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Marine Propeller Selection and the Stochastic Sea

by

Reynaldo B. Vea*

Introduction

The traditional procedure of marine propeller selection inherently assumes that the ship encounters only calm water throughout her lifetime. This assumption invariably departs from reality – the degree of departure depending upon the intended route of the vessel. In the North Atlantic, for example, calm water conditions may obtain for only 40 percent of a ship's annual steaming time.

This paper shows a method by which the traditional procedure may be generalized to incorporate other sea conditions as inputs to the propeller selection process.

Ship speed and hull resistance are major input variables to such a process. In a disturbed sea, a ship encounters additional resistance due to wave and wind action, resulting in loss of speed. On top of this involuntary decrease in speed, the ship's master may order a further reduction to limit ship motions to acceptable levels. A change in heading or a combination of course-change and speed-reduction are also among his options.

The traditional procedure is based on a single point, namely, ship design speed and the corresponding resistance in calm water. The consideration of other sea conditions can be done by including multiple combinations of speed and associated resistance which the ship may be expected to experience in service. By also considering the probability of occurrence of each such speed-resistance combination, the approach more closely approximates the nature of the sea.

Triantafyllou (1) has worked out a procedure to incorporate involuntary speed loss. This paper extends this work by including voluntary speed reduction. Change in ship's heading is not considered.

The new procedure becomes practicable only if computer-aided. 64 K RAM is more than sufficient.

Since propeller efficiency ultimately affects the fuel bill, a comparison of the projected fuel consumption using the traditional method and the new approach is carried out.

The Traditional Approach

An optimum propeller may be found starting from either of two constraints: 1) diameter is restricted due to the geometry of the stern or 2) RPM (revolutions per minute) is set to match the RPM of an engine contemplated to be used.

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Inputs. For diameter-constrained optimization, propeller diameter (D) is an input whereas propeller RPM (n) is an input to RPM-constrained optimization. The maximum diameter that can be accommodated is always chosen since propeller efficiency increases with diameter.

Among the inputs common to both types of design problem are:

1. *Design speed* of ship (V).
2. *Hull resistance* (R). This is the total force exerted by seawater on a towed hull. In the preliminary stage of design, prior to model testing in a towing tank, an estimate of hull resistance at various speeds is made by extrapolating model resistance data for a standard series of ship forms which have been systematically varied. All such data are based on calm water conditions.

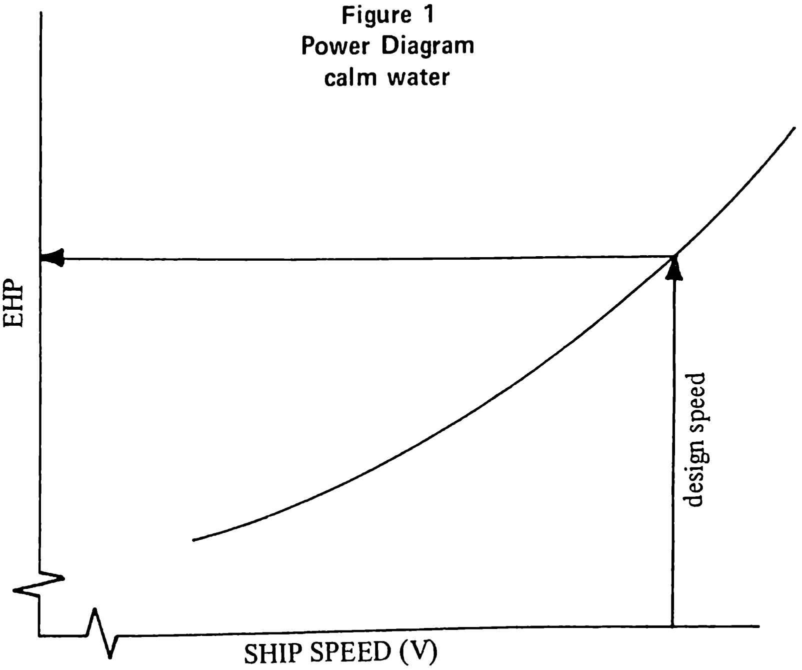
   The effective horsepower (EHP), which is the power necessary to tow the ship at a certain speed, can be calculated from the relation:

   $$EHP = R * V * \text{factor} \qquad (1)$$

   the factor depending upon the units used.

   The usual representation of ship resistance is by means of the power diagram as shown in Figure 1.

Figure 1
Power Diagram
calm water



3. *Wake fraction* (w). As a ship plows through water, the water around its stern acquires a forward motion due to frictional drag and reduced relative velocity of the water as the streamlines close in at the stern. This forward-moving water is called the wake.

   Due to the wake, the propeller's forward speed is not the same as the ship's speed. It is a lower speed called the speed of advance (Va). The wake fraction is defined as follows:

   $$w = \frac{V - Va}{V} \qquad (2a)$$

   $$Va = (1 - w)V \qquad (2b)$$

Reference 2 gives data on w.

4. *Thrust deduction factor* (t). With a self-propelled hull, the pressure over the stern is reduced by the acceleration of the water flowing into the propeller. This reduction of pressure (and hence its forward component) is manifested as an increase in resistance. Therefore, the thrust (T) necessary to propel the ship will be greater than the resistance of a towed ship.

   It is the usual practice to view this increase in resistance as a reduction in thrust available at the propeller. Thus:

$$t = \frac{T - R}{T} \qquad (3a)$$

$$R = (1 - t)\,T \qquad (3b)$$

   Reference 2 gives data on t.

Outputs. Among the outputs of both diameter-constrained and RPM-constrained optimization are:

1. *Open-water efficiency* (η_o). "Open-water" conditions refer to running the propeller without any hull ahead of it. This is the set-up when the performance characteristics of a model propeller are measured. Open-water efficiency is defined as

$$\eta_o = \frac{TVa}{2\pi nQo} \qquad (4)$$

   where Qo is the open water torque at the propeller end of the shaft.

2. *Pitch ratio* (P/D). In the simplest case, the surface of a propeller facing aft is a portion of a true helical surface. When the generating line of a helical surface has made a complete revolution, the distance it has advanced is called the pitch (P). In practice, the pitch varies with radii. In such cases, the pitch at 0.7* radius is often taken as the representative mean pitch because this is approximately the point at which maximum thrust is generated.

   The ratio of pitch to propeller diameter is termed as the pitch ratio.

