"the solutions that are presented are straightforward, analytical and in closed form."

Kinematic Synthesis and Analysis of Planar Crank-Slider and Slider-Slider Mechanisms

by

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ABSTRACT

Synthesis and analysis equations are formulated for the planar crank-slider and the slider-slider mechanisms using motion parameters based on displacements from the initial position of the mechanisms. The synthesis equations are derived explicitly in terms of these parameters and also the parameters describing the mechanisms at their initial positions. These synthesis equations are generalized for Multiply-Separated Position synthesis. The solutions to the resulting systems of non-linear equations are shown for different cases of number of positions and unknowns and specified parameters describing the mechanisms at their initial positions.

INTRODUCTION

The approach developed by Hernandez [1] for the synthesis of planar four-bars as applied to the crank-follower mechanism is extended for the crank-slider and the slider-slider mechanisms. With these two additional mechanisms considered, the synthesis and analysis of planar four-bar function generators are completed. The coordination of a rotational motion with another rotational motion is covered by the crank-follower (CF), that of a rotational motion with a linear motion is covered by the crank-slider (CS) while the coordination of a linear motion with another linear motion is covered by the slider-slider (SS).

Although the number of positions that these planar four-bars can be synthesized is limited (5 for the CF and CS and 4 for the SS) as compared to mechanisms with greater number of links (e.g., six-bar mechanisms), a very high percentage of synthesis requirements can be satisfied by these four-bars mentioned.

As a review, one can look at references [2] and [1], in that order, for a study of some of the concepts related to the development and derivation of the equations.

Note that there has been previous work, as in references [3] and [4], in the subject of planar four-bar synthesis. This new approach, however, has reduced some of the difficulties encountered previously and has also eliminated some of the seeming inconsistencies in these earlier methods.

The result of this new approach gives very explicit relations with respect to the chosen parameters required to describe the mechanisms. In addition, the solutions that are presented are straightforward, analytical and in closed form.

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THE CRANK-SLIDER MECHANISM

One of the first requirements in the synthesis of an already predetermined mechanism is to be able to represent the mechanism in its "basic" configuration. This "basic" configuration means that a unique mechanism is located and oriented in the plane and is described by the minimum number of parameters (MNP). This means that no matter how one will eventually locate and orient the mechanism in the plane and describe it by more than this MNP, only a subset of this total number of parameters is independent as indicated by this minimum number of parameters. There is no exact method of configuring a mechanism to its basic form. A check at the outset is that one cannot reduce the number further and still describe a unique mechanism. There are also mathematical ramifications that lead to inconsistencies and/or dependencies in some of the synthesis equations. This happens when considering synthesis at a certain number of positions that is already exceeding the true maximum number of positions. In another context, one is trying to solve for unknowns in a system of equations in which some of these unknowns are not independent.

As an illustration, Figure 1 shows the crank-slider as described by seven parameters, which are, the two coordinates each of the fixed pivot, the moving pivot and the slider pivot and also the angle describing the direction of the slider axis.

