"Available correlations were obtained mostly through models that lump the effects of each significant mechanism of heat transfer in packed beds."

Heat Transfer Parameters for Two-Phase Models of Packed Beds at Low Reynolds Numbers

by
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ABSTRACT

A review, consolidation and summary of the available correlations and data from various literatures were made for the parameters representing the mechanisms of heat transfer for fluid flows in packed beds at low Reynolds numbers. These parameters are the axial dispersion coefficient, the effective solid phase conductivity, the particle and effective heat transfer coefficient. Values of these parameters are required in the two-phase models of packed bed. Due to lack of data from heat transfer experiments, the possibility of obtaining these parameters from mass transfer experiments was explored on the assumption of direct analogy between mass and heat transfer.

INTRODUCTION

When a fluid flows through a packed bed of solid particles, the transfer of heat is due to a number of interacting mechanisms in the fluid phase, in the solid phase and in the stagnant film surrounding the particles. In the fluid phase, heat is transferred due to the mechanisms of molecular conduction and eddy-mixing of the fluid. These mechanisms are represented by a single-parameter called axial dispersion coefficient. In the solid phase, transfer of heat is due to the mechanisms of solid conduction within the particle and, to a lesser extent, of particle-to-particle conduction. Heat transfer across the stagnant film surrounding the particle is then represented by the usual convective heat transfer coefficient.

Models which consider the exchange of heat occurring between the solid and fluid phases are called two-phase models. The more realistic two-phase models which consider the above mechanisms separately are the Extended Rosen Model [35] (ERM) and the Conduction-Dispersion Model [29] (CDM). These two models differ in the treatment of the mechanisms of heat transfer in the solid phase. In the ERM, the solid thermal conductivity represents the solid phase mechanism of heat transfer. The CDM, on the other hand, makes use of an effective solid phase conductivity to represent inter-particle and intra-particle conduction. On the assumption of solid phase continuity, it may also include conduction through a stagnant fluid connecting the particles. In the casé of ERM, this conductivity as well as inter-particle conductivity, if significant, may only be lumped together with the axial dispersion coefficient.

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In the pioneering work of Schumann [38], the mechanisms of heat transfer in the solid phase and the fluid phase are neglected. This approach essentially leads to an effective heat transfer coefficient which lumps together the effects of the mechanisms in the solid and liquid phases. The effective heat transfer can then become considerably different from the particle heat transfer coefficient especially at low Reynolds numbers.

In this paper, a review, consolidation and summary of the available correlations and data from various literatures were made for the above parameters representing the mechanisms of heat transfer; namely, the axial dispersion coefficient, the effective solid phase conductivity and the particle and effective heat transfer coefficient. Values of these parameters are required in the above two-phase models of packed bed. Due to lack of data from heat transfer experiments, the possibility of obtaining these parameters from mass transfer experiments was explored on the assumption of direct analogy between mass and heat transfer.

AXIAL DISPERSION IN THE FLUID PHASE

The effects of longitudinal eddy-mixing and molecular diffusion of fluids in flow through packed beds are usually combined and generally characterized by a single parameter D_{ef}, the axial dispersion coefficient. This characterization leads to a dimensionless parameter, P_{ea}, the axial Peclet number defined as

$$P_{ea} = \frac{V_i d_p}{D_{ef}} \tag{2-1}$$

Mass Transfer Studies

In mass dispersion experiments, it is possible to measure fluid-phase dispersion independent of the other transfer mechanisms. This is done through the use of "inert" and "non-porous" particles involving no surface nor intra-particle diffusion. More consistent measurements of fluid phase dispersion are found in this "pure mixing approach".

Based on the results of mixing-cell theory and extensive data from the literature, Perkins and Johnston [31] arrived at the following equation describing longitudinal dispersion written here in terms of P_{ea} as

$$\frac{1}{P_{ea}} = \frac{(1/F_e)p}{(V_i d_p/D_f} + 0.5F_h \tag{2.1-1}$$

The term $1/F_e$ above is based on an analogy between electrical conductivity and porous media. It is equivalent to the well-known tortuousity parameter, F_t, which indicates the reduction of the effects of molecular conduction by virtue of the tortuous nature of the passages between the particles [15]. F_t is 0.7 for a typical randomly packed bed.

The factor F_h is a measure of the increased dispersion due to the packing caused by the differences in particle shapes and sizes and their distribution. F_h has a theoretical minimum value of 1 for regular packing and about 3.5 for a randomly packed bed.