Design procedure. There are two main methods by which a marine propeller may be designed. The first involves the use of circulation theory and is resorted to when the propeller is heavily loaded and liable to cavitation. The second involves the use of charts containing the results of open-water tests on a series of model propellers. The latter method is used in this paper.

The B-series propellers. Open-water tests on a methodically varied series of propellers cover variations in a number of design parameters: pitch ratio, blade area, number of blades (Z) and section shapes.

The Netherlands Ship Model Basin (NSMB) at Wageningen has conducted the most extensive series testing. The second set it tested, called the series B, is the most popular worldwide. Figure 2 shows the typical geometry for a four-bladed propeller in the series.

A designation like B.4.40 indicates a four-bladed propeller with a 40 percent expanded area ratio (EAR), which is the ratio of the expanded surface area of all the blades to the disk area. The series extends from B.3.35 to B.7.85.

Figure 3 shows a typical chart of performance characteristics in terms of nondimensional quantities discussed below.

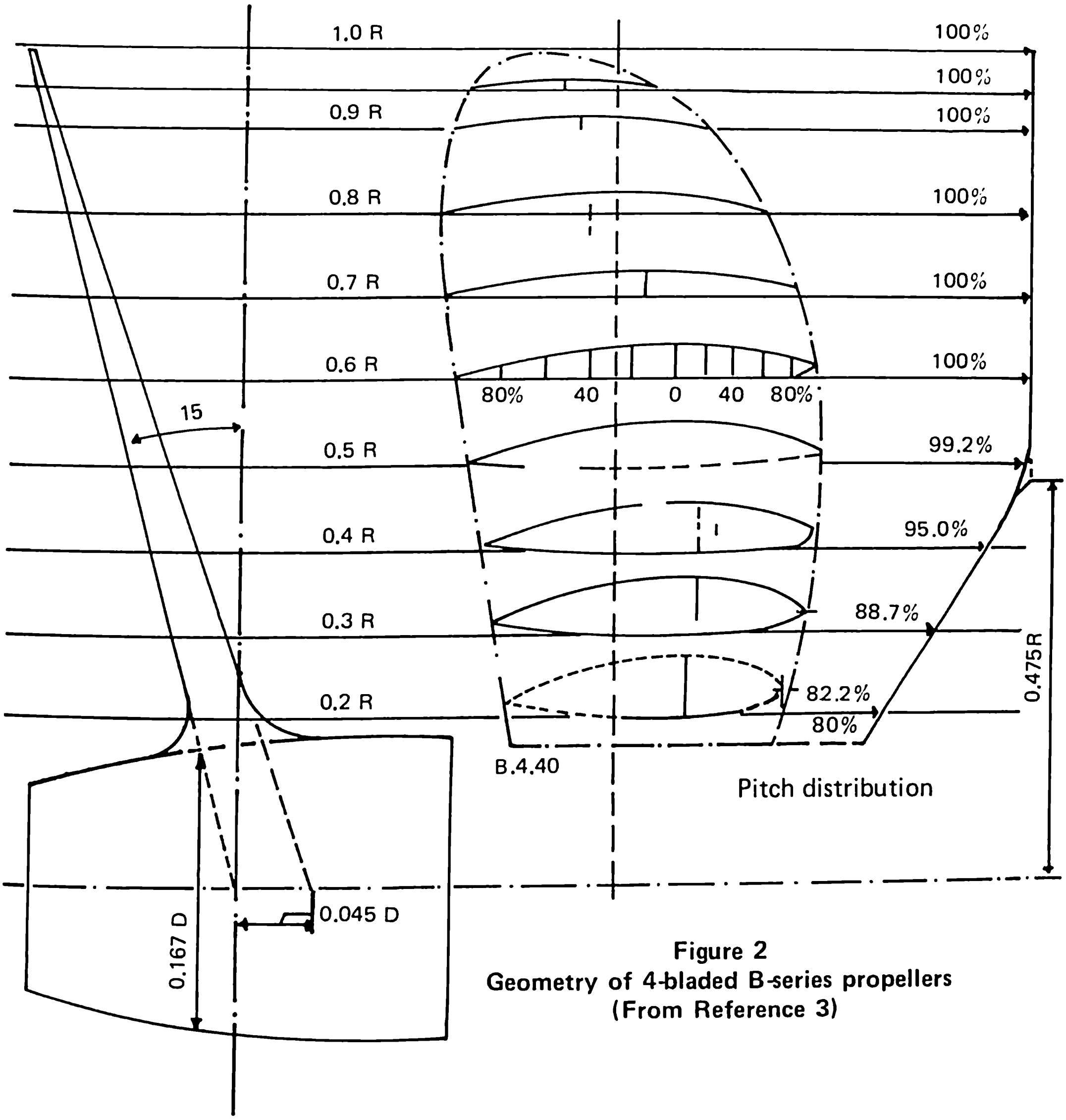


Figure 2
Geometry of 4-bladed B-series propellers
(From Reference 3)

Advance coefficient(J). This is defined as:

$$J = \frac{Va}{nD} \tag{5}$$

Dimensional analysis reveals that for kinematic similarity J must be the same for both model and ship.

Thrust coefficient (Kt). This is defined as:

$$Kt = \frac{T}{\rho n^2 D^4} \tag{6}$$

where ρ is the fluid density.

Torque coefficient (Kq). This is defined as:

