jb_n)) + v_n^-\exp(jq_n)\exp(-x(a_n+jb_n))]\exp(-\alpha_n + j\beta_n)t + \sum_{n=0}^{\infty}[(v_n^+/2)\exp(-jp_n)\exp(x(a_n-jb_n)) + v_n^-\exp(-jq_n)\exp(-x(a_n-jb_n))]\exp(-\alpha_n - j\beta_n)t$$

$$= \sum_{n=0}^{\infty}(v_n^+/2)\exp(a_n x)\,[\exp(jp_n + jb_n x + j\beta_n t) + \exp(-jp_n - jb_n x - j\beta_n t)]\exp(-\alpha_n t) +$$

$$\sum_{n=0}^{\infty} (v_n^-/2) \exp(-a_n x) \, [\exp(jq_n - jb_n x + j\beta_n t) +$$

$$\exp(-jq_n + jb_n x - j\beta_n t)] \exp(-\alpha_n t)$$

$$= \sum_{n=0}^{\infty} v_n^+ \exp(a_n x) \cos(\beta_n t + b_n x + P_n) \exp(-\alpha_n t) +$$

$$\sum_{n=0}^{\infty} v_n^- \exp(-a_n x) \cos(\beta_n t - b_n x + q_n) \exp(-\alpha_n t)$$

$$= \sum_{n=0}^{\infty} [u_n^+(x, t) + u_n^-(x, t)] \, .$$

Where,

$u_n^+(x, t) = v_n^+ \exp(a_n x) \cos(\beta_n t + b_n x + p_n) \exp(-\alpha_n t)$.

$u_n^-(x, t) = v_n^- \exp(-a_n x) \cos(\beta_n t - b_n x + q_n) \exp(-\alpha_n t)$.

They are so called forward and backward direction travelling-wave respectively.