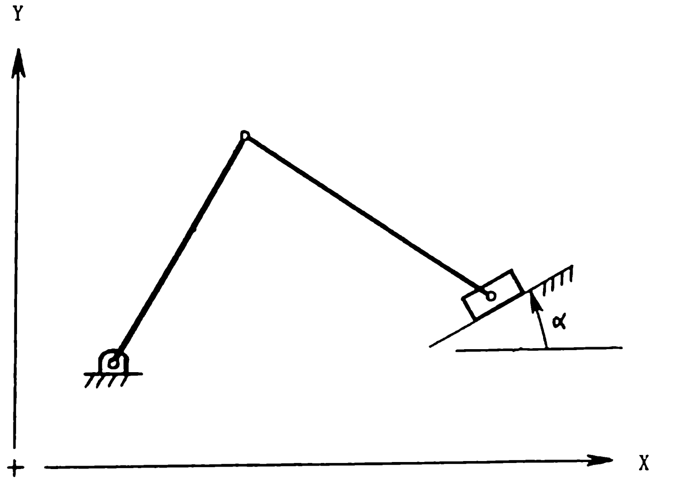

Figure 1. The Planar Crank-Slider

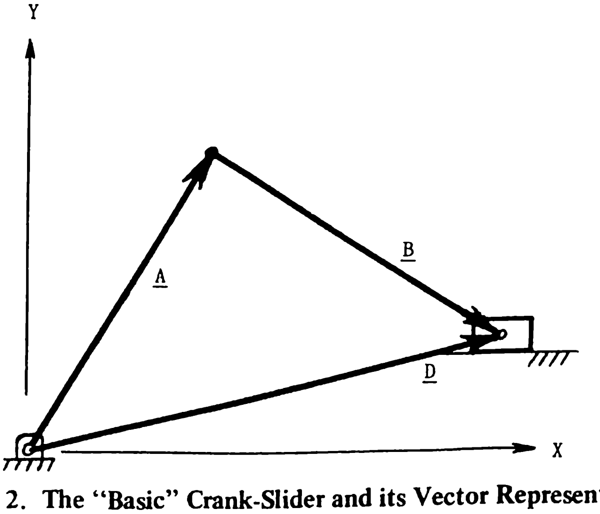

Figure 2. The "Basic" Crank-Slider and its Vector Representation

One might be misled into believing that the crank-slider can be synthesized for these seven parameters. Figure 2, however, shows the "basic" crank-slider which is described by only four parameters -- two coordinates each for the moving pivot and the slider pivot. Also, these four parameters are the components of the vectors $\underline{A}$ and $\underline{D}$ as shown in the same figure. In most literature and textbooks, the crank-slider shown is also called an off-set slider-crank.

EQUATION OF MOTION FOR THE CRANK-SLIDER

The loop closure equation for the crank-slider at its initial position is

$$\underline{A} + \underline{B} = \underline{D} \text{ or } \underline{B} = \underline{D} - \underline{A} \tag{1}$$

Figure 3 shows the crank-slider at its displaced position due to the motion parameters θ_j and S_j. This time, the loop closure equation becomes

$$\underline{A}_j + \underline{B}_j = \underline{D} + S_j\underline{C} \tag{2}$$

where S_j describes the linear displacement of the slider from its initial position
and $\underline{C}$ is a unit vector in the direction of the x-axis.

Equation (2) is now rearranged as

$$\underline{B}_j = \underline{D} + S_j\underline{C} - \underline{A} \tag{3}$$

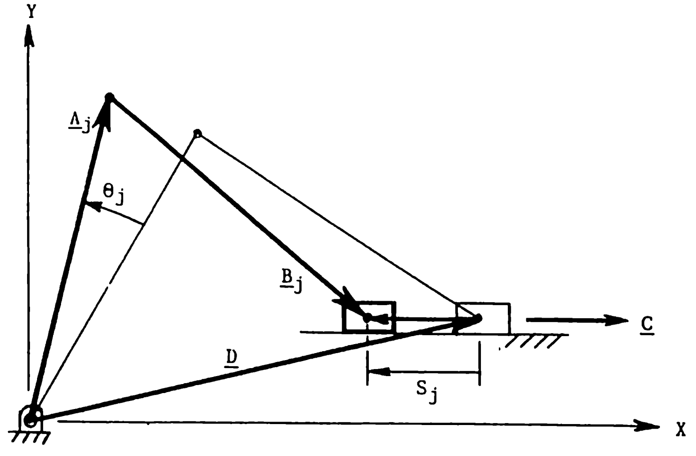

Figure 3. The Crank-Slider at its Displaced Position

The concept of the linkage constraint is now applied to the coupler which is simply that its length remains the same throughout. In an equation, this constraint is written as

$$\underline{B}_j \cdot \underline{B}_j = \text{constant} = \underline{B} \cdot \underline{B} \tag{4}$$

The expression for the vector rotation of $\underline{A}$ is (see reference 2)

$$\underline{A}_j = \underline{A}\cos\theta_j + (\underline{k} \times \underline{A})\sin\theta_j \tag{5}$$

where θ_j is the angle of rotation of vector $\underline{A}$
and, $\underline{k}$ is the unit vector in the z axis.

Taking the dot product of $\underline{B}$ in equation (1) gives us

$$\underline{B}\cdot\underline{B} = \underline{A}\cdot\underline{A} + \underline{D}\cdot\underline{D} - 2\underline{A}\cdot\underline{D} \tag{6}$$

Substituting equations (3) and (6) into equation (4) will result in

$$(\underline{D} + S_j\underline{C} - \underline{A}\cos\theta_j - (\underline{k}\times\underline{A})\sin\theta_j)(\underline{D} + S_j\underline{C} - \underline{A}\cos\theta_j - (\underline{k}\times\underline{A})\sin\theta_j)$$
$$= \underline{A}\cdot\underline{A} + \underline{D}\cdot\underline{D} - 2\underline{A}\cdot\underline{D} \tag{7}$$

Carrying out the dot product operation and simplifying will give us the equation of motion for the crank-slider as

$$S_j^2/2 + \underline{C}\cdot(\underline{D} - \underline{A}_j)Sj + \underline{D}\cdot(\underline{A} - \underline{A}_j) = 0 \tag{8}$$

Note the difference of the above equation of motion with that of equation (28) in reference [2]. The reason for this slight change is to make the equation of motion more convenient for synthesis.

