"This paper demonstrates the applicability of the finite element method in the solution of typical boundary value problems in geotechnical engineering."

Finite Element Studies of Ambuklao and Caliraya Dams*

by

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ABSTRACT

In the last two decades the finite element method has experienced tremendous growth in both theoretical developments and applications. This paper explores the usefulness of this numerical method in the area of geotechnical engineering, incorporating into the implementation complex factors that cannot readily be handled by closed-form procedures such as dynamic behavior, material nonhomogeneities, plasticity, and creep.

Subjects of the studies were two hydroelectric power-generating dams in the Philippines, namely, Ambuklao and Caliraya. Eigenvalue studies were performed of Ambuklao Dam to investigate its dynamic response to the ground shaking induced by the earthquake of April, 1985. A creep analysis of Caliraya Dam was also carried out using the conventional incremental plasticity theory and a recently developed constitutive model to predict the dam's time-dependent deformation behavior in the next 40 years.

INTRODUCTION

Engineers are often faced with a variety of complex engineering problems for which no standard solutions are available. Particularly in the area of geotechnical engineering, the degree of difficulty of problems normally encountered can be enormous considering that a large number of complex factors such as material nonlinearities and nonhomogeneities may be involved.

In the past, solutions of so-called nonstandard engineering problems were done either on a purely empirical basis or via simplifying assumptions aimed at reducing the problem to one where a standard solution is available. Oftentimes, the results obtained from any of these approaches were either highly inaccurate or totally unrealistic.

With the advent of high-speed computers and exceptional growth in the development of numerical procedures, engineers are now able to incorporate as many details as are necessary into the solution. Among the existing numerical procedures the finite element method seems to have gained the most favorable acceptance, perhaps because of its reliable accuracy as well as its versatility when applied to different classes of engineering problems.

This paper demonstrates the computational capability of the finite element method in solving typical boundary value problems in the area of geotechnical engineering. Subjects of the studies are

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the Ambuklao and Caliraya Hydroelectric Dams on a remedial project research aimed at assessing the dams' present conditions. The behavior of the dams when subjected to seismic excitation and long-term sustained loadings are investigated.

HOW AMBUKLAO DAM SWAYS AS THE GROUND SHAKES

Background

The Ambuklao Dam is a rockfill hydroelectric power-generating dam and is part of the Ambuklao Hydroelectric Project owned and operated by the National Power Corporation of the Philippines. The dam serves a drainage area of about 265 sq. miles above its site on the Agno River, island of Luzon.

The dam has a crest height of 131 meters relative to its base, the foundation consisting primarily of metamorphic rocks and diorite. Figure 1 shows a section through the dam and reveals two basic material zones represented by the central clay core and outer rockfill shells with layers of filter materials in between.

On April 24, 1985 a strong earthquake shook the damsite. As part of a remedial project undertaken to ensure its safe operation, a numerical study aimed at investigating the dam's dynamic response to ground shaking was conducted.

The study was confined to the determination of the dam's fundamental periods of vibration and mode shapes as no time history of ground shaking was recorded to make a complete analysis possible.