Equation (2.1-1) is only valid for $V_i d_p/D_f$ less than 50. At higher values of $V_i d_p/D_f$, dispersion is greater than that which could be explained by the mixing-cell theory. Moreover, diffusion is not entirely equalizing concentration within each pore space. From this point, an increase in fluid dispersion may be expected, resulting in lower P_{ea}.

At higher particle Reynolds number (R_e); wherein eddy-mixing is the dominant mechanism of dispersion, values of P_{ea} are quite well established by the above "pure-mixing" approach. McHenry and Wilhelm [30] obtained a value of P_{ea} equal to 1.88 + 0.15 for R_e from 100 to 400 using frequency response techniques. Aris and Amundson [1] theoretically supported these findings when they obtained for high R_e numbers a limiting value of 2.

For comparison with equation (2.1-1), reported values [14] covering a wide range of R_e (0.008 to 50) for gases and large particles in the range (0.0377 to 0.60 cm) were plotted in Figure 2.1-1. In terms of P_{ea}, the correlating equation was obtained in the form [14]

$$\frac{1}{P_{ea}} = \frac{0.73\ {}_pD_f}{V_i d_p} + \frac{0.5}{1 + 9.7\ {}_pD_f/(V_i d_p)} \qquad (2.1\text{-}2)$$

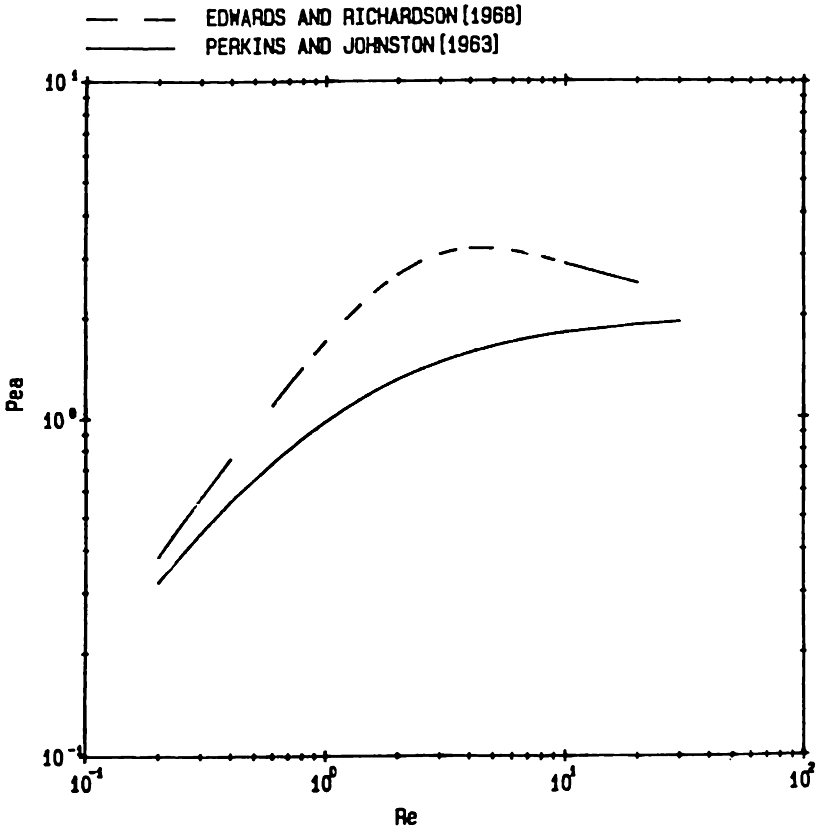

Figure 2.1-1. Axial Peclet Numbers for Gases: Pr-0.7, Fh = 1.0, Ft = 0.73

Figure 2.1-1 shows that the value of P_{ea} higher than the theoretical value of 2 is obtained from the above equation. This result was explained on the basis suggested by Hiby [25] and Giddings [17]. According to Edwards and Richardson [14], on the scale of particle size, a radial concentration gradient was set-up due to uneven flow through the pores of varying sizes. When issuing streams of different concentrations from the neighboring pores come into contact again, radial diffusion takes place, thus reducing longitudinal dispersion. This reduction in dispersion shown in their experiments was taken into account by introducing empirical factor $1 + 9.7{}_pD_f/(V_i d_p)$ in equation (2.1-1) and using an $F_t = 0.73$ and $F_h = 1$, resulting in equation (2.1-2).

