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Microcomputers and Static Equilibrium

by
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INTRODUCTION

Early applications in engineering analysis of what were then in the mid '50s relatively costly computer resources were essentially "high-end" in nature and limited to work which required much computation. In contrast, the microcomputer now offers, at far lower costs, the convenience of a personal computing device along with power which rivals that of the early mainframes. The impact on society of this "ultimate computing tool for the democratic world" (Hall-Sheehy, 1987) is widely acclaimed to be comparable to that of the automobile.

In engineering, the availability of microcomputers means that no serious computations need to be manually done any longer. The implications in engineering education and practice are bound to be far reaching because the essential requirements of effective computer-aided problem solving are not necessarily met by extant elementary methods which had been developed for manual calculations.

This paper attempts to illustrate how traditional problem solving techniques in engineering statics might have to be modified or even supplanted by approaches which, though less than optimal for manual calculations, are actually the most suitable for microcomputers.

STATICS OF RIGID BODIES

The analysis of a system of rigid bodies at rest is based on the assumption that the condition of equilibrium holds for any part of the system. Therefore, algebraic equations involving unknown forces of interaction can be deduced by isolating suitable parts as "free bodies" and applying Newton's laws. Enough independent equations can be developed in this manner if, as assumed herein, the problem is statically determinate.

For hand calculations, the selection and sequencing of freebody diagrams (and associated equations) seek to minimize simultaneous equation solving. Assuming planar forces, the horizontal and vertical components of the resultant of known forces on the freebody at hand are denoted as X and Y, respectively. An uncoupled equation may then be obtained by summing forces along a direction, α_o, which is orthogonal to all except one unknown force, P. If so, this force is

$$P = \frac{x \sin \alpha_o - Y \cos \alpha_o}{\sin (\alpha_o - \alpha_P)} \qquad (1)$$

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Another type of uncoupled equation is obtained by summing moments about a point (x', y') through which the lines of action of all but one of the unknown forces pass. That force is given by

$$P = \frac{M + X y' - Y x'}{(y_p - y) \cos \alpha_p - (x_p - x') \sin \alpha_p} \tag{2}$$

where M is the couple component of the resultant of known forces, α_p is the orientation of P, and (x_p, y_p) are coordinates of its point of applications.

The components of the resultant are themselves computed in the following manner. If F_j is a typical known force at (x_j, y_j) and has orientation, α_j,

$$\begin{aligned} X &= \sum_j F_j \cos \alpha_j \\ Y &= \sum_j F_j \sin \alpha_j \\ M &= \sum_j F_j (x_j \sin \alpha_j \cdot y_j \sin \alpha_j) \end{aligned} \tag{3}$$

Equations (1) to (3) are highly suited for coding on programmable calculators (Reyes, 1978). Using spreadsheet software, the solution of typical problems in engineering statics such as that shown in Figure 1 can be developed as illustrated in Plate 1. The pedagogical value of using a micro-computer and the well known "what-if" capabilities of spreadsheets is the relative ease with which certain key data could be considered as parameters and varied at will. For example, the relation of geometry with the forces of interaction between components of a system can thereby be studied free of tedious calculations.

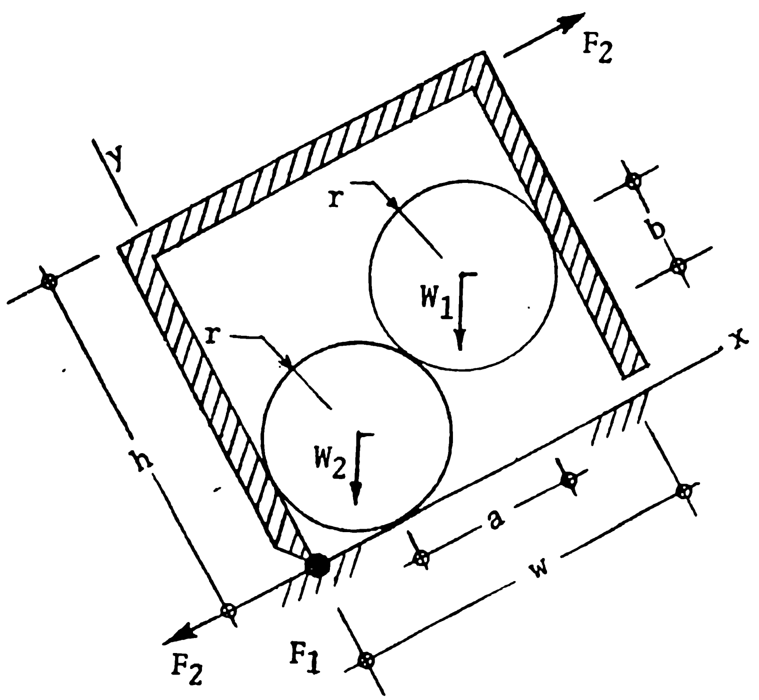

Figure 1. Rigid Body Statics

Plate 1

Illustrative Problem 1
STATICS OF RIGID BODIES

Given Data (Parameters):