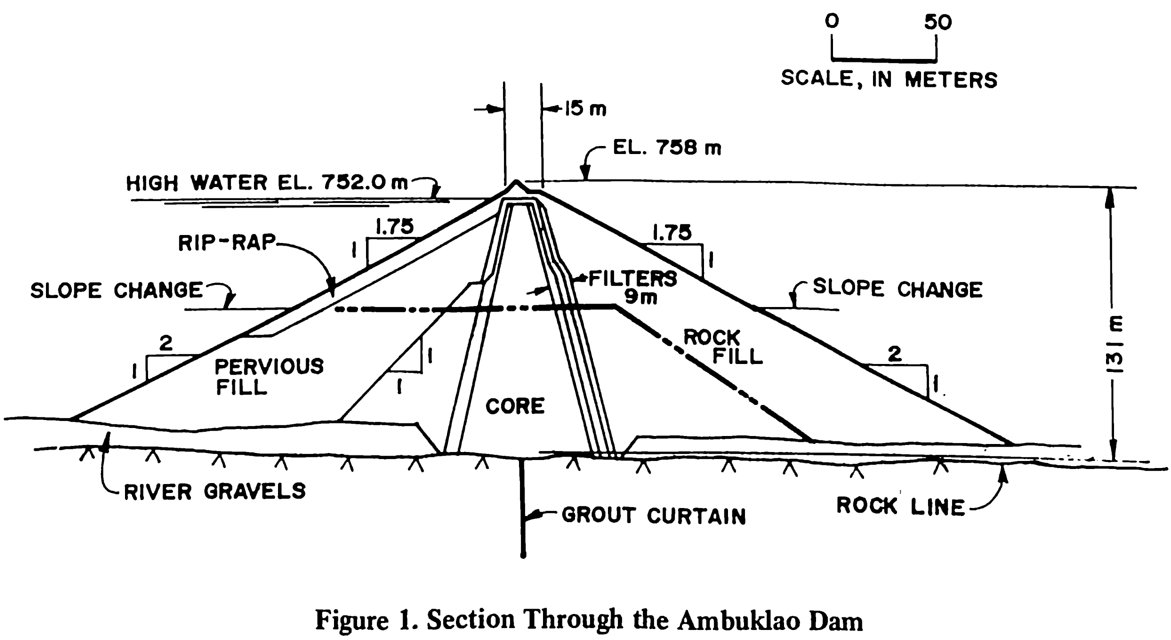


Figure 1. Section Through the Ambuklao Dam

Theoretical Formulation

For dynamic equilibrium of a body in motion the governing finite element-based matrix equation at time station t_n can be written as follows:

$$\mathbf{M}\ddot{\mathbf{d}}_n + \mathbf{C}\dot{\mathbf{d}}_n + \mathbf{K}\mathbf{d}_n = \mathbf{f}_n \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the consistent global mass, damping, and stiffness matrices, respectively, $\ddot{\mathbf{d}}_n$ is the global vector of nodal accelerations, $\dot{\mathbf{d}}_n$ is the global vector of nodal velocities, $\mathbf{d}_n$ is the global vector of displacements, and $\mathbf{f}_n$ is the vector of consistent nodal forces for the applied surface tractions and body forces (including the inertia term $-\mathbf{M}\ddot{\mathbf{u}}_g$ due to seismic excitation) grouped together. If the forcing functions on the right hand side of (1) are given explicitly, the displacement, velocity, and acceleration profiles can be determined by temporal discretization (e.g., Newmark integration scheme) or by modal superposition. For a complete treatment of the subject, the reader is referred to Owen and Hinton (1980).

Let the displacement, velocity, and acceleration vectors be of the form

$$\mathbf{d} = \mathbf{a}e^{iwt} \tag{2a}$$
$$\dot{\mathbf{d}} = iw\mathbf{a}e^{iwt} \tag{2b}$$
$$\ddot{\mathbf{d}} = -w^2\mathbf{a}e^{iwt} \tag{2c}$$

where w is the angular frequency associated with mode shape $\mathbf{a}$, $i = \sqrt{-1}$, and t is time. The eigenvalue problem for an undamped finite element system requires the determination of the non-trivial solutions to the homogeneous equations

$$(-w^2\mathbf{M} + \mathbf{K})\mathbf{a} = 0 \tag{3}$$

by setting the determinant of the coefficient matrix

$$\left| -w^2\mathbf{M} + \mathbf{K} \right| = 0 \tag{4}$$

In general, equation (4) will result in n_{eq} roots or vibration modes corresponding to the total number of free nodal degrees of freedom for a given finite element mesh.

Assuming that $\mathbf{M}$ admits the factorization

$$\mathbf{M} = \mathbf{L}\cdot\mathbf{L}^{\mathrm{T}} \tag{5}$$

in which $\mathbf{L}$ is either upper triangular or diagonal, equation (3) can be reduced to the standard form

$$(\mathbf{S} - \mathbf{I}w^2)\mathbf{b} = 0 \tag{6}$$

where $\mathbf{S} = \mathbf{L}^{-1}\cdot\mathbf{K}\cdot\mathbf{L}^{-\mathrm{T}}$ is a real symmetric matrix, $\mathbf{b} = \mathbf{L}\cdot\mathbf{a}$, and $\mathbf{I}$ is the identity matrix. The same roots of (4) can be obtained by setting

$$\left| \mathbf{S} - \mathbf{I}w^2 \right| = 0, \tag{7}$$

solving for the eigenvalues w_i^2, and evaluating the eigenvectors $\mathbf{b}_i$ of the matrix $\mathbf{S}$. The back-substitution process $\mathbf{a}_i = \mathbf{L}^{-1}\mathbf{b}_i$ leads to the original eigenvectors or mode shapes of equation (3).

If $\mathbf{M}$ is a diagonal, the factorization step indicated in (5) is rendered trivial since $\mathbf{L} = \mathbf{L}^{\mathrm{T}}$ as $\mathbf{L}$ also becomes diagonal. Consequently,

$$\mathbf{L} = \sqrt{\mathbf{M}} \tag{8}$$

which implies that $L_i = \sqrt{M_i}$ where L_i and M_i are the ith diagonal elements of $\mathbf{L}$ and $\mathbf{M}$, respectively.