$$Kq = \frac{Qo}{\rho n^2 D^5} \tag{7}$$

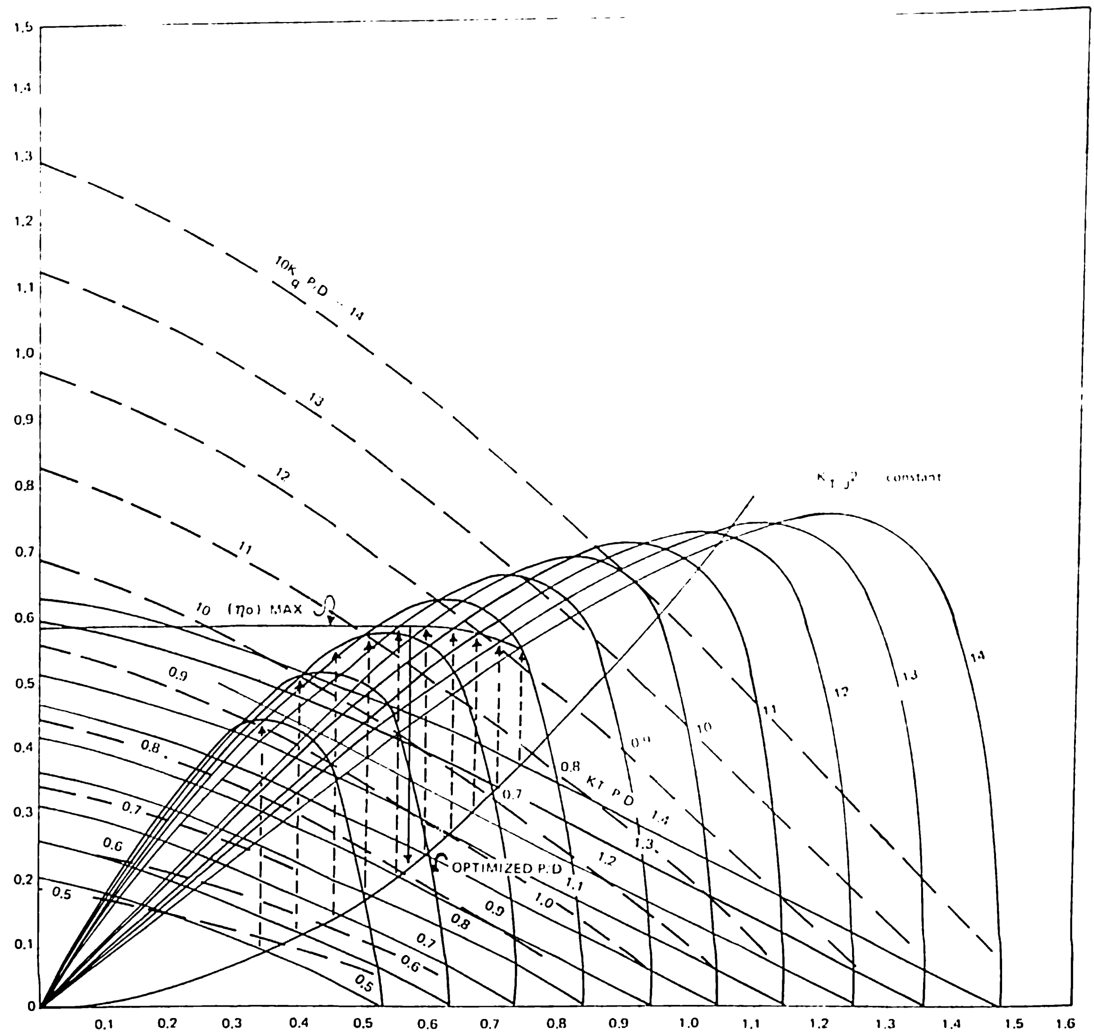


Figure 3. Typical performance characteristics curves B-series propellers

From equations 4, 5, 6 and 7, the following relation may be derived:

$$\eta_o = \frac{J}{2\pi}\,\frac{K_t}{K_q}$$

Thus, if Kt, Kq and J are known, so is ηo. In the charts, Kt, Kq and ηo are functions of J and P/D is a parameter.

A polynomial regression analysis (up to the sixth order) has been performed on the propeller data (4) and Kt and KQ are expressible as:

$$\begin{matrix} K_t \\ K_q \end{matrix} = \sum_{k}\sum_{l}\sum_{m}\sum_{n} (EAR)^k (P/D)^l Z^m J^n \qquad (8)$$

To correct for three-dimensional effects (blade interaction) and for difference in Reynold's Number (Rn), correction polynomials have likewise been formulated:

$$\begin{matrix} \Delta K_t \\ \Delta K_q \end{matrix} = \sum_{k}\sum_{l}\sum_{m}\sum_{n} (EAR)^k (P/D)^l Z^m (\log Rn - 0.301)^n \qquad (9)$$

These polynomials serve as the basis for the computerization of the procedure.

Diameter-constrained optimization. Given R, V, w, t and D, the following relationship holds:

$$\frac{Kt}{J^2} = \frac{T}{\rho n^2 D^4}\,\frac{n^2 D^2}{Va^2} = \frac{R}{\rho D^2 V^2\,(1-t)\,(1-w)^2} = \text{constant} \quad (10)$$

For any ship then, Kt/J^2 is a constant.

To carry out the procedure, a choice will first have to be made about the number of blades and EAR. The choice of the number of blades is influenced by vibration and strength considerations while the choice of EAR is governed by cavitation. The higher the EAR, the less likely is cavitation but the less efficient is the propeller. It is assumed here that such a choice has been made – cavitation, vibration and strength being outside the scope of this paper.

On the appropriate chart B.Z.EAR, the parabola $Kt = \text{constant} * J^2$ may be plotted (See Figure 4.).

The intersection of the parabola with the family of Kt curves represents the possible operating points of the propeller. Each of the intersection points corresponds to a certain value of ηo, which can be determined by projecting vertically upward to the ηo curve of the same P/D. After all points have been projected, a curve may be drawn through all the values of ηo. The maximum ηo may then be determined graphically. The value of P/D corresponding to the maximum ηo is the optimized pitch ratio and may be determined by projecting back to the parabola and visually interpolating for P/D.

Several other charts may be tried and a comparison of open-water efficiencies made, although experience usually points the way towards the appropriate chart.