r = 0.5 w = 1.6 h = 3 theta = 30

Derived Data:

a = 0.6 b = 0.8

Forces:

Label	x	y	Orient	cos	sin	Value	Include	Sum-x	Sum-y	Sum-M
W1	0.5	0.5	−120	−0.5	−0.86602	100	1	−50	−86.6025	−18.3012
W2	1.1	1.3	−120	- 0.5	- 0.86602	100	1	−50	−86.6025	−30.2627
Fa	0.5	0	90	0	1	NA	0	0	0	0
Fb	0	0.5	0	1	0	NA	0	0	0	0
Fc	0.8	0.9	53.13010	0.6	0.8	NA	0	0	0	0
Fd	1.6	1.3	0	1	0	NA	0	0	0	0
								−100	−173.205	−48.5640

Data for Force Calculation (1-force; 2-moment):

Alph-0 =		0	1	0	
Alph-p =	0	0	1		
xp =	1.6			−14.9519	
yp =	1.3				
x' =	0.5				
y' =	0.5				
Meth =	2				

Result – – – – – – – – – – – – – – – – – – −14.9519

PLANAR TRUSSES

The above procedure can also be applied to the practical problem of analyzing statically determinate and non-complex planar trusses. Equations (1) and (2) in this case are equivalent to the method of joints and the method of sections, respectively (Norris et al., 1976). However, with a spreadsheet software capable of matrix arithmetic, it is more convenient to proceed as follows.

The set of joint equilibrium equations can be written in matrix form as

$$Z\,R + p = 0 \tag{4}$$

where R is the set of bar forces and p the joint forces (given loads and reactions). Rules for generating the elements of Z are easily worked out. For example, in any row of Z representing the equilibrium of horizontal force components at some joint, the m^{th} term is either + cos (θ_m) or − cos (θ_m), where θ_m is the orientation of bar, m, depending on whether the start or the end of the bar is connected to the joint. But if bar m is not connected to the joint, the term vanishes.

If the elements of p are sorted into a subvector, p_F,, consisting of known forces and a subvector, ps, of reactions, Equation (4) can be recast in the form.

$$\begin{matrix} Z_F & O \\ Z_s & I \end{matrix} \quad \begin{matrix} R \\ P_s \end{matrix} = \begin{matrix} -P_F \\ O \end{matrix} \tag{5}$$

and solved for the unknowns.

A positive value in any component of R signifies tension in the bar. Plate 1 illustrates the analysis of the truss shown in Figure 2. Note that the reactions are represented as fictitious links for convenience. The table of joint coordinates along with the table of bar incidences are used to calculate the bar orientations which, in turn, are utilized for the generation of the elements of Z.