Matrix diagonalization of **M** can be achieved by lumping masses at the nodes. Consider the following finite element expression for **M** written on the element level (consult Owen and Hinton, 1980):

$$\mathbf{m}^e = [\mathbf{m}^e_{ab}] = \int_{\Omega^e} \mathbf{N}_a^{\mathrm{T}}\, m\, \mathbf{N}_b \, d\Omega \tag{9}$$

where $\mathbf{m}^e_{ab}$ is a typical submatrix of $\mathbf{m}^e$ associated with element nodes a and b, $\mathbf{N}_a$ and $\mathbf{N}_b$ are the nodal shape function matrices, m is mass density, and Ω^e is the element domain. It can be seen that if the numerical integration of (9) is performed at the element nodes, $\mathbf{m}^e$ diagonalizes since

$$\begin{aligned} \mathbf{N}_a^{\mathrm{T}} \mathbf{N}_b &= \mathbf{I}\,, \quad \text{when } a = b \\ &= \mathbf{O}\,, \quad \text{otherwise,} \end{aligned} \tag{10}$$

where **I** and **O** are identity and null matrices, respectively.

Numerical Solution

The computer program used in the analysis is a general multipurpose finite element program called SPIN 2D. A module of this program performs the matrix operations outlined in the preceding section for eigenvalue and eigenvector determination. The consistent mass and stiffness matrices are computed and assembled internally for linear elastic or elasto-plastic materials obeying the associative flow rule. Element types can be any of the standard four-, eight-, or nine-noded isoparametric quadrilateral elements.

The finite element mesh used for the dam problem is shown in Figure 2. Ignoring some minor material and geometric irregularities indicated in Figure 1, the cross section of the dam was assumed to be symmetric about its centerline.

The mesh consists of 48 nodes and 35 isoparametric 4-noded quadrilateral elements. Six nodes were assumed completely fixed at the base, leaving a total number of 2x (48 – 6) = 84 degrees of freedom or vibration modes.

Numerical integration is facilitated by a standard 2x2 Gauss quadrature rule on each element. The material constants for each element were evaluated at the element centroid. Masses for the elements were lumped at the nodes via selective numerical integration discussed in the preceding section.

An eigenvalue solver of the iterative-type was used for evaluating the vibration periods and mode shapes. The mass matrix factorization/back substitution steps of the preceding section were employed in the computation.

Material Properties

The thirty five elements of the mesh shown in Figure 2 were divided into two material groups representing the inner clay core and the outer rock fill materials. Material properties assigned each group were estimated from the following:

1. *Clay Core.* Estimates of the soil's index properties, compressibility, and strength were based on laboratory test results on undisturbed samples taken from test pits dug at the clay core section of the dam.

Table 1 summarizes the average values of compressibility parameters used in the analysis. These values, obtained from five-one dimensional consolidation tests, exhibit a consistently stiff behavior characteristic of a well-compacted middle clay core material.