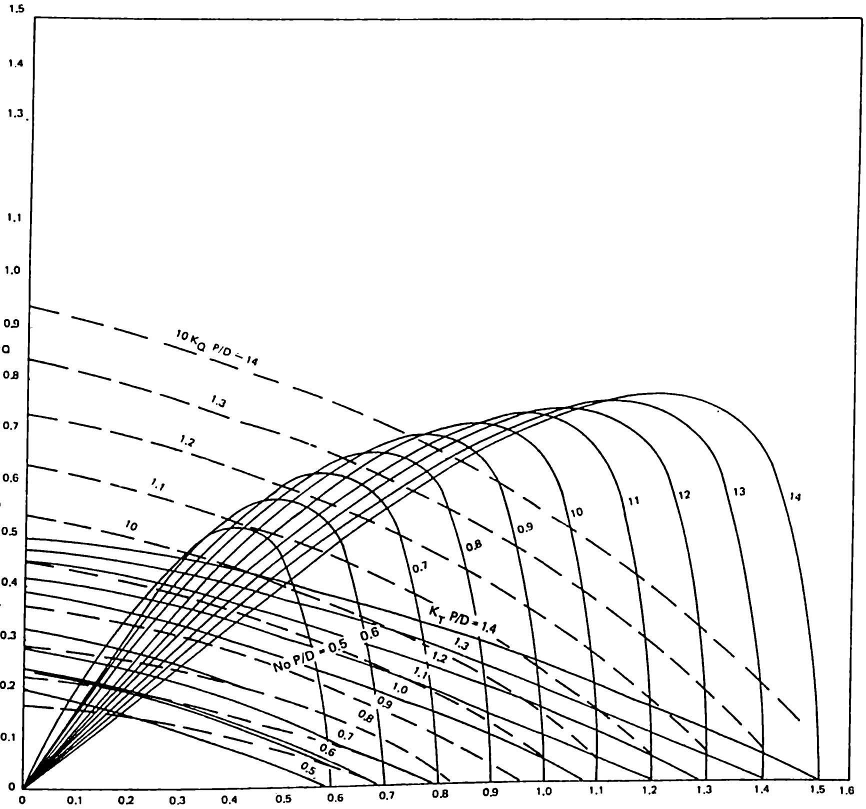


Figure 4. Propeller optimization graphical procedure

RPM-constrained optimization. The following relation holds:

$$\frac{Kt}{J^4} = \frac{R * n^2}{\rho V^4 (1-t)(1-w)^4} = \text{constant} \tag{11}$$

Essentially the same graphical procedure is used as in diameter-constrained optimization.

Engine selection. To work towards the selection of the engine, the brake horsepower (BHP) required and the engine RPM have to be determined.

For diameter-constrained optimization, the value of propeller RPM is obtained from the value of J corresponding to the optimized P/D. To get the engine RPM for either design problem, only the value of the reduction gear ratio is needed.

To get the BHP, the following equation is used:

$$BHP = \frac{EHP}{\eta o\, \eta h\, \eta_{rr}\, \eta tr} \tag{12}$$

where $\eta h = \frac{(1-t)}{(1-w)}$ = hull efficiency

$\eta rr = \frac{Qo}{Q}$ = relative rotative efficiency

Q = behind-the-hull, in contrast to open-water, torque
ηtr = shaft transmission efficiency

The value of η rr is usually taken as 1.02 for cargo ships. The value of ηtr is usually taken as 0.98. The entire denominator of equation 12 is known as the propulsive coefficient (PC).

Computerization. Using the Wageningen polynomials, equations 8 and 9, the optimization procedure is the solution to the optimizing equation

$$\frac{d(\eta o)}{d(P/D)} = 0 \tag{13}$$

subject to the constraint

$$\frac{Kt}{J^p} = \text{constant} \tag{14}$$

where p = 2 for diameter-constrained optimization and p = 4 for RPM-constrained optimization.

It may be shown, by introducing Equation 14 implicitly into Equation 13, that the following equation has to be satisfied, assuming Z and EAR to be fixed:

$$\frac{\partial Kt}{\partial (P/D)} [(p+1)Kq - J\frac{\partial Kq}{\partial J}] + \frac{\partial Kq}{\partial (P/D)} [J\frac{\partial Kt}{\partial J} - pKt] = 0 \tag{15}$$

With the polynomials for Kt and Kq known, this equation can easily be solved by computer.

The Probabilistic Approach

The basic task of the new approach is the determination of the different combinations of ship speeds and associated resistances corresponding to different sea states and the probability of occurrence of each such combination. The success of the method then hinges on the capability to estimate added resistance and ship motions in a seaway and the availability of ocean wave data.

At present, the variation of w, t and ηrr with sea state is not clearly understood. It is thus assumed here that their values at calm water conditions remain invariant with changes in sea conditions.

Figure 5 shows a schematic design of the procedure to be followed.

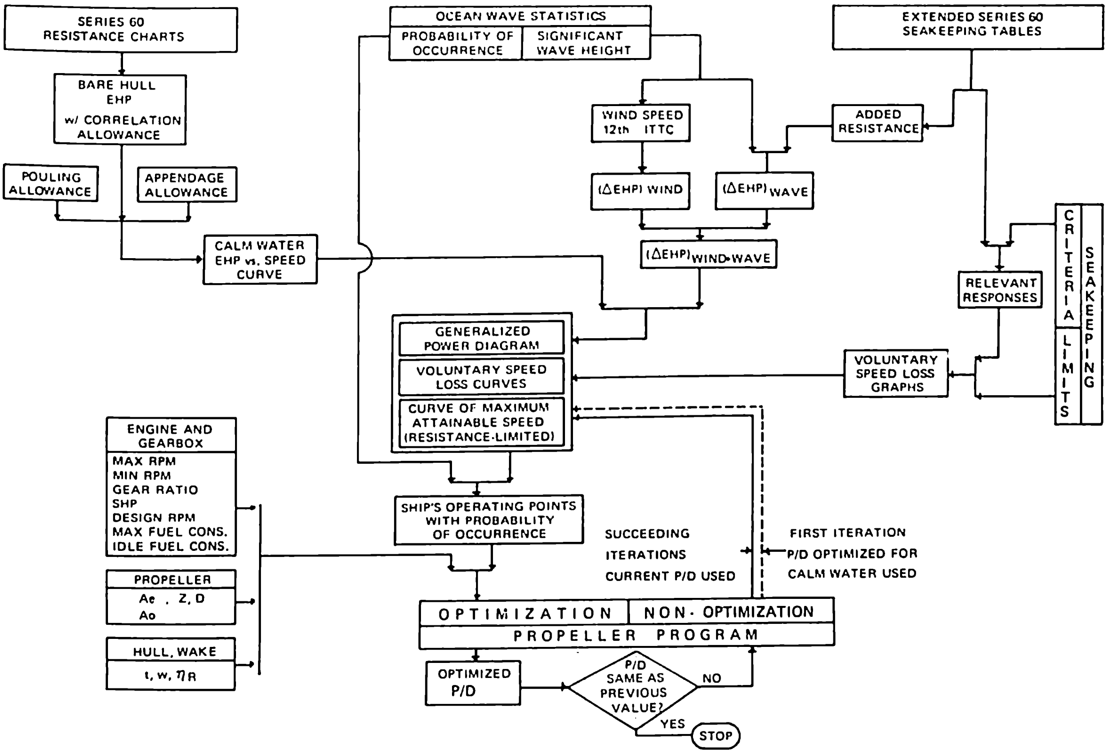


Figure 5. Propeller selection procedure (probabilistic approach)

Added resistance and ship motions. Spectral techniques are used in the determination of added resistance and ship motions in a seaway. Ship response operators, which depend on the physical characteristics of the ship, are theoretically calculated. These are then combined with seaway spectra using spectral techniques to predict ship responses. Appendix A discusses seaway spectra and some formulas used for determining ship responses.