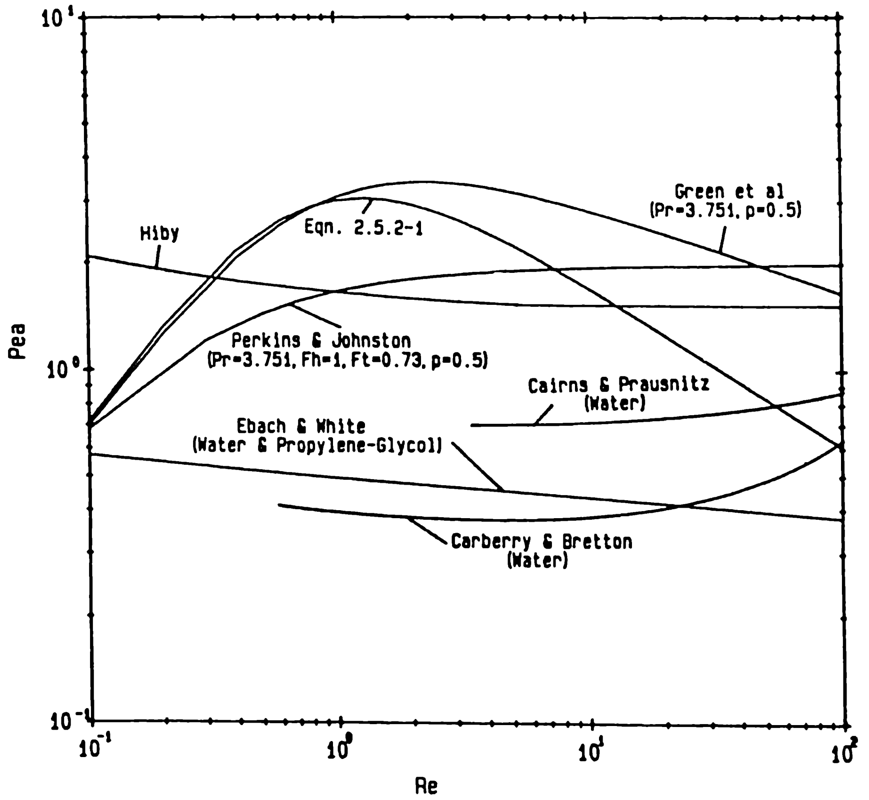

Figure 2.1-2. Axial Peclet Numbers for Liquids

A number of investigators have also studied dispersion of liquids and their results were summarized in the work of Green et al. [19], Babcock et al [3] and Edwards and Richardon [14]. among others. The reported values of P_{ea} for the case of liquid mass dispersion as shown in Figure 2.1-2 are smaller than the corresponding case of gas mass dispersion with the exception of Hiby's [25] data.

Green et al. [19] explained the above difference between liquid and gas dispersion based on the results of Taylor [39] for dispersion in flow through a capillary tube. Stagnant pockets are usually formed when a fluid flows through a packed bed. This by-passing process increases fluid phase dispersion, the dispersion becoming greater for smaller molecular diffusivity. Liquids which have smaller molecular diffusivities than gases will exhibit increased dispersion.

Following the above explanation, porous particles should result in lower fluid dispersion since in addition to molecular diffusion in the stagnant pockets, concentration gradients normal to the flow will be further diminished by intra-particle or pore diffusion. The decrease in dispersion, on the other hand, may be offset by the magnitude of surface diffusion if the particle is "active" as well.

Measurements of fluid phase dispersion in experiments involving "porous" and "active" particles were, however, only few. For gas-porous particle systems, Deisler and Wilhelm [12] using a closely similar analytical system and frequency response techniques as that later used in the "pure mixing" studies of McHenry and Wilhelm [30] obtained higher values of P_{ea}. Although, only pore diffusion and fluid phase dispersion were considered, their results indicate the likely effects of pore diffusion on fluid phase dispersion.

Chao and Hoelscher [10] using "active" particles obtained for gases at low R_e, values of P_{ea} were generally smaller than 2 when only surface diffusion was considered. Although no explanation was given to these results, they noted that the lowest possible values for pore diffusivity were assumed. An increase in the assumed value of pore diffusivity, however, would lead to a decrease in fluid phase dispersion and, thus, higher P_{ea}.

Mass transfer experiments involving "porous" and "active" particles are not as conclusive as those in "inert" and "non-porous" particles. Extension of their results to the case of heat transfer, therefore, cannot be made with confidence.