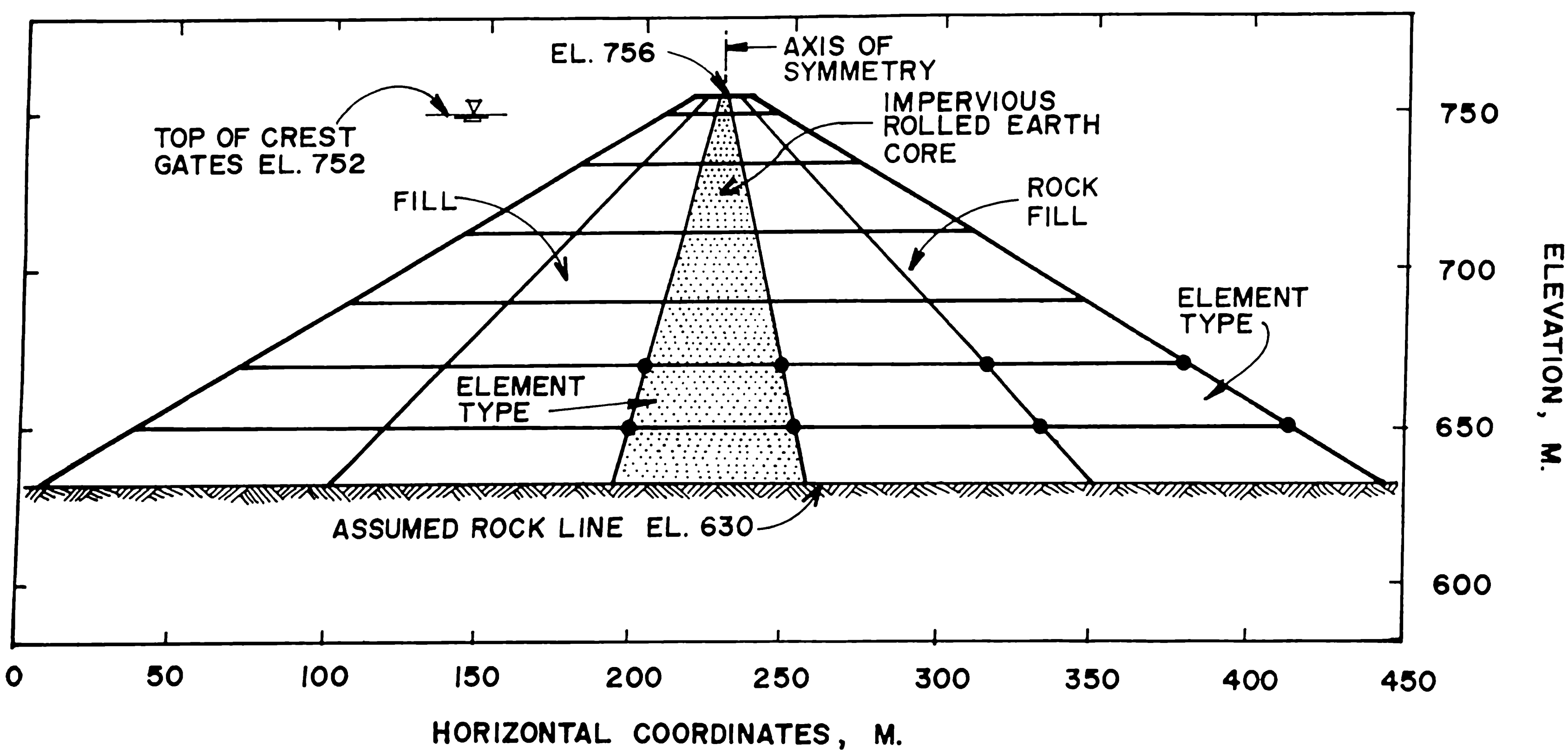


Figure 2. Finite Element Mesh for Ambuklao Dam

Triaxial compression tests were also performed for shear strength determination, with cohesion C and friction angle Ø adopted as strength indicators. The average values of C and Ø are tabulated in Table 2 for various types of testing; namely, unconsolidated undrained (UU), consolidated undrained (CU), and consolidated drained (CD). Also shown in Table 2 are the strength properties obtained from saturated, consolidated-drained, direct shear tests.

The clay's deformation behavior was modelled using the Cam Clay theory presented by Roscoe and Burland (1968). This model assumes an elasto-plastic strain-hardening stress-strain behavior in which the yield surfaces are a family of ellipsoids centered about the hydrostatic axis. Although the clay core material may have been preconsolidated to a certain degree due to aging, the inertia loads were assumed large enough to initiate yielding. An elasto-plastic stress-strain matrix of the form presented by Borja and Kavazanjian (1985) was used in the stiffness matrix formation.

2. *Rock Fill.* Rock-fill materials were assumed to be isotropic and linearly elastic with the following material constants:

Poisson's ratio $\nu = 0.30$

Young's modulus $E = kp$

where p is the mean normal stress and k is a constant (Lambe and Whitman, 1969).

Table 1
Ambuklao Dam: Central Clay Core
Compressibility Parameters

1.	Swelling/Recompression Index	C_r	0.027
	Cam Clay Index $\varkappa = 0.43C_r$		0.012
2.	Virgin Compression Index	C_c	0.153 ± 0.018
	Cam Clay Index $\lambda = 0.43C_c$		0.066 ± 0.008
3.	Average Void Ratio at p = 1 ton/sq.m. on the isotropic consolidation line		0.70

Table 2
Ambuklao Dam: Central Clay Core
Strength Parameters

Type of Test	Friction Angle Ø, deg	Cohesion, tons/sq.m.	Initial Tangent modulus E_i, $x\sigma_c$
A. Triaxial			
1. UU	21	15.2	variable
2. CU	24	6.4	128
3. CD	29	3.8	103
B. Direct Shear	40	1.2	variable

The above estimate for E necessitates an estimate of the stress distribution within the dam due to gravity loads. A preliminary statical analysis was performed to obtain these self weight-induced stresses for use as input in the eigenvalue analysis.

The constant k was determined from a calibration study based on a recorded horizontal movement of 4 inches of a bench mark in the middle of the dam's crest when the reservoir was gradually filled to spillway crest Elevation 740 between the wet season of 1955 to early 1956 (Fucik and Edbrooke, 1958). From this study, the constant k was estimated to be about 85.