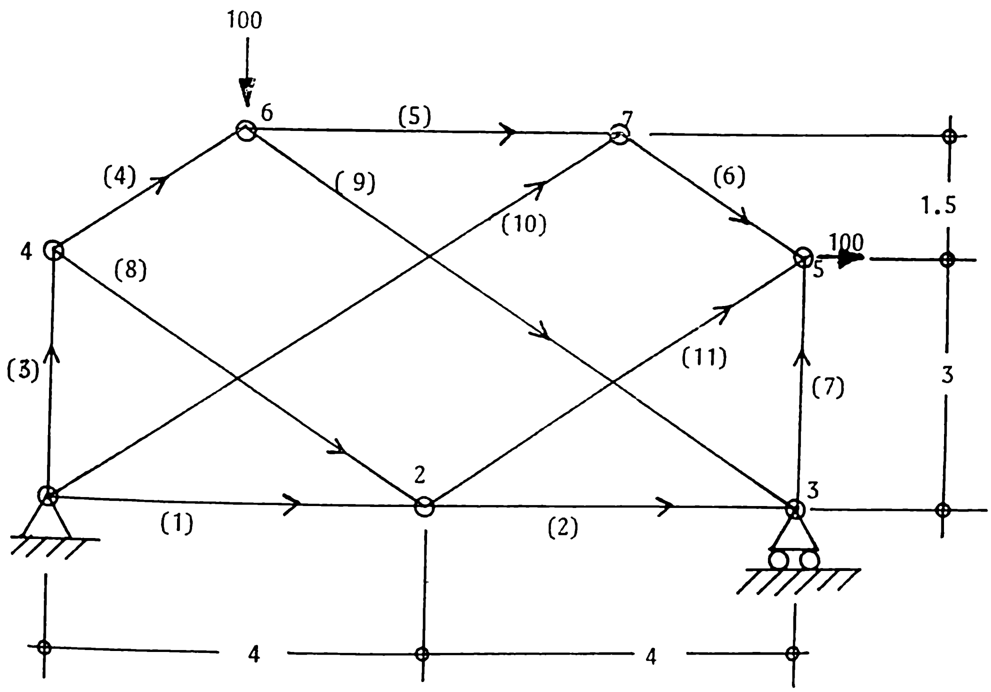

Figure 2. Planar Truss Analysis

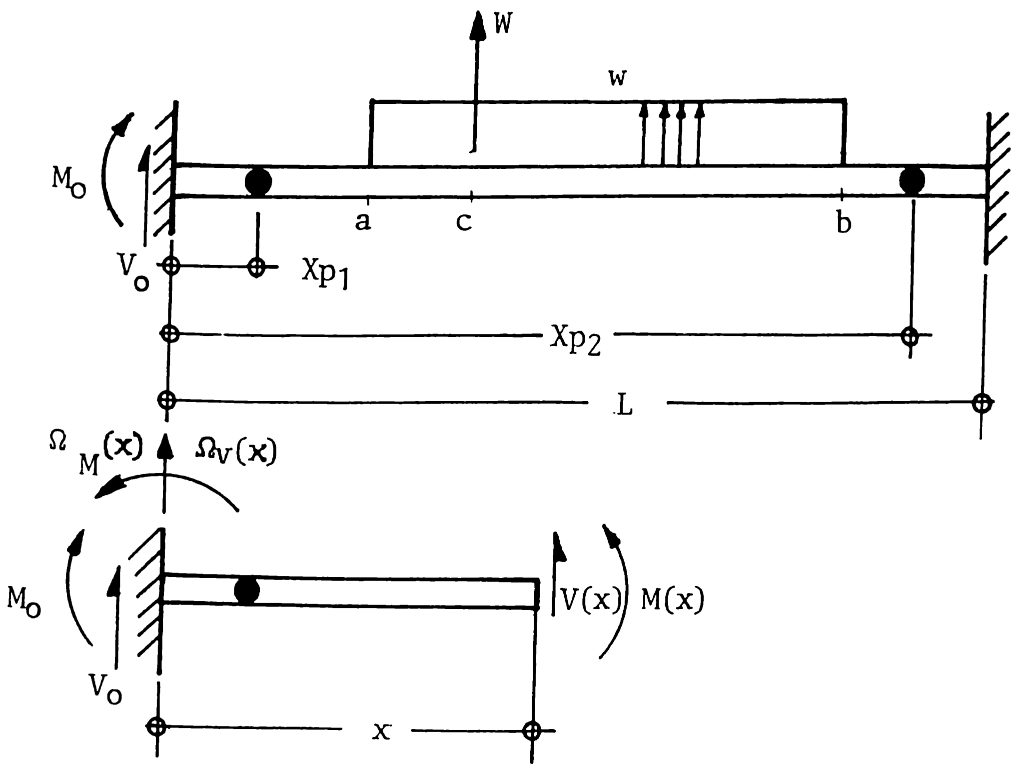

Figure 3. Beam Statics

Again, data for problems of this type are readily treated as parameters for learning purposes as well as in actual practice.

STATICS OF BEAMS

Shear (V) and bending moment (M) in a beam supported at its ends (Figure 3) are determined by statics if two releases are provided. For example, the common simply supported case has a pin at each end, while cantilevered case has a pin and a guide at one end. More generally, a guide at $x=x_V$, say, provides the statical condition.

$$V(x_V) = 0 \tag{6}$$

while a pin at $x = x_M$ provides

$$M(x_M) = 0 \tag{7}$$

Loads can usually be represented analytically with the help of distribution functions for purposes of representing loads throughout the beam span, L; i.e., for $0 \leq x \leq L$. For example, a concentrated transverse force (W) at some point (x = c) on the beam axis can be written

$$p(x) = W\,\delta(x-c) \tag{8}$$

where δ is the delta function. Similarly, a uniform transverse load (w) applied in some segment $(a \leq x \leq b)$ can be represented as

$$p(x) = [u(x-a) - u(x-b)]\,w(x) \tag{9}$$

where u is the unit step function.