As an analysis equation for S_j given a θ_j, the solution is obtained from the following

$$Sj = -b \pm \sqrt{b^2 - 2c} \tag{9}$$

$$\text{where } b = \underline{C}\cdot(\underline{D} - \underline{A}_j)$$
$$\text{and } c = \underline{D}\cdot(\underline{A} - \underline{A}_j) \tag{10}$$

To complete the analysis for velocity and acceleration, we refer to equation (4) and take its first time derivative to get

$$\dot{\underline{B}}_j\cdot\underline{B}_j = 0 \tag{11}$$

$$\text{where } \dot{\underline{B}}_j = \dot{S}_j\underline{C} - A'_j\dot{\theta}_j \tag{12}$$

$$\text{and } \underline{A}'_j = -\underline{A}\cos\theta_j + (\underline{k}\times\underline{A})\cos\theta_j \tag{13}$$

Substituting equations (12) and (13) into equation (11), noting that $\underline{A}'_j$ is perpendicular to $\underline{A}_j$ and after simplifying will result in

$$\dot{S}j = [\underline{A}'j\cdot(\underline{D} + S_j\underline{C})/(S_j + \underline{C}\cdot(\underline{D} - \underline{A}_j)\,]\,\dot{\theta}_j \tag{14}$$

For the acceleration of the slider, the constraint equation (4) is differentiated twice with respect to time to give us

$$\ddot{\underline{B}}_j\cdot\underline{B}_j + \dot{\underline{B}}_j\cdot\dot{\underline{B}}_j = 0 \tag{15}$$

$$\text{where } \ddot{\underline{B}}_j = \ddot{S}_j\underline{C} + A_j\dot{\theta}_j^2 - \underline{A}'_j\ddot{\theta}_j \tag{16}$$

Substituting equations (12) and (16) into equation (15) will give us the acceleration of the slider block as

$$\ddot{S}_j = [\,2\,(\underline{A}'_j\cdot\underline{C})\,\dot{\theta}_j\dot{S}_j - \underline{A}_j\cdot(\underline{D} + S_j\underline{C})\,\dot{\theta}^2{}_j - \dot{S}^2{}_j + \underline{A}'_j\cdot(\underline{D} + S_j\underline{C})\,\ddot{\theta}_j\,]\,/\,(S_j + \underline{C}\cdot(\underline{D} - \underline{A}_j) \tag{17}$$

where $\dot{\theta}_j$ and $\ddot{\theta}_j$ are the angular velocities and angular accelerations of the input crank at the particular position θ_j.

CRANK-SLIDER SYNTHESIS EQUATIONS

Writing the design vectors $\underline{A}$ and $\underline{B}$ as

$$\underline{A} = a_1\hat{i} + a_2\hat{j} \tag{18}$$

and $$\underline{D} = d_1\hat{i} + d_2\hat{j} \tag{19}$$

and substituting these equations into (8) will give us the synthesis equation as,

$$S^2_j/2 - S_j\cos\theta_j a_1 + S_j\sin\theta_j a_2 + S_j d_1 + (1 - \cos\theta_j)a_1 d_1 - \sin\theta_j a_1 d_2$$
$$+ \sin\theta_j a_2 d_1 + (1 - \cos\theta_j)a_2 d_2 = 0 \tag{20}$$

Equation (20) is the motion equation of the crank-slider mechanism in terms of the design parameters a_1, a_2, d_1 and d_2. This time, for synthesis, the motion parameters θ_j and S_j are specified. Since we are able to write one equation for each displaced position (from the initial position), we can therefore synthesize the CS for a maximum of five positions. The synthesis equation (20) is now written for the generalized MSP synthesis as,

$$L_1{}^n_j a_1 + L_2{}^n_j a_2 + L_3{}^n_j d_1 + L_4{}^n_j a_1 d_1 + L_5{}^n_j a_1 d_2 + L_6{}^n_j a_2 d_1 + L_7{}^n_j a_2 d_2$$
$$+ L_8{}^n_j = 0 \tag{21}$$

$j = 2, 3$ up to 5
$n = 0$ or 1 or 2

For reference, the expressions for the $L_i{}^{n'}_j$s, n = 0, 1 or 2 are listed in the appendix.