Discussion of Results

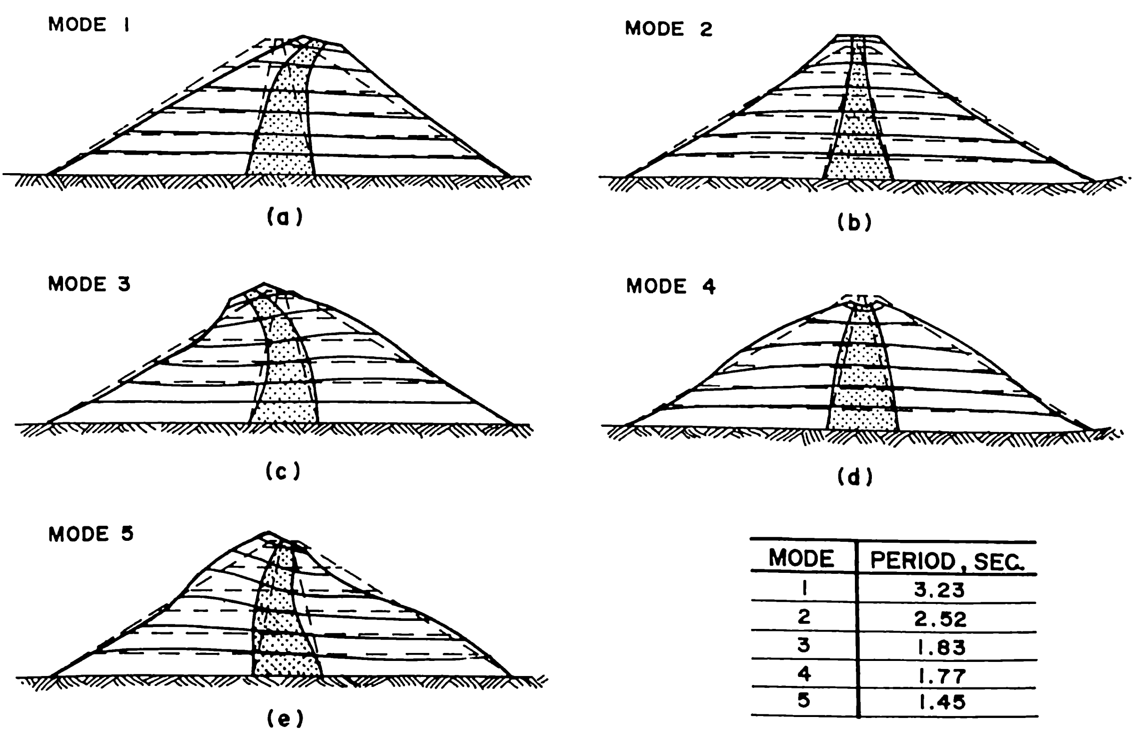
Shown in Figure 3 are the mode shapes defining the first five fundamental modes of vibration.

Mode 1 has a fundamental period of 3.23 seconds and appears to be a sidesway mode (Figure 3a). This mode does not seem to result in significant volume change. The resulting displacements are primarily distortional.

Mode 2 has a fundamental period of 2.52 seconds (Figure 3b). Oscillations are primarily vertical, resulting in significant volume change. The horizontal components of displacements are symmetric about the centerline, causing lateral expansion to develop at the crest during the time instant shown in Figure 3b. At another time instant not shown, the top zone of the dam may also contract during oscillation as the nodal displacements are reversed.

Modes 3 through 5 appear to be higher level vertical and sidesway modes (Figure 3c, d, e). Mode 4 is a higher level vertical mode having a period of 1.77 seconds (Figure 3d).

The two vertical mode shapes depicted in Figure 3b and d are worthy of note. The deformed shapes shown are clearly reflective of the relative difference in compressibilities between the inner clay core and the outer rock-fill materials, the more compressible inner material apparently tending to "pull" the less compressible rock-fill to a state of larger deformation.



MODE	PERIOD, SEC.
1	3.23
2	2.52
3	1.83
4	1.77
5	1.45

Figure 3. Ambuklao Dam: The First Five Modes of Vibration

CALIRAYA DAM: PREDICTION OF CREEP AND CYCLIC MOVEMENTS

Background

The Caliraya Hydroelectric Dam of Lumban, Laguna is part of the Kalayaan Power Plant System operated by the National Power Corporation of the Philippines.

The 45 meter-high dam sits on a volcanic bedrock consisting primarily of agglomerate and basalt with its main dike spanning about 300 meters across the Caliraya River. Constructed of rolled earth fill of clay to silty clay composition, the dam was completed about forty years ago and has since served as a water impounder until the operation of the pumped storage system in 1982.

As part of an on-going project study to assess the dam's present condition, a finite element deformation analysis was conducted. The results presented in this paper cover the prediction of dam movements due to the reservoir drawdown/filling as well as creep deformations expected to develop in the next forty years.