Heat Transfer Studies

In packed bed heat transfer, axial dispersion cannot be observed experimentally, independent of other heat transfer mechanisms. Except probably in the case wherein an "inert" particle with almost negligible thermal conductivity is used, no heat transfer study is directly comparable to that of mass transfer which involves only fluid dispersion. Thus, fluid phase dispersion coefficients for heat transfer will be expected to be smaller than the corresponding mass transfer case due to unavoidable inter- and intra-particle conduction.

Gunn and de Souza [20] and Gunn et al. [21], however, reported lower values of P_{ea} for gases than the "pure-mixing" mass transfer case for R_e less than 100. The explanation for this difference was made in terms of the relative magnitudes of the thermal and mass conductivities in the gas and solid phase. Since the thermal conductivity of the particles used in their study was larger than that of the fluid (air), Gunn et al. noted that higher fluid dispersion was obtained. This contradicts physically the explanation discussed previously as to the effect of solid phase conduction, i.e., conduction in the solid phase suppresses axial dispersion. It may also be pointed out that due to experimental difficulties and significant parameter interaction in R_e less than 100, Gunn et al. noted the scatter in their reported values.

For liquids, the only available data [19, 2] were obtained in terms of the effective axial dispersion coefficient, D_e, which, unlike the axial dispersion coefficient, D_{ef}, in equation (2-1), lumps together the effects of all the heat transfer mechanisms in the fluid phase, in the solid phase and in the stagnant film surrounding the particle. From these data, a correlating equation for D_m was derived and given in the form [19]

$$\frac{D_m}{D_f} = 0.115 \left(\frac{V_i d_p}{D_f}\right)^{1.25} \qquad (2.2\text{-}1)$$

The above equation as shown in Figure 2.1-2 results in higher values of P_{ea} than the "pure-mixing" liquid data of Ebach and White [13], Cairns and Prausnitz [8] and Carberry and Bretton [9]. It is about the same order of magnitude as the liquid data of Hiby and the predicted curve of equation (2.1-1) based on the mixing-cell theory with $F_h = 1$. Note also the similarity in shape between the predicted curve of equation (2.2-2) and that of equation (2.1-2) for the case of $Pr = 0.73$ as shown in Figure 2.1-1. This suggests that the experimental curves tend to show a region of theoretically unexpected high values of P_{ea} which for the case of $F_h = 1$ is higher than 2.

Babcock et al. [3] obtained more liquid data for various solid particles in terms of D_e and also derived values of D_m from them including the above mentioned data of Green et al. [19]. Values of P_{ea} obtained were similarly higher than mass transfer data. A correlation for D_m was not given. Correlation of the above only source of liquid heat transfer data was attempted later [6,7]. The correlating equation was obtained similar in form to equation (2.2-2).

$$\frac{D_m}{D_f} = 0.078 \left(\frac{V_i d_p}{D_f}\right)^{1.46} \qquad (2.2\text{-}3)$$

FLUID-PARTICLE HEAT TRANSFER

The enormous amount of data on heat transfer coefficients found in the literature has been reviewed extensively by various investigators [5,24,4]. In these reviews, a surprisingly large amount of scatter was noted and was generally attributed to differences in the definition of parameters and the experimental techniques.

At low R_e, the discrepancies between reported results become more significant and it appears that different limiting values of Nusselt or Sherwood numbers are approached as the R_e goes to zero. Various investigators have attempted to explain this behavior of packed beds at low R_e. Glicksmann and Joos [18] critically reviewed their attempts and noted that low values of Sherwood or Nusselt numbers approaching zero at low R_e result when the effective Sherwood number (S_h') or effective Nusselt number (N_u') is based on the overall concentration or temperature change across the bed and the total surface area of the particles in the bed. Unlike the particle Sherwood numbers (S_h) or particle Nusselt numbers (N_u) which are based on local concentration or temperature differences, S_h' or N_u' will be influenced by bed non-uniformities and back-mixing. Glicksmann and Joos demonstrated, using a steady-state axial dispersion model which involves an effective axial dispersion and fluid-particle transfer coefficients as parameters, that the presence of dispersion results in values of S_h' lower than Sherwood numbers for a single particle in an infinite media.

A similar conclusion was obtained earlier by Gunn and de Souza [20] who showed that neglecting axial dispersion results in an expression for Nusselt numbers that generally represent the trend of the reported low values of Nusselt numbers.

It is, therefore, very important that available data or correlations for particle-fluid transfer coefficients be carefully identified as to being particle or effective values. Some models proposed to describe packed bed dynamics consider separately the effects of each significant mechanism of heat or mass transfer in packed beds. In these models, the correct parameter is the N_u or S_h. Examples of these models are the ERM and the CDM. In models such as the Schumann model wherein the heat transfer coefficient is used as the sole parameter to describe heat transfer, the N_u' or S_h' must be used.