SYNTHESIS CASES FOR THE CRANK-SLIDER

The solutions for the four- and five-position synthesis problems will now be shown. The two- and three-position problems do not pose any difficulty and will not be presented here. There are four possible cases of the four-position problem. These are when any one of a_1, a_2, d_1 and d_2 is the unknown. The case where the parameter d_1 is specified differs from the cases when the specified parameters are either a_1, a_2 or d_2. When d_1 is specified, the synthesis equations are written as,

$$(A_j + B_j d_2)a_1 + (C_j + D_j d_2)a_2 + E_j = 0 \qquad j = 2, 3, 4 \tag{22}$$

where: the A_j, B_j . . . . . E_j are determined from the $L_i j$'s and d_1.

The solutions for d_2 are obtained from the roots of the quadratic equation expressed below as the eliminant of equation (22).

$$\left| (A_j + B_j d_1) \quad (C_j + D_j d_1) \quad E_j \right| = 0 \qquad j = 2, 3, 4 \tag{23}$$

These two roots are then substituted back into any two equations of (22) and this will result in a linear system of equations with unknowns a_1 and a_2. There are therefore, two sets of solutions for this particular case of the four-position problem.

The cases when any of a_1, a_2 or d_2 are the specified parameters give a cubic as the eliminant. As an illustration, the case where d_2 is specified will be shown. The synthesis equations are written as,

$$(A_j + B_j d_1)a_1 + (C_j + D_j d_1)a_2 + (E_j + F_j d_1) = 0 \qquad j = 2, 3, 4 \tag{24}$$

The cubic eliminant is the following determinant set to 0.

$$\begin{vmatrix} (A_j + B_j d_1) & (C_j + D_j d_1) & (E_j + F_j d_1) \end{vmatrix} = 0 \qquad j = 2, 3, 4 \tag{25}$$

The one or three real roots of the eliminant are then substituted into any two equations of (24) to get a_1 and a_2. This gives a possible maximum of three sets of solutions to this four-position problem.

For the five-position problem, all of the design parameters a_1, a_2, d_1 and d_2 are unknowns. To solve this case, the synthesis equations are first rearranged in the following form:

$$(A_j + B_j d_1) + (C_j + D_j d_1)\, a_1 + (E_j + F_j d_1)\, a_2 + O d_2 + G_j a_1 d_2 + H_j a_2 d_2 = 0 \qquad j = 2, 3, 4, 5 \tag{26}$$

where: the A_j, B_j, . . . H_j are the corresponding $L_{ij}{}^{n}$'s of the synthesis equations.

By multiplying all of the four equations of (26) by d_2, we can assemble a homogenous "linear" system of eight equations in the 8 unknowns 1, a_1, a_2, d_2, $a_1 d_2$, $a_2 d_2$, $a_1 d_2^2$ and $a_2 d_2^2$. The solutions for d_1 are obtained from the roots of the following eliminant expressed as the determinant below set to 0.

$$\begin{vmatrix} (A_j + B_j d_1) & (C_j + D_j d_1) & (E_j + F_j d_1) & 0 & G_j & H_j & 0 & 0 \\ 0 & 0 & 0 & (A_j + B_j d_1) & (C_j + D_j d_1) & (E_j + F_j d_1) & G_j & H_j \end{vmatrix} = 0 \qquad j = 2, 3, 4, 5 \tag{27}$$

The eliminant from the 8 x 8 determinant is a 5th degree polynomial. The solutions to d_1 are the roots of this polynomial and there can be 1, 3 or 5 real roots of meaning to the synthesis problem. To get the solutions for the other parameters a_1, a_2 and d_2. we consider any seven of equations (26) and equation (26) multiplied by d_2. Then, solve this linear system for the unknowns a_1, a_2, d_2 and for verification purposes, also $a_1 d_2$, $a_2 d_2$, $a_1 d_2^2$ and $a_2 d_2^2$.