Theoretical Formulation

The numerical algorithm employed in the analysis is based on an elasto-plastic creep-inclusive rate constitutive equation of the form (summation implied on repeated subscripts)

$$\dot{\sigma}_{ij} = C_{ijkl}\dot{\epsilon}_{kl} - \dot{\sigma}^t_{ij} \tag{11}$$

where σ_{ij} is the Cauchy stress tensor, ϵ_{kl} is the small strain tensor, C_{ijkl} is the material tangent stress-strain tensor, and the overdots denote a material time differentiation. The quantity $\dot{\sigma}^t_{ij}$ is the stress relaxation rate due to creep which is converted into element nodal pseudo-forces via

$$\dot{\mathbf{F}}^e_{creep} = -\int_{\Omega^e} \mathbf{B}^T \dot{\sigma}^t d\Omega \tag{12}$$

where **B** is the strain-displacement matrix and Ω^e is the element domain. For a complete discussion of the development of (11), the reader is referred to Borja and Kavazanjian (1985).

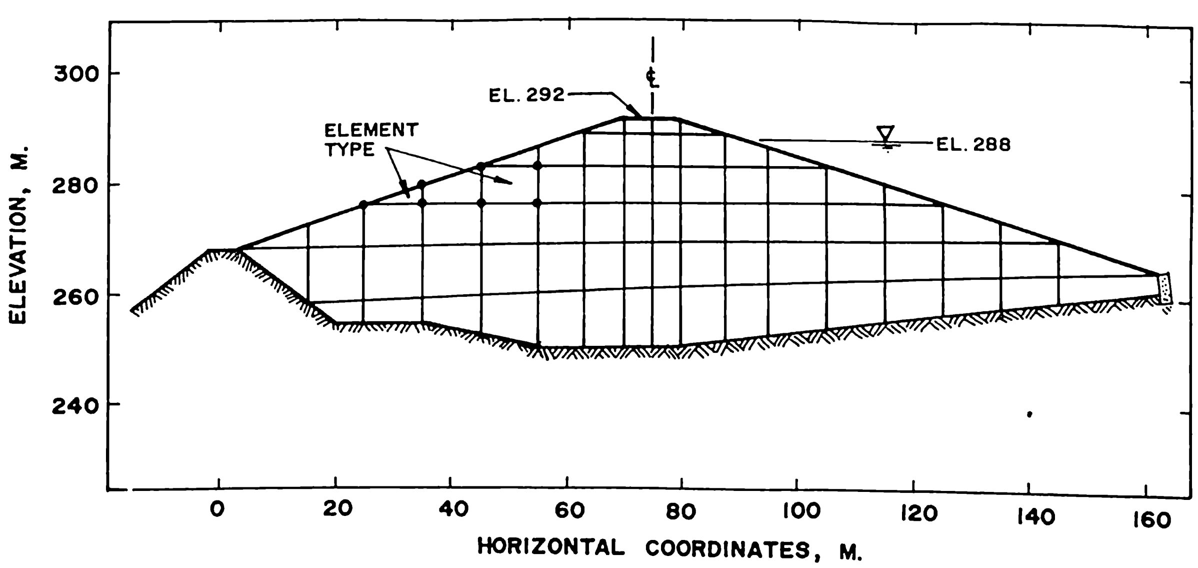


Figure 4. Finite Element Mesh for Caliraya Dam Cross Section at Sta. 0+380

Numerical Solution

The finite element program used in the analysis is a module of SPIN 2D which performs the time integration of (11) by the conventional incremental theory of plasticity.

The finite element mesh for the dam is shown in Figure 4 for cross section at Sta. 0+380. The mesh consists of 87 nodes of which 20 are fixed and 73 isoparametric 4-noded quadrilateral elements, resulting in a total number of 2x (87–20) = 134 unknown degrees of freedom. The numerical integration rules employed are as discussed in Section 2. Material properties used are as summarized in Tables 3 and 4.

Initial stresses are required as input data to enable complete definition of initial states of stress and their positions relative to the corresponding yield surfaces. Initial stresses are due to both the dam's self weight and hydrostatic pressure, acting normal to the upstream face of the dam, as the reservoir was filled to design capacity.

Table 3
Caliraya Earth Fill Dam
Compressibility & Creep Parameters

1.	Swelling/Recompression Index	C_r	0.0250
	Cam Clay Index $\kappa = 0.43\, C_r$		0.0109
2.	Virgin Compression Index	C_c	0.2500
	Cam Clay Index $\lambda = 0.43\, C_c$		0.1090
3.	Secondary Compression Index	C_a	0.0067
	Natural Log Index $\psi = 0.43\, C_a$		0.0029
4.	Average Void Ratio at p = 1 ton/sq.m. on the isotropic consolidation line		1.55

Table 4
Caliraya Earth Fill Dam
Strength Parameters

Type of Test	Friction Angle ϕ, deg.	Cohesion c, tons/sq.m.
1. UU	25	4.9
2. CU	15	10.6
3. CD	18	6.2

To obtain the self weight-induced stresses an elastostatic analysis was performed using an average soil unit weight of 1.76 tons/sq.m. (standard deviation = 0.16 ton/sq.m. from lab test results). Superimposing the additional stresses due to hydrostatic pressure as the reservoir was filled to Elev. 288, the total principal stresses and their directions are as depicted in Figure 5.