At x=0, the resultant transverse force on a segment (0, x) of a beam may be denoted as the following function of x

$$\Omega_V = \int_0^x p(\alpha)\,d\alpha\,. \tag{10}$$

and the resultant couple as

$$\Omega_M = \int_0^x \alpha \cdot p(\alpha)\,d\alpha\,. \tag{11}$$

To express these functions in terms of ordinary integrals, note that if f (x) is any continuous function and F (x) its integral, then

$$\int_0^x f(\alpha)\,\delta(\alpha-c)\,d\alpha = u(x-c)\,f(c) \tag{12}$$

and

$$\int_0^x f(\alpha)\,[u(\alpha-a) - u(\alpha-b)]\,d\alpha = u(x-a)\,[F(x)-F(a)] - u(x-b)\,[F(x)-F(b)]\,. \tag{13}$$

Using Equations (12) and (13), Ω_V and Ω_M can be spelled out for most loading types in forms convenient for computer coding. For example, for a series of downward concentrated loads, $W_1, \ldots, W_n$,

$$\Omega_V(x) = \sum_{k=1}^{n} -u(x - x'_K)\, W_K \tag{14}$$

$$\Omega_M(x) = \sum_{k=1}^{n} -x\, u(x - x'_K)\, W_K \; . \tag{15}$$

Evaluation of these functions is easily coded using conditional branching or if-statements based on the definition of the unit step function.

Letting V_o and M_o be the initial values (at x = 0) of the internal force functions, the equilibrium conditions now lead to

$$V(x) = V_o - \Omega_V(x) \tag{16}$$

and

$$M(x) = M_o - x\, V_o - \Omega M(x) + x\, \Omega_V(x) \; . \tag{17}$$

Recalling Equations (6) and (7), these equations provide the statical or release conditions to determine V_o and M_o. For a guide release at $x = x_v$, the condition is

$$0 = V_o - \Omega V(xv) \tag{18}$$

while a pin at $x = x_M$ implies

$$0 = M_o - x_M V_o - \Omega_M(x_M) + X\, \Omega_V(x_M). \tag{19}$$

For example, with pins at x_{M1} and x_{M2}.

$$V_o = \frac{x_{M1}\, \Omega V_{M1} - \Omega_{MM1} - x_{M2}\, \Omega V_{M2} + \Omega_{MM2}}{x'_{M1} - x_{M2}} \tag{20}$$

$$M_o = \frac{x_{M1}\, x_{M2}\, (\Omega_{VM2} - \Omega_{VM1}) + x_{M2}\, \Omega_{MM1} - x_{M1}\, \Omega_{MM2}}{x_{M2} - x_{M1}} \tag{21}$$

(where $\Omega_{VM1} = \Omega_V(x_{M1})$, etc.). The combination of a guide at X_V and pin at X_M leads to

$$V_o = \Omega_{VV} \tag{22}$$

$$M_o = \Omega_{MM} - x_M(\Omega_{VM} - \Omega_{VV}) \; . \tag{23}$$

Plate 3 illustrates the general formulation of the analysis of a beam supporting three concentrated loads. Magnitudes of the loads, beam span, and types and locations of releases can be varied in this layout. Then, for any specified x, the shear and bending moment are calculated. The layout is easily modified to calculate these functions at a set of nodes along the beam axis.

Plate 2

Illustrative Problem 2:
STATICALLY DETERMINE PLANAR TRUSS

Joint Coordinates and Loads:

Joint	x	y	P-x	P-y
1	0	0	0	0
2	4	0	0	0
3	8	0	0	0
4	0	3	0	0
5	8	3	100	0
6	2	4.5	0	−100
7	6	4.5	0	0
8	−5	0	0	0
9	0	−5	0	0
10	8	−5	0	0

Bar/Reaction Data:

Bar/React	Start	End	Length	cos	sin
1	1	2	4	1	0
2	2	3	4	1	0
3	1	4	3	0	1
4	4	6	2.5	0.8	0.6
5	6	7	4	1	0
6	7	5	2.5	0.8	−0.6
7	3	5	3	0	1
8	4	2	5	0.8	−0.6
9	6	3	7.5	0.8	−0.6
10	1	7	7.5	0.8	0.6
11	2	5	5	0.8	0.6
12	8	1	5	1	0
13	9	1	5	0	1
14	10	3	5	0	1