THE SLIDER-SLIDER MECHANISM

Any slider-slider mechanism can be oriented and located in the x-y coordinate system as shown in Figure 4 with the vectors $\underline{A}$, $\underline{B}$ and $\underline{D}$. For this "basic" slider-slider, the parameters describing a unique SS are the vectors locating the two moving pivots, that is, $\underline{A}$ and $\underline{D}$. After a series of attempts, a convenient representation of the parameters for synthesis and also analysis is the case of a polar notation for vector $\underline{A}$. Thus, we let r (the length of $\underline{A}$), θ and d (the length of $\underline{D}$) be the parameters for synthesis. Using the said representation, the following are obtained and used in the derivation of the equation of motion of the slider-slider:

$$\begin{aligned} \underline{A} &= r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} \\ \hat{A} &= \cos\theta\, i + \sin\theta\, j \\ \underline{D} &= d\,\hat{i} \\ \underline{C} &= 1\,\hat{i} \end{aligned} \tag{28}$$

EQUATION OF MOTION OF THE SLIDER-SLIDER MECHANISM

At the initial position of the mechanism, we can get the expression for the vector $\underline{B}$ as,

$$\underline{B} = \underline{D} - \underline{A} \tag{29}$$

At the jth position when the sliders have moved from their initial positions by corresponding S_j's, we can get the following relations as seen in Figure 5.

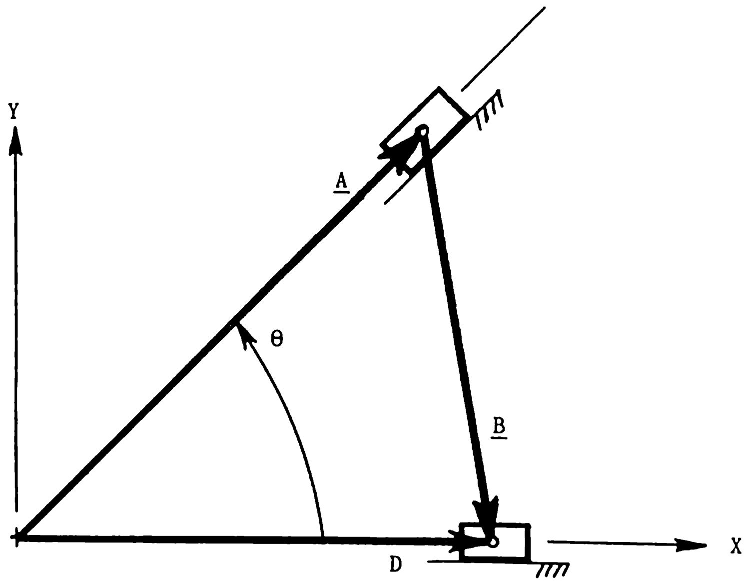

Figure 4. The Slider-Slider and its Vector Representation

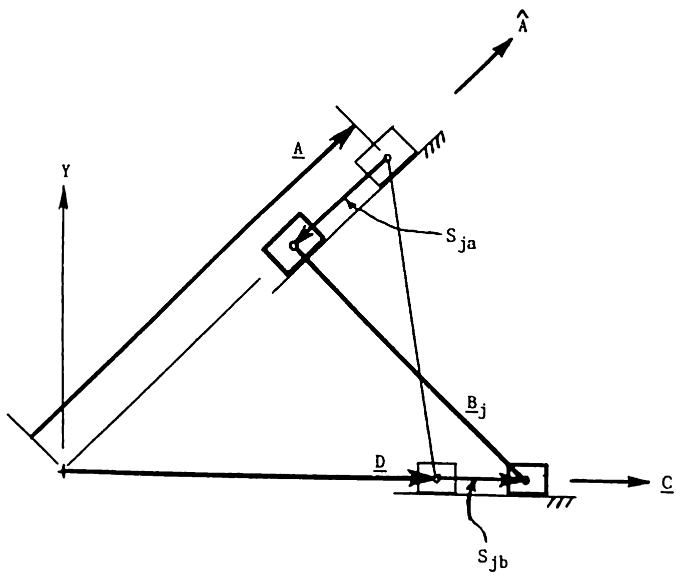

Figure 5. The Displaced Slider-Slider

$$\underline{A} + S_{ja}\hat{A} + \underline{B}_j = \underline{D} + S_{jb}\underline{C} \tag{30}$$

$$\text{or, } \underline{B}_j = \underline{D} + S_{jb}\underline{C} - \underline{A} - S_{ja}\hat{A} \tag{31}$$