Discussion of Results

1. *Predicted movement due to creep.* Under sustained initial stresses resulting from gravity and hydrostatic loads each finite element was allowed to creep according to the secondary compression equation. A complete definition of all the components of creep strain was made by employing the normality rule on the same ellipsoidal yield surfaces as in the time-independent plasticity model.

Figure 6 illustrates the displacement profile due to creep predicted to develop over the next forty years. A maximum crest settlement of about 10 centimeters can be seen from this profile.

A time history of movement due to creep of a point located at the center of the dam's crest is also plotted in Figure 6. As a rough check on the correctness of the above estimate, the soil was assumed to have aged by t_v = 40 years from the time that the dam was completed. Over the next Δt_v = 40 years, the settlement due to secondary compression of a dam 45 meters high is estimated to be

$$\delta_v = \frac{C_\alpha}{1+e} \log_{10} \left(1 + \frac{\Delta t_v}{t_v}\right) \times 45.0 \text{ meter}$$

$$= \frac{0.0067}{1 + 1.0} \log_{10} \left(1 + \frac{40.0}{40.0}\right) \times 45.0$$

$$= 0.050 \text{ meter.}$$

With movements due to shearing distortion still to be included, the above check verifies the predicted settlement of 0.10 meter obtained from the computer analysis.

2. *Cyclic movements due to reservoir drawdown and filling.* High and low water elevation of 289 m. and 286 m., respectively, were recorded for Caliraya Lake during the period January 1983 to October 1984.

Choosing Elev. 288 m. as a reference elevation, predicted displacement profiles due to reservoir drawdown and filling are as plotted in Fig. 7. The results of Fig. 7 were obtained from elastic analyses with elastic constants derived from the Cam Clay expressions. As permanent strains are expected to become reasonably small after several cycles of loading, a stable hysteresis loop was assumed.

It can be seen from Fig. 7 that a maximum displacement of about 1 cm is likely to occur in the neighborhood of nodes 19 and 21 as the water elevation fluctuates from Elev. 286 to Elev. 289. This movement is considered small to cause serious cracking in the dam's upstream concrete lining provided that the drawdown/filling rates are slow enough.