There exist a number of computer seakeeping programs to predict ship motions in six degrees of freedom. For preliminary or even detailed design of commercial ships, use of such large programs is currently hard to justify.

To arm the designer with a rapid and economical way of estimating ship responses, Loukakis and Chryssostomidis (5) generated standard seakeeping tables for the series 60 ships. This series represents a family of single-screw, cruiser-stern ship forms whose principal characteristics vary in a systematic manner. The tables were computer-generated and assumed long-crested head seas. By using the tables, it is possible to determine, for a range of Froude numbers and nondimensionalized significant wave heights, the values of added resistance, heaving motion amidships, pitching motion and acceleration at different locations and relative motion and velocity at the bow and stern – all in nondimensionalized form.

For preliminary design purposes, the results are considered sufficiently accurate for ships of a form very roughly similar to the series 60 ships.

Ocean wave data. Hogben and Lumb's *Ocean Wave Statistics* (6), a compilation of data from all oceans of the world, enables the determination of significant wave heights ($\bar{H}_{1/3}$) to be encountered along a vessel's intended route. For this study, the wave observed period is ignored. All observed sea states of the same significant wave height, regardless of period, are considered as a single sea condition.

The generalized power diagram. Building up on the EHP vs. speed diagram for calm water (Figure 1), a generalized power diagram showing the added resistance in a seaway may be drawn. (See Figure 6.) Significant wave height appears as a parameter.

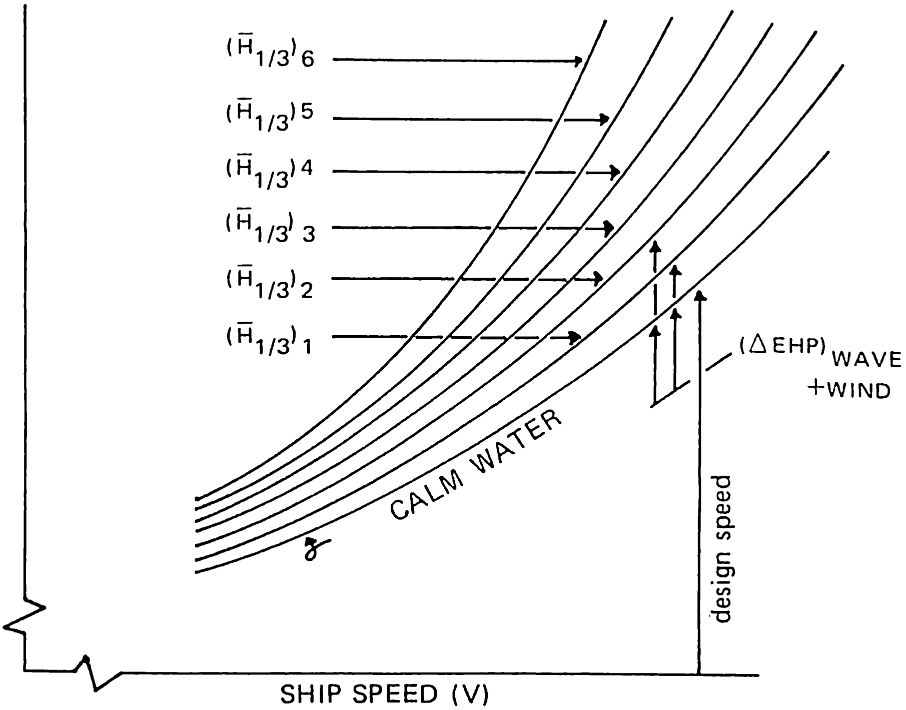


Figure 6
Generalized power diagram

To generate such a diagram, the standard seakeeping tables are used. The additional horsepower necessary to overcome wave resistance, $(\Delta \text{EHP})_{\text{wave}}$, may be computed for different wave heights and speeds. By interpolation, this additional power may be computed for the significant wave heights obtained from ocean wave statistics.

For each significant wave height and Froude Number, the following equations are used:

$$\text{mean added resistance} = \rho * g * (\text{LBP})^3 * (\text{added resistance}) * 10^{-7} \qquad (16)$$

where: mean added resistance = dimensionalized added resistance added resistance = nondimensionalized item taken from tables g = gravitational constant

LBP = length of ship between perpendiculars (the forward perpendicular being a vertical line through the intersection of the stem and the waterline; the after perpendicular normally being a line through the rudder stock)

$$(\Delta \text{EHP})_{\text{wave}} = \text{mean added resistance} * V * \text{factor}$$

the factor depending upon the units used (17)

In addition to $(\Delta \text{EHP})_{\text{wave}}$, $(\Delta \text{EHP})_{\text{wind}}$ is calculated. To relate significant wave height to wind velocity, the 12th ITTC (International Towing Tank Conference) Standard is used (7). (See Figure 7.)

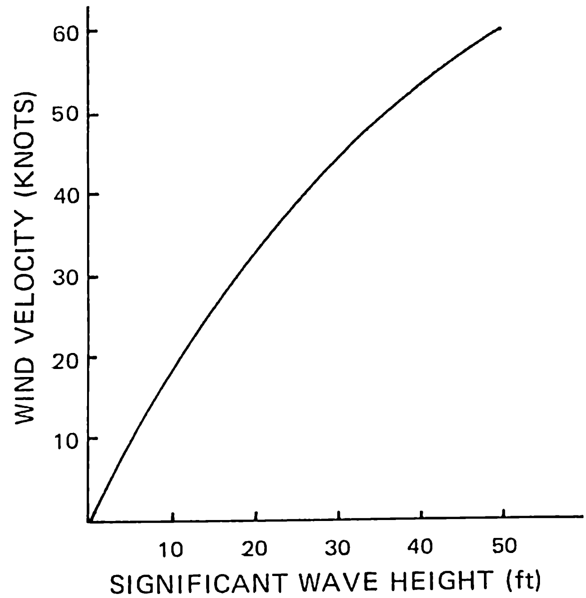


Figure 7
12th ITTC Standard

The following equation is used (7).

$$(\Delta\ \text{EHP})_{\text{wind}} = \frac{0.00435 * B^2 * Vr^2 * V}{2 * 325.66} \quad \text{in British units} \qquad (18)$$

where; B = ship's beam (ft)
Vr = velocity of wind relative to ship (knots)
V = ship's speed (knots)

Since an assumption about head seas has been made, it is clear that wind velocity has to be added to ship speed to get Vr.