Using the constraint of the coupler having the same length, that is $\underline{B}_j \cdot \underline{B}_j$ is the same as $\underline{B} \cdot \underline{B}$ we get,

$$\begin{aligned} &½S_{ja}^2 + ½S_{jb}^2 + S_{jb}(\underline{C} \cdot \underline{D}) - S_{ja}(\hat{A} \cdot \underline{D}) - S_{jb}(\underline{A} \cdot \underline{C}) - \\ &\quad S_{ja}S_{jb}(\hat{A} \cdot \underline{C}) + S_{ja}(\hat{A} \cdot \underline{A}) = 0 \end{aligned} \tag{32}$$

Substituting the equations of (28) into (32) and rearranging will result in the slider-slider equation of motion in terms of the parameters r, d and $\cos\theta$.

$$\tfrac{1}{2}S_{ja}^2 + \tfrac{1}{2}S_{jb}^2 + S_{jb}d - S_{ja}d\cos\theta - S_{jb}r\cos\theta - S_{ja}S_{jb}\cos\theta + S_{ja}r = 0 \tag{33}$$

For use as an analysis equation, we can get S_{ja} when S_{jb} is given from the equations below.

$$S_{ja} = -b_b \pm \sqrt{b_b^2 - 2c_b} \tag{34}$$

$$\text{where: } b_b = r - d\cos\theta - S_{jb}\cos\theta$$
$$\text{and } c_b = S_{jb}\,(\tfrac{1}{2}S_{jb} + d - r\cos\theta) \tag{35}$$

Or, when S_{ja} is specified or known and we would like to get S_{jb}, we have,

$$S_{jb} = -b_a \pm \sqrt{b_a^2 - 2c_a} \tag{36}$$

$$\text{where: } b_a = d - r\cos\theta - S_{ja}\cos\theta$$
$$\text{and } c_a = S_{ja}\,(\tfrac{1}{2}\dot{S}_{ja} + r - d\cos\theta) \tag{37}$$

For the velocity and acceleration equations, we take the derivatives of equation (33) and rearrange into two forms, that is, corresponding to the cases when either of S_{ja} or S_{jb} is the unknown and the other is specified.

For the case when the slider A is the output, we have,

$$\dot{S}_{ja} = -(n/m)\,\dot{S}_{jb} \tag{38}$$

$$\text{and } \ddot{S}_{ja} = (1/m)\,(2S_{ja}S_{jb}\cos\theta - \dot{S}_{ja}^2 - \dot{S}_{jb}^2 - n\ddot{S}_{jb}) \tag{39}$$

$$\text{where: } m = S_{ja} + r - d\cos\theta - S_{jb}\cos\theta \tag{40}$$
$$n = S_{jb} + d - r\cos\theta - S_{ja}\cos\theta$$

For the case when slider B is this time the output, we have,

$$\dot{S}_{jb} = -(m/n)\,\dot{S}_{ja} \tag{41}$$

$$\text{and } \ddot{S}_{jb} = (1/n)\,(2S_{ja}S_{jb}\cos\theta - \dot{S}_{ja}^2 - \dot{S}_{jb}^2 - m\ddot{S}_{ja}) \tag{42}$$

SYNTHESIS OF THE SLIDER-SLIDER MECHANISM

The equation of motion (33) is rearranged and written as,

$$(M_1{}^n{}_j + M_2{}^n{}_j\cos\theta)\,r + (M_3{}^n{}_j + M_4{}^n{}_j\cos\theta)\,d + (M_5{}^n{}_j + M_6{}^n{}_j\cos\theta) = 0 \tag{43}$$

The $M_i{}^n{}_j$'s were derived and are listed in the appendix for n = 0, 1 and 2.

For the four position problem, the maximum number of positions for the slider-slider, the solution for the design parameters are obtained by first getting the roots for $\cos\theta$ from the following determinant set to zero.

$$\left|(M_1{}^n{}_j + M_2{}^n{}_j\cos\theta)\quad(M_3{}^n{}_j + M_4{}^n{}_j\cos\theta)\quad(M_5{}^n{}_j + M_6{}^n{}_j\cos\theta)\right| = 0 \qquad j = 2, 3, 4 \tag{44}$$

The roots of the cubic eliminant are the solutions for $\cos\theta$. Note here that roots for $\cos\theta$ obtained which are less than -1 or greater than +1 would correspond to the case of getting complex roots in the previous mechanisms considered. To get the solutions for the other design parameters r and d, we first substitute the roots for $\cos\theta$ in any two equations of (43). This will result in a linear system of two equations in r and d.