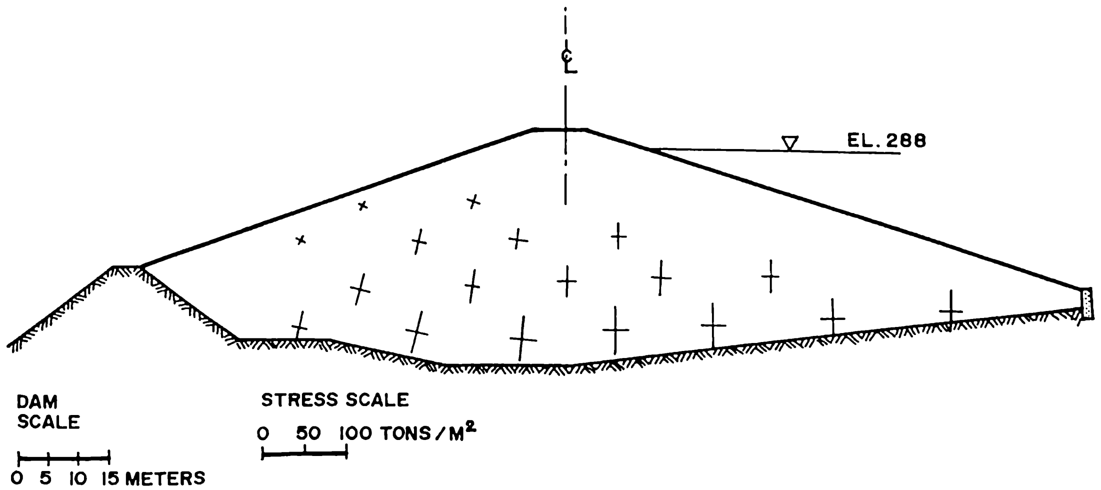


Figure 5. Caliraya Dam: Total Principal Stresses and Directions

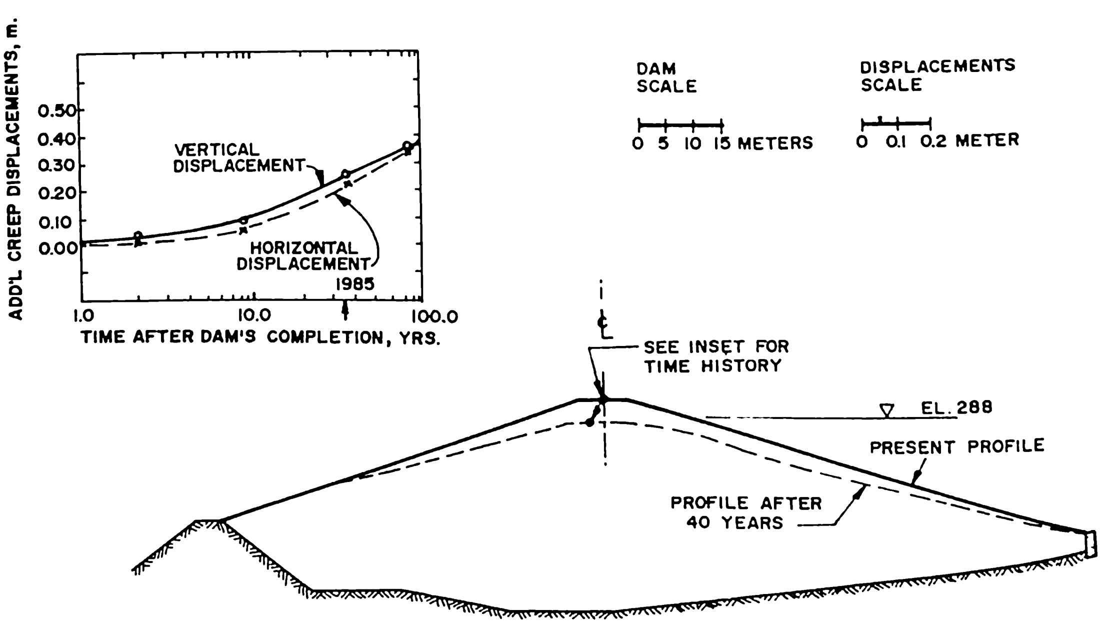


Figure 6. Caliraya Dam: Predicted Creep Displacement Profile

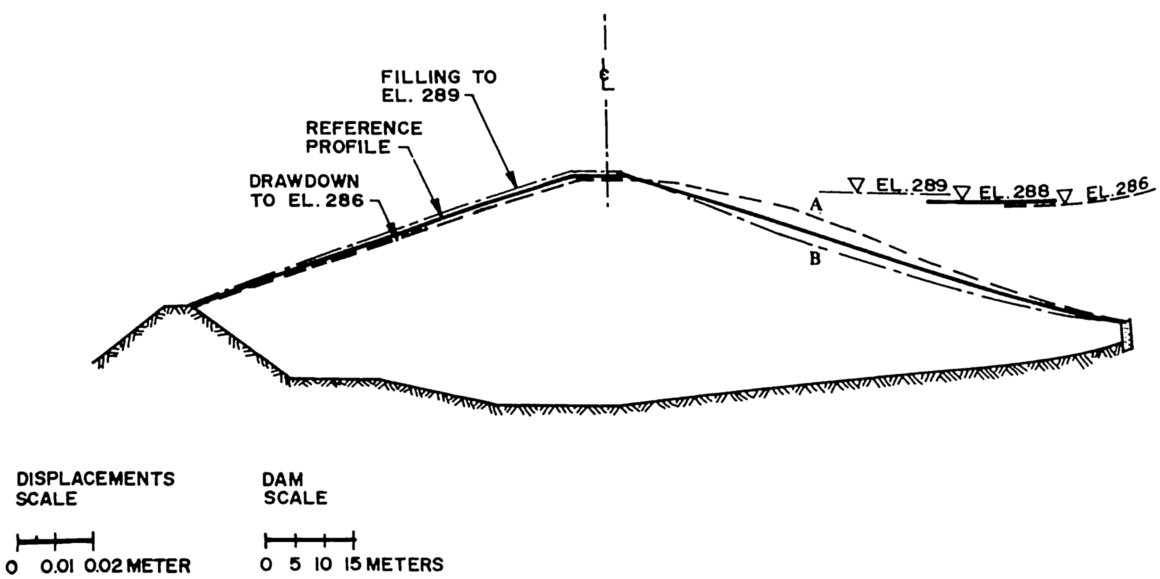


Figure 7. Caliraya Dam: Cyclic Movement Due to Reservoir Drawdown and Filling

CONCLUDING REMARKS

This paper demonstrates the applicability of the finite element method in the solution of typical boundary value problems in geotechnical engineering.

Although the results presented in this paper were established *a priori* and remain subject to verification by actual field instrumentation, it was demonstrated that the above numerical method can be extremely useful in accounting for complex factors such as material nonlinearities and non-homogeneities into the solution.

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