Direct Analogy Between Heat and Mass Transfer

In terms of the J-factors, analogy between heat and mass transfer can be expressed as [11,16]

$$J_h = \frac{N_u P_r^{2/3}}{P_e} \tag{3.1-1}$$

$$J_d = \frac{S_h S_c^{2/3}}{P_e} \tag{3.1-2}$$

Theoretically, the above analogy is valid only for particle Sherwood and Nusselt numbers which represent solely the effects of fluid-particle mass or heat transfer rates. It cannot be safely extended to analysis in terms of effective Nusselt number or Sherwood numbers since the effects of the other significant mechanisms of heat and mass transfer may be present and the relative importance of these effects may be different for heat and mass transfer.

Particle Nusselt Number

Heat or mass transfer experiments on packed beds were mostly done by measuring temperature or concentration change across the length of the bed. These measurements in most cases will, therefore, result in effective values of Nusselt or Sherwood numbers (N_u' and S_h') unless the effects of the other mechanisms for heat and mass transfer are very small or, if significant, are taken into account. Unfortunately, the only available data which may closely approximate particle Sherwood numbers are those obtained from liquid mass transfer experiments. In mass transfer experiments, the only other significant parameter is fluid phase dispersion which, in the case of liquids, is very small. Examination of available mass transfer data for liquids indicates a finite constant limiting value as shown in Figure 3.2-1.

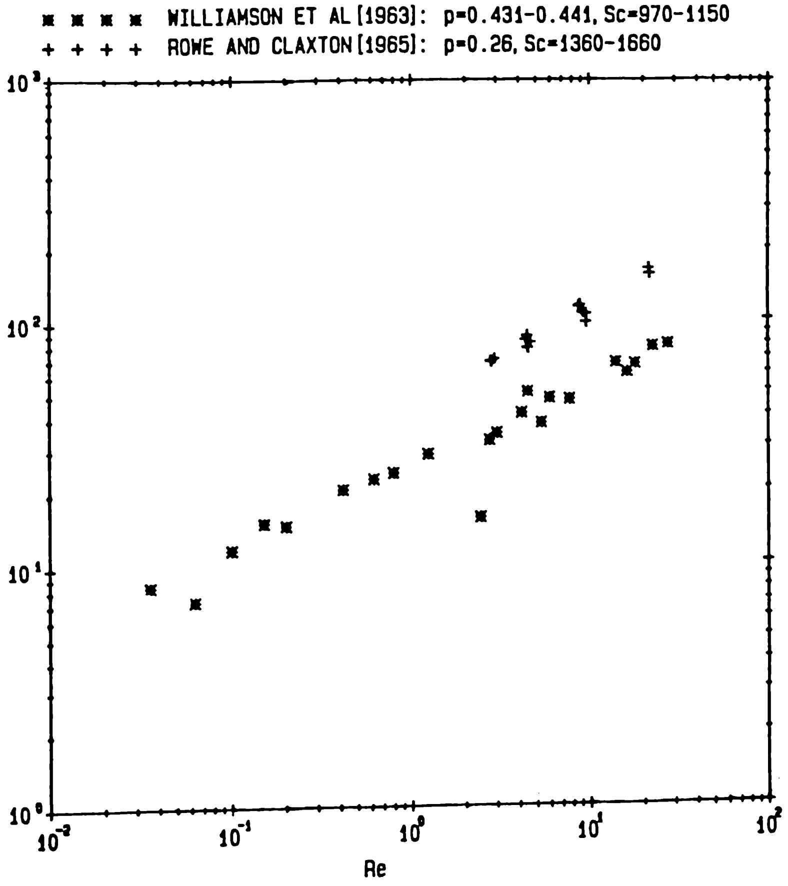

Figure 3.2-1. Sherwood Numbers for Liquids

In contrast, gas mass transfer data [26] based on concentration change across the bed have indicated a fairly large discrepancy although a limiting value towards zero appears to be being approached. This discrepancy may be attributed to the fact that for gas systems, fluid dispersion plays a more significant role at low R_e. The situation is made complicated by pore diffusion in the case of mass transfer involving "porous" and "active" particles or, in the case of heat transfer, by inter- and intra-particle conduction. Techniques which take into account these additional mechanisms for mass or heat transfer had been therefore attempted. The frequency or pulse response methods used by some investigators [8,20,21,29,34] in which all significant parameters are simultaneously obtained had resulted, unfortunately, in a fairly large scatter and no correlating equation was obtained. The results, however, indicate fairly large values of Nusselt numbers at low R_e suggesting a finite constant limiting value.