By adding $(\Delta\ \text{EHP})_{\text{wave}}$ and $(\Delta\ \text{EHP})_{\text{wind}}$ to the calm water power curve at various ship speeds, the power curve for each significant wave height may be generated (Figure 6).

Curve of maximum attainable speed. On the generalized power diagram, a curve of maximum attainable speed may be drawn. (See Figure 8.)

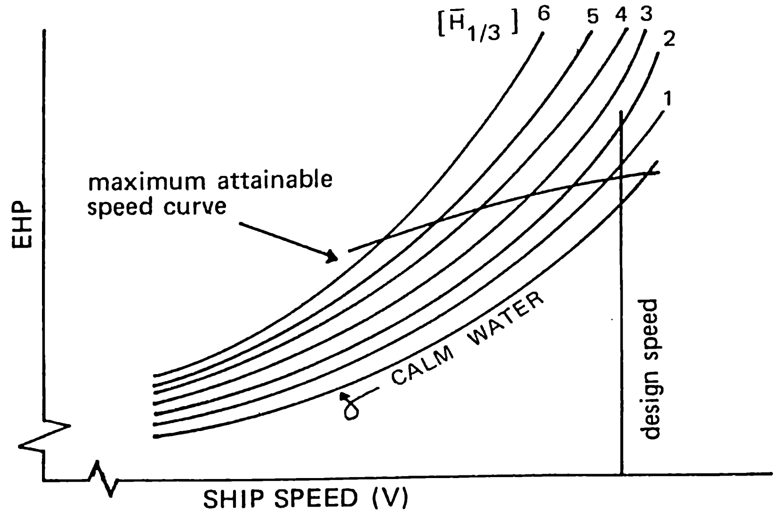


Figure 8. Maximum attainable speeds

The point of intersection of this curve with the power diagram for any significant wave height represents the highest possible speed the vessel may attain at full engine power in that sea condition. This is sometimes called a resistance-limited speed.

To determine the maximum attainable speeds, an initial selection of the propeller must have been made. Iteration is thus necessary as shown in the schematic design in Figure 5. The traditional method of propeller selection may be used to determine a propeller at the start, and the propeller chosen by the new method is used to determine the maximum attainable speeds in the next iteration.

The resistance-limited speed is determined by working the propeller charts (or equations) essentially backwards. This routine is best done by computer since it involves trial and error.

It may be noted in Figure 8 that for low significant wave heights as well as the calm water condition, the maximum attainable speed exceeds the design speed. It is the usual practice to aim for attainment of the design speed in calm water conditions at 90 percent of the engine's maximum continuous rating to prevent excessive wear and tear of the engine. It may be assumed that at slight departure from calm water conditions, the master will stick to the design speed. Thus, a vertical line may be drawn through the design speed to get the appropriate ship operating points in the generalized power diagram.

Motion-limited speeds. Appendix B discusses seakeeping indices and criteria.

The ship responses and seakeeping events relevant to the case at hand will first have to be determined, and judgment used to set the limiting values of ship motion. Once this has been decided, the standard seakeeping tables may once again be used to determine the limiting speeds corresponding to the maximum allowable limit of ship response at any significant wave height. Sets of values of $(V, \bar{H}_{1/3})$ for any seakeeping criterion may be obtained and graphed as voluntary speed loss curves. (See Figure 9.)

Since different seakeeping indices and criteria may be involved, the governing criterion (or criteria) has to be determined. One criterion may govern over a certain range of significant wave heights while another criterion may govern over another range. (See Figure 10.) The lower envelope of the curves represent the most speed-restraining conditions, and would, therefore, define the motion-limited speeds.

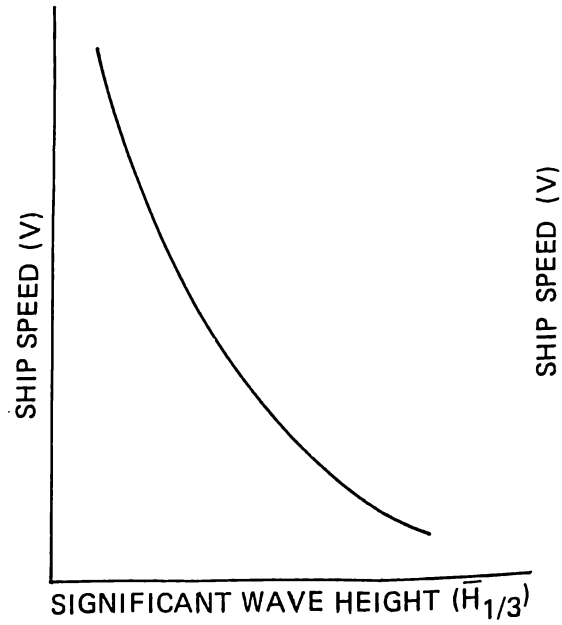


Figure 9. Voluntary speed loss curve

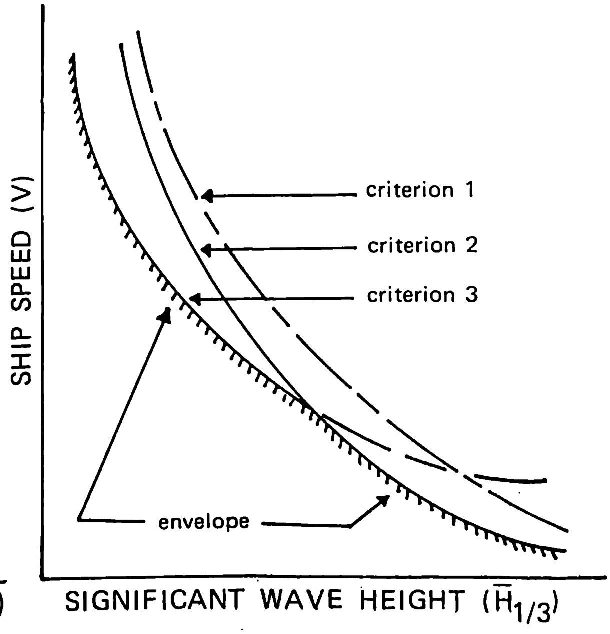


Figure 10. Governing criteria

The lower envelope may now be transferred to the generalized power diagram as the motion-limited speed curve. (See Figure 11.)