After entering or updating data, press "/X"

2–Matrix:

Joint:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Jt Forces	Bar Forces/React
1	−1	0	0	0	0	0	0	0	0	−0.8	0	1	0	0	0	183.3333
1	0	0	−1	0	0	0	0	0	0	−0.6	0	0	1	0	0	150
2	1	−1	0	0	0	0	0	0.8	0	0	−0.8	0	0	0	0	25
2	0	0	0	0	0	0	0	−0.6	0	0	−0.6	0	0	0	0	20.83333
3	0	1	0	0	0	0	0	0	0.8	0	0	0	0	0	0	166.6666
3	0	0	0	0	0	0	−1	0	−0.6	0	0	0	0	1	0	104.1666
4	0	0	0	−0.8	0	0	0	−0.8	0	0	0	0	0	0	0	50
4	0	0	1	−0.6	0	0	0	0.6		0	0	0	0	0	0	−20.8333
5	0	0	0	0	0	0.8	0	0	0	0	0.8	0	0	0	−100	−187.5
5	0	0	0	0	0	−0.6	1	0	0	0	0.6	0	0	0	0	−104.166
6	0	0	0	0.8	−1	0	0	0	−0.8	0	0	0	0	0	0	20.83333
6	0	0	0	0.6	0	0	0	0	0.6	0	0	0	0	0	100	100
7	0	0	0	0	1	−0.8	0	0	0	0.8	0	0	0	0	0	−37.5
7	0	0	0	0	0	0.6	0	0	0	0.6	0	0	0	0	0	−62.5

2–inverse

0	0	1	0.666666	1	0	0.5	0	1.5	0	0.75	−0.33333	0.75	−1
0	0	0	0.666666	1	0	−0.5	0	0.5	0	0.25	−1	0.25	−0.33333
0	0	0	1	0	0	−9.3E−18	1	0.75	0	0.375	0.5	0.375	−0.5
0	0	0	0.833333	0	0	−0.625	0	0.625	0	0.3125	0.416666	0.3125	−0.41666
0	0	0	1.333333	0	0	−1	0	1	0	−0.5	−0.66666	0.5	−0.66666
0	0	0	0.833333	0	0	−0.625	0	0.625	0	−0.3125	−0.41666	−0.3125	0.416666
0	0	0	1	0	0	−0.75	0	9.3E−18	1	−0.375	−0.5	−0.375	0.5
0	0	0	−0.83333	0	0	−0.625	0	−0.625	0	−0.3125	−0.41666	−0.3125	0.416666
0	0	0	−0.83333	0	0	0.625	0	−0.625	0	−0.3125	1.25	−0.3125	0.416666
0	0	0	−0.83333	0	0	0.625	0	−0.625	0	0.3125	0.416666	0.3125	1.25
0	0	0	−0.83333	0	0	0.625	0	0.625	0	0.3125	0.416666	0.3125	−0.41666
1	0	1	−1.2E−16	1	0	1	0	1	0	1	7.9E−17	1	−6.0E−17
0	1	0	0.5	0	0	0.375	1	0.375	0	0.5625	0.75	0.5625	0.25
0	0	0	0.5	0	1	−0.375	0	−0.375	1	−0.5625	0.25	−0.5625	0.75

Plate 3

Illustrative Problem 3
END SUPPORTED BEAM

Span = 9

Concentrated Loads (positive downward):

W:	100	200	100
x :	3	4.5	6

Two Releases (1—guide, 2—pin):

Type:	2	2
Loc:	0	9

Calculation of Vo and Mo:

			Sums		Rel—2,2	Rel1—1, 2	Rel—2, 1
Release 1	0	0	0	0	200	0	0
	0	0	0	0	0	0	0
Release 2	-100	-200	-100	-400			
	-300	-900	-600	-1800			

Vo = -200

Mo = 0

Shear and Bending Moment at x = 4 V — — -100

M — — 700

-100	0	0	-100
-300	0	0	-300

CONCLUSION

More systematic approaches to elementary problems in engineering statics have been shown to lead to solutions which are precise, general, and well suited for programmed or coded implementation particularly on microcomputers. Such implementations are evidently desirable for practical computations. They also facilitate instructive parametric studies and are therefore completely in line with Hamming's (1962) highly apt motto — that the purpose of computing is insight and not numbers.

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