The method involving measurements of Nusselt or Sherwood numbers for a single "active" sphere in a packed bed of spheres appears to be the only direct method that can closely result in values of N_u or S_h. Rowe and Claxton [36] obtained values of Nusselt and Sherwood numbers at low and high R_e for water and air but only mass transfer measurements for water at low R_e were taken. Their measured values of S_h at low R_e were in good agreement with previously reported Sherwood numbers for liquid at low R_e. This suggests that for liquid mass transfer, the experimental technique is not so critical unlike in gas mass transfer and in liquid or gas heat transfer.

By combining their results for low and high R_e with previous results for a single particle in an infinite media and the limiting value suggested by Zabdrodsky [41] and Rowe [37], Rowe and Claxton obtained a correlating equation in the form

$$N_u \text{ (or } S_h) = A_p + B_p P_r^{1/3} (\text{or } S_h^{1/3}) R_e^n \qquad (3.2\text{-}1)$$

Pffefer and Happel [32] also derived theoretically N_u or S_h at low Peclet numbers using the so-called "free surfance model" of Happel [23], similar to the picture of packed beds proposed by Zabdrodsky. They solved the energy equation applied to the fluid shell assumed to move past the sphere with creeping motion. Examination of their tabulated Nusselt numbers for p greater or equal to 0.422 and extrapolated values for p less than 0.422 shows good agreement with the limiting values given by equation (3.3-2). For p less than 0.422, equation (3.2-2) predicts slightly higher values.

At higher values of P_r, Pffefer and Happel found that N_u (or S_h) is only a function of R_e for a given p and in terms of J-factors

$$J_h \text{ (or } J_d) = B_e R_e^{-2/3} \qquad (3.2\text{-}3)$$

Similar results were obtained by Pffefer [33] for high P_e when the thin boundary solution of Levich [28] was combined with "free surface" model in which B_e was obtained as a function of p as

$$B_e = 1.26 \ \frac{1 - (1-p)^{5/3} \ {}^{1/3}}{W} \qquad (3.2\text{-}4)$$

For high S_c or P_r, Figure 3.2-2 shows excellent agreement between liquid mass transfer data and the theoretical and empirical equations given by Rowe and Claxton [36] and Pffefer and Happel [32].

J-factors for gases (P_r = 0.7) were also calculated and plotted in Figure 3.2-2. Unfortunately, no data are available for gas J_h or J_d which can be used to verify the validity of these predicted values. Comparison of the "free surface" model results with the empirical equation of Rowe and Claxton shows good agreement at low R_e ($<$10). At R_e greater than around 10, the latter equation deviates significantly from the model. The empirical equation of Rowe and Claxton physically should yield the proper order of N_u or S_h since previous results for single particle measurements for both gas and liquids and their own measurements at high R_e were used in its formulation.

Effective Nusselt Numbers

Unlike the particle heat transfer coefficients, h, the effective heat transfer coefficients, h′, is affected by diffusion and to a large extent by the length of the bed. Kato et al. [27] examined available experimental data on mass transfer coefficient and found that the Sherwood number, presumably the effective Sherwood number, is also a function of the ratio of particle diameter and bed height. Based on their results and available literature data, they successfully obtained a correlation of S_h' against $R_e \, (d_p/L_b)^{0.6}$.

Based on a similar point of view, Kato and Wen [26] also correlated available data for gas heat transfer as follows

$$N_u' = 1.05P_r^{1/3}[R_e(d_p/L_b)]^{1.1} \qquad (3.3\text{-}1)$$

for $0.1 < R_e(d_p/L_b)^{0.6} < 2.5$ and

$$N_u' = 1.5P_r^{1/3}[R_e(d_p/L_b)]^{0.63} \qquad (3.3\text{-}2)$$

for $2.5 < R_e(d_p/L_b)^{0.6} < 2.5$

For liquids, measurements of effective heat transfer coefficients which may be used to obtain a similar correlation are not available. Based on correlations for effective thermal conductivity for which measurements are available [19,2,3], a more general expression applicable to liquids and gases for effective heat transfer coefficient was derived and given in the form [6,7]

$$\frac{1}{h'} = \frac{(K_e^0 + {}_pK_m)a}{({}_pR_fC_fV_i)B'^2} + \frac{1 + B_i/5}{h} \qquad (3.3\text{-}3)$$