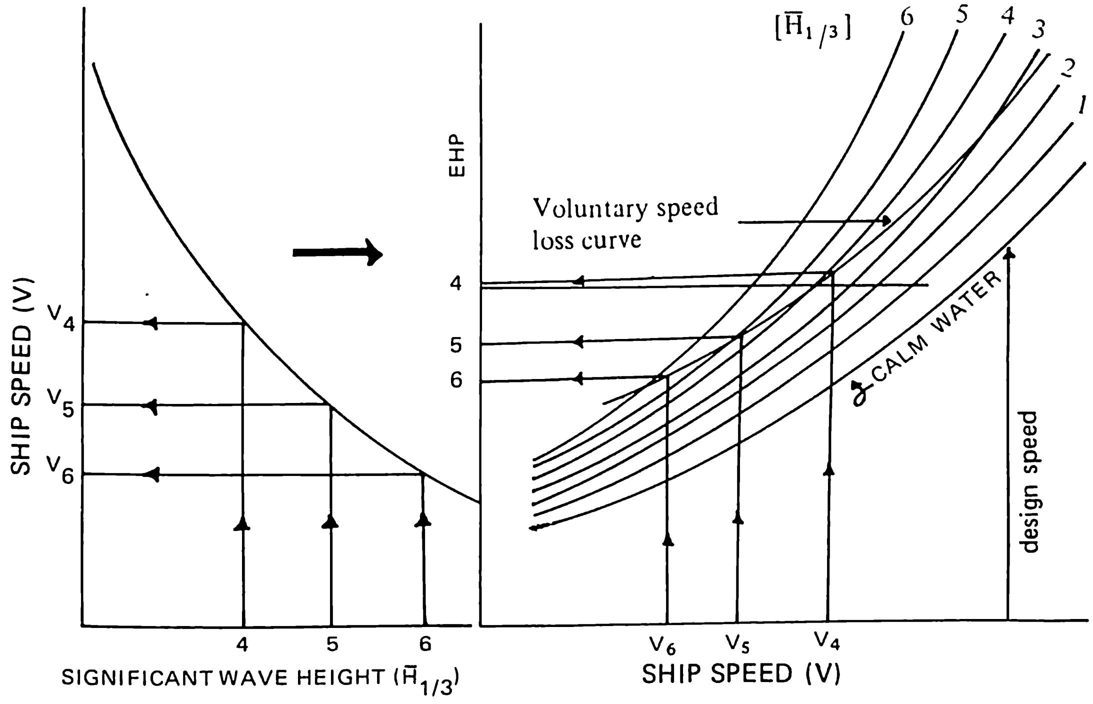


Figure 11. Transfer of voluntary speed loss curve to generalized power diagram

Ship operating points. Figure 12 shows the generalized power diagram with the resistance-limited speed curve and the motion-limited speed curve.

The encircled points represent the estimated operating points of the ship. In the region of high significant wave heights, motion-limited speeds govern. In the region of low significant wave heights, resistance-limited speeds govern. For calm water and very low significant wave heights, ship speed is limited to design speed.

A tabulation may now be made of (EHP, V, probability of occurrence) corresponding to each of the ship operating points. These constitute the input to the propeller selection process.

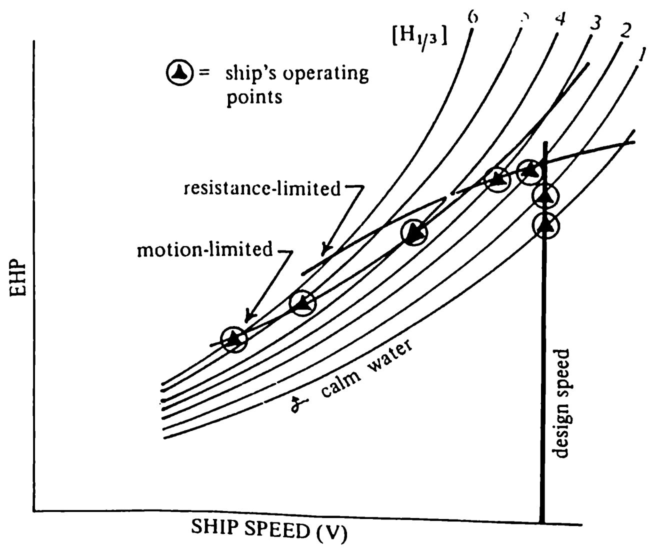


Figure 12. Ship's operating points

Generalized optimization procedure. If there are N ship operating points, the propeller selection problem becomes one of maximizing the expectation of ηo, $E(\eta o)$, subject to N constraints

$$\frac{Kt_i}{J_i^2} = C_i \tag{19}$$

where C_i = constant, i = 1, 2, . . . N. Only diameter-constrained optimization is considered here.

The optimization condition becomes

$$\frac{d}{d(P/D)} E(\eta o) = 0 \tag{20a}$$

or

$$\frac{d}{d(P/D)} \sum_{i=1}^{N} (\eta o_i \; p_i) = \sum_{i=1}^{N} \frac{d}{d(P/D)} (\eta o_i \; p_i) = 0 \tag{20b}$$

where p_i is the probability of occurrence of each ship operating point.

By introducing Equation 19 implicitly into Equation 20b, the following expression is obtained 1

$$\sum_{i=1}^{N} \frac{p_i}{kq_i^2} \left[\left(3KtKq - JKt \frac{\partial Kq}{\partial J} \right) \frac{\partial Kt / \partial(P/D)}{2Kt/J - \partial Kt/ \partial J} - JKt \frac{\partial Kq}{\partial(P/D)} \right] = 0 \tag{21}$$

The solution of the system of N + 1 equations, with N + 1 unknowns, J_i and P/D, will give the optimum propeller. It may be shown that 15 is a special case of 21 with N = 1.