CONCLUSIONS

The synthesis and analysis of the crank-slider and the slider-slider have been presented. With these and the results of a previous work [1], the synthesis and analysis of the possible cases of planar four-bar function generators are covered.

A worthwhile and useful effort will now be to develop a software package for the synthesis and analysis of these function generators. This package can then be used by teachers, students and practicing engineers. Professor Manuel G. Ibanez of the Mechanical Engineering department of UP and the author are presently undertaking this effort. Although requiring more refinements, a partially completed package has already been used in a Machine Design class as a test case. More extensive use and consideration of this and maybe more packages in the future will be carried out when the new curriculum for ME in UP which considers these topics is implemented.

After these efforts for function generators, attention can then be focused on the planar four-bar motion and path generation synthesis. Although many methods have already been developed, one can look at either refinements of these methods by reformulations or modifications, or, one can devote attention or efforts in the implementation of these methods, specially with the availability and the utility of micro computers.

APPENDIX

A. Expressions for $K_i{}^n{}_j$ for i = 1, 2, . . . 8 and n = 0, 1 and 2

$K_{1j} = - S_j\cos\theta_j$

$K_1{}^1{}_j = S_j\sin\theta_j\theta_j - S_j\cos\theta_j,$

$K_1{}^2{}_j = S_j\,(\cos\theta_j\theta_j{}^2 + \sin\theta_j\theta_j) + 2\sin\theta_j\theta_jS_j - \cos\theta_jS_j$

$K_{2j} = S_j\sin\theta j$

$K_2{}^1{}_j = S_j\cos\theta_j\theta_j + S_j\sin\theta_j$

$K_2{}^2{}_j = -S_j\,(\sin\theta_j\theta^2{}_j + \cos\theta_j\theta_j) + 2\cos\theta_j\theta_jS_j + \sin\theta_jS_j$

$K_{3j} = S_j \qquad K_3{}^1{}_j = S_j \qquad K_3{}^2{}_j = S_j$

$K_{4j} = 1 - \cos\theta j$

$K_4{}^1{}_j = \sin\theta_j\theta_j$

$K_4{}^2{}_j = \cos\theta_j\theta_j{}^2 + \sin\theta_j\theta_j$

$K_{5j} = -\sin\theta_j$

$K_5{}^1{}_j = -\cos\theta_j\theta_j$

$K_5{}^2{}_j = \sin\theta_j\theta_j{}^2 - \cos\theta_j\theta_j$

$K_6{}^n{}_j = -K_5{}^n{}_j$

$K_7{}^n{}_j = K_4{}^n{}_j$

$K_{8j} = \frac{1}{2} S_j{}^2 \qquad K_8{}^1{}_j = S_j \dot{S}_j \qquad K_8{}^2{}_j = \dot{S}_j{}^2 + S_j \ddot{S}_j$

B. Expressions for $M_i{}^n{}_j$ for i = 1, 2, . . . 6 and n = 0, 1 and 2

$M_{ij} = S_{jb} \qquad M_i{}^1{}_j = \dot{S}_{jb} \qquad M_i{}^2{}_j = \ddot{S}_{jb}$

$M_{2j} = S_{ja} \qquad M_2{}^1{}_j = \dot{S}_{ja} \qquad M_2{}^2{}_j = \ddot{S}_{ja}$

$M_3{}^n{}_j = -M_2{}^n{}_j$

$M_4{}^n{}_j = -M_1{}^n{}_j$

$M_{5j} = \frac{1}{2}(S_{ja}{}^2 + S_{jb}{}^2)$

$M_5{}^1{}_j = S_{ja}\dot{S}_{ja} + S_{jb}\dot{S}_{jb}$

$M_5{}^2{}_j = S_{ja}\ddot{S}_{ja} + \dot{S}_j{}^2{}_a + S_{jb}\ddot{S}_{jb} + \dot{S}_{jb}{}^2$

$M_{6j} = -S_{ja}S_{jb}$

$M_6{}^1{}_j = -S_{ja}\dot{S}_{jb} - \dot{S}_{ja}S_{jb}$

$M_6{}^2{}_j = -(2\dot{S}_{ja}\dot{S}_{jb} + \ddot{S}_{ja}S_{jb} + S_{ja}\ddot{S}_{jb})$

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