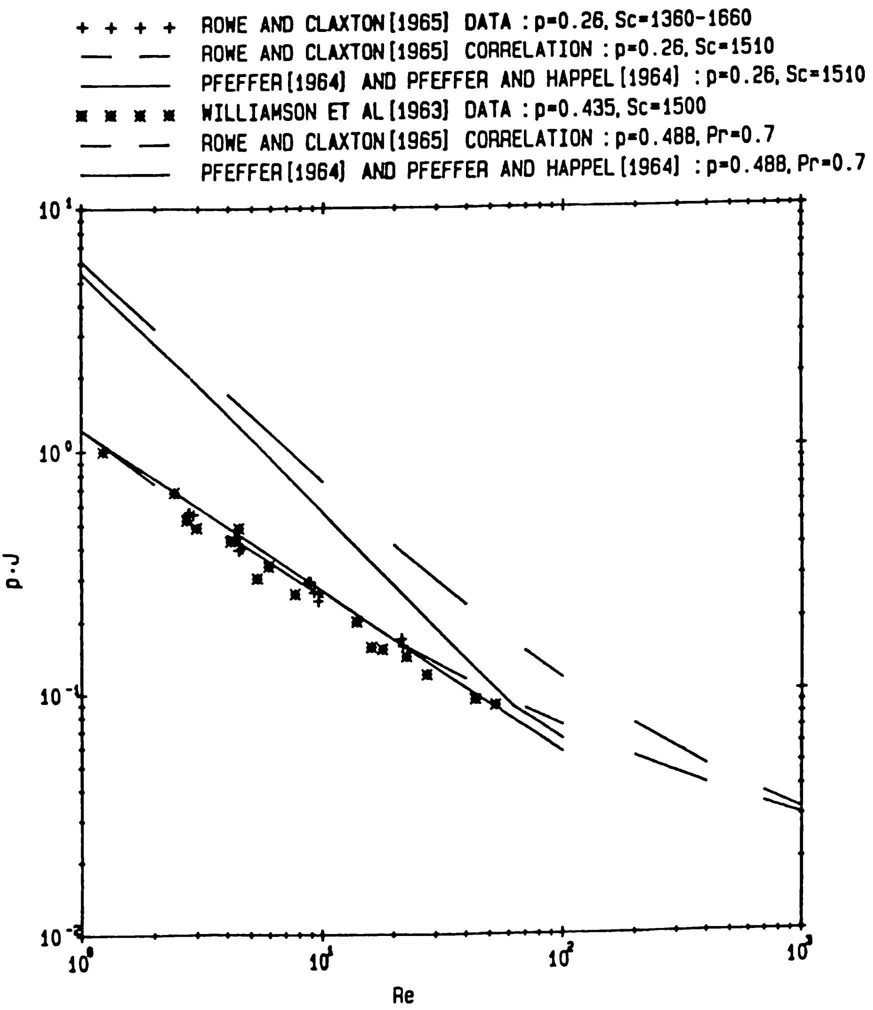

Figure 3.2-2. Comparison of Theoretical Equations for J-factors with Experimental Data

CONDUCTION THROUGH THE SOLID PHASE

Two different pictures of conduction through the solid phase are usually assumed in the models describing packed bed heat transfer. In the Rosen Model [35], a simple radial conduction process is used to describe the thermal behavior of each particle. The relative effects of point-to-point contacts of the particles and the stagnant film connecting them are, however, not taken into consideration. These effects are usually called fluid-solid-fluid conduction. In Rosen model, therefore, it is likely that the effects of fluid-solid-fluid conduction will be lumped with fluid phase dispersion.

In the so-called Conduction-Dispersion Model (CDM) developed independently by several investigators [29], conduction in the solid phase occurs via an "effective solid phase" and is represented by an effective solid phase conductivity, K_{fs}. The precise physical meaning of K_{fs} is, however, not well-defined. From the manner K_{fs} has been used in the CDM, it appears to represent conduction through the solid particle connected with the other particles by a stagnant fluid film and by point-to-point contact in the direction of fluid flow.

Littman et al. [29] obtained values of K_{fs} for air systems in the low R_e range. No correlating equation for K_{fs} was, however, given. They only noted that $(1\text{-}p)K_{fs}$ is in the order of magnitude as the static bed conductivity, $K_e{}^o$, and that it is a function of fluid flow.

For liquids, no measurements of K_{fs} are available.

For this lack of correlating equation for K_{fs}, an expression was derived from measurements of effective thermal conductivity data. It is given in the form[6,7]

$$(1 - p)K_{fs} = (K_e{}^o - pK_f) + Kp \tag{4-1}$$

The above expression combines the effects of a fraction of static bed conductivity, $K_e{}^o$, due to the solid phase and the effects of radial conduction in the solid particle in terms of the intra-particle thermal conductivity, Kp.

CONCLUSION

Considerable amount of work has been done on the subject of packed beds. Most of these work, however, concentrated on high R_e such that there is a surprising lack of data on heat transfer measurements for gases and, more so, with liquids at low R_e.

Equation (2.2-2) of Green et al. for fluid dispersion in gases and liquids is in good agreement with equations (2.1-1) and (2.1-2) and with liquid data. Using additional liquid data of Babcock et al., an equation similar in form is given in equation (2.2-3).

Equation (3.2-1) appears to be the most appropriate correlation for particle-to-fluid heat or mass transfer due to its excellent agreement with theoretical derivations and measurements at low and high R_e numbers.

The effective heat transfer coefficient is difficult to obtain since their measurements are strongly affected by axial thermal or mass diffusion in the fluid phase and by bed non-uniformities. Satisfactory correlations were obtained for gases in equations (3.3-1) and (3.3-2) in which the effects of bed lengths are taken into account. A more general expression applicable to gases and liquids is given in equation (3.3-3).

An expression for the effective solid phase conductivity derived from measurements of effective thermal conductivity data is given in (4-1). It combines the effects of a fraction of static bed conductivity due to the solid phase and the effects of radial conduction in the solid particle in terms of intra-particle thermal conductivity.

NOMENCLATURE

a = particle area per unit volume of bed
= $6/dp$ for spherical particles
A_p = $2/[1-(1-p)^{1/3}]$
B_p = $(2/3)/p$
B_i = Biot number
= hd_p/K_s
B' = $V_h/(1+V_h)$
B_e = constant depending on the value of p in equation (3.2-3)
C_f = fluid specific heat
C_s = solid particle specific heat
D_e = effective axial dispersion coefficient
= $K_e/(R_fC_f)$
D_{ef} = axial thermal or mass dispersion coefficient
= $K_{ef}/(R_fC_f)$
= $D_m + Df$
D_f = molecular diffusivity
= $K_f/(R_fC_f)$
D_m = eddy-mixing conductivity
= $K_m/(R_fC_f)$
d_p = particle diameter
F_e = formation electrical resistivity factor
F_h = inhomogeneity factor
F_t = tortuousity parameter
h = particle heat transfer coefficient
h' = effective heat transfer coefficient
J_h, J_d = Colburn J-factors for heat and mass transfer, respectively
K_e = effective thermal conductivity
K_{ef} = effective axial fluid phase thermal conductivity
K_e^o = static thermal conductivity of the bed from Hamilton [22]

$$= K_f \cdot \frac{K_s+2K_f-2(1-p)(K_f-K_s)}{K_s+2K_f+(1-p)(K_f-K_s)}$$

K_f = fluid thermal conductivity
K_{fs} = effective solid phase conductivity
K_m = eddy-mixing fluid conductivity of the bed
K_p = intra-particle thermal conductivity

$$= \frac{[R_sC_s(1-p)V_bd_p]^2}{60K_s(1-p)}$$

L_b = bed length

$$\frac{2-3n}{3n-1} = 4.65R_e^{-0.28}$$

N_u = particle Nusselt number
= hd_p/K_f

N_u' = effective Nusselt number
= $h'd_p/K_f$
p = void fraction
P_r = Prandtl number
P_e = Peclet number
= V_id_p/D_f
= $R_eP_r = R_eS_c$
P_{ea} = axial Peclet number
= V_id_p/D_{ef}
R_e = particle Reynolds number
= $pV_id_pR_f/U_f$
R_f = fluid density
R_s = solid particle density
S_c = Schmidt number
S_h = particle Sherwood number
S_h' = effective Sherwood number
U_f = dynamic viscosity
V_b = heat-front velocity
= $\dfrac{pR_fC_fV_i}{pR_fC_f + (1 - p)\,R_sC_s}$
V_h = heat capacity ratio
= $\dfrac{(1 - p)R_sC_s}{pR_fC_f}$
V_i = interstitial velocity
W = $2 - 3Gp + 3Gp^5 - 2Gp^6$
G_p = ratio of inner sphere to outer sphere radii
= $(1-p)^{1/3}$

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