With Z and EAR as initial inputs, the system of equations can easily be solved by computer.

Fuel consumption. Having chosen a propeller, fuel consumption can be estimated by working backwards – finding RPM and BHP at each operating point. This can then be compared with the estimated fuel consumption if the propeller had been selected using the traditional approach.

Numerical example. See Appendix C.

Conclusions

Table 1 shows that the projected dollar savings in fuel consumption becomes significantly high for high values of horsepower. Thus it will be worthwhile to use the method discussed in this paper for big ships.

Table 1. FUEL SAVINGS FOR DIFFERENT SHIPS

BHP	Engine Type	Fuel Savings (kg/day)	Dollar Savings (oil at) 205 $/tonne) $/yr	Source
4,500	diesel	17.3	1,200	Vea (8)
9,000	diesel	316.8	22,300	This paper
30,500	gas turbine	717.0	50,500	Triantafyllou (1) (resistance-limited only)

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Appendix A

Sea Conditions and Ship Responses

The spectrum used in the Standard Seakeeping Tables, upon which the calculations in this paper were based, is the two-parameter Pierson-Moskowitz family.

$$\Phi_{\zeta\zeta}(\omega) = \alpha\, \omega^{-5} \exp\left(- \beta\, \omega^{-4} \right) \tag{A.1}$$

where $\Phi_{ss}(\omega)$ is the one-sided spectral density function and ω is the circular frequency.

The curve of this function contains one peak, which corresponds to ω_p, the modal frequency.

The parameters α and β necessary for the calculation of the function are given in terms of ω_p and significant wave height, $\bar{H}_{1/3}$, which refers to the average 1/3 highest heights (crest-to-trough). For an ideally narrow band process, $\bar{H}_{1/3}$ is four times the root-mean-square (rms) value of the wave time history. This rms value is the square root of the area under the spectral density function integrated for all frequencies. Mathematically,

$$m_o = \int_0^\infty \Phi_{\zeta\zeta}(\omega)\, d\omega \text{ (area under)} \qquad \text{(A.2)}$$

$$\bar{H}_{1/3} = 4(m_o)^{1/2} \qquad \text{(A.3)}$$

To determine the sea spectrum, it is thus necessary only to define the location of the spectral peak (ω_p) and the area under the curve ($\bar{H}_{1/3}$).

The Bretschneider family is defined by:

$$\alpha = 0.31225\, (\bar{H}_{1/3})^2\, \omega_p^4 \qquad \text{(A.4a)}$$

$$\beta = 1.25\, \omega_p^4 \qquad \text{(A.4b)}$$

For fully-developed seas:

$$\alpha = 0.0081 g^2 \qquad \text{(A.5a)}$$

$$\beta = 0.0324 g^2 \qquad \text{(A.5b)}$$

$$\omega_p = 0.4013 g^{1/2} \qquad \text{(A.5c)}$$

where g is the acceleration due to gravity. The Standard Seakeeping Tables assume fully-developed seas.

For ship responses (except added resistance), the results in the seakeeping tables are expressed in terms of rms values, i.e., $(m_o)^{1/2}$. Here, m_o is the mean square value of the response and equals the area under the response spectrum.

Below are formulae used for determining the frequency of occurrence of certain responses and seakeeping events. See Appendix B also for more discussion of these responses.

The frequency N(occurrences/hr) of keel emergence, deck wetness or propeller emergence is given by

$$N = \frac{3600}{2\pi} \cdot \exp\left(\frac{-f^2}{\hat{A}_{rm}^2}\right) \left[\frac{m_{o\,(rv)}}{m_{o(rm)}}\right]^{1/2} \qquad \text{(A.6)}$$

where f = local draft, freeboard or distance to propeller

$m_{o\,(rv)}$, $m_{o\,(rm)}$ = mean square values of relative velocity and relative motion, respectively

$\hat{A}_{rm}^2 = 2m_{o\,(rm)}$ = mean square apparent amplitude of relative motion at the same location

To calculate the probability of deck wetness (in lieu of occurrences per hour), the following equation is used:

$$\text{probability} = \exp\left(\frac{-f^2}{2m_{o\,(rm)}}\right) \qquad \text{(A.7)}$$

To account for static swell, the effective freeboard (f_e) is used instead of the geometric freeboard (f). Tasaki's formula may be used (9):

$$f_e = f - 0.75\, B \frac{L}{Le} Fn^2 \qquad \text{(A.8)}$$

where B = vessel's beam; L = length between perpendiculars; Le = hull length of entrance and Fn = Froude No.

To determine the probability of bow acceleration exceeding a certain value, equation A.7 may be used with f now the value of acceleration to be exceeded and m_o is for bow acceleration.

The frequency of slamming, N_s (slams/hr), can be calculated from

$$N_s = \frac{3600}{2\pi} \exp \left[- \left(\frac{f^2}{A_{rm}^{\ 2}} + \frac{V_{cr}^{\ 2}}{A_{rv}^{\ 2}} \right) \right] \left[\frac{m_{o\,(rv)}}{m_{o\,(rv)}} \right]^{1/2} \qquad \text{(A.9)}$$

where f is the draft at the location in question, Vcr is the critical relative velocity between ship and sea for a slam to occur and $A_{rv}^{\ 2} = 2m_{o(rv)}$ is the mean square apparent amplitude of relative velocity in the same location.

Another formula giving the probability of a slam is

$$\mathrm{In}\, N = \frac{f^2}{2m_{o\,(rm)}} + \frac{Vcr^2}{2m_{o\,(rv)}} \qquad \text{(A.10)}$$

where N = number of oscillations after which another slam may be expected. 4 slams/100 pitch oscillations is taken to be equivalent to a 4 percent probability of occurrence. This also corresponds to N = 25 oscillations between slams in equation A.10.

Appendix B

Seakeeping Indices and Criteria

Seakeeping indices refer to ship responses and seakeeping events critical to the performance of a ship's mission. In general, ship responses include linear and rotational displacements, velocities and accelerations and sea-induced loads and stresses. Seakeeping events such as slamming of the bow onto the water, shipping of water aboard ship, bow emergence and propeller emergence are related to the relative motions between ship and sea.

Seakeeping criteria refer to the allowable limits of seakeeping indices. The major seakeeping indices and some associated criteria are listed below.

1. *Deck wetness.* shipping of green water on board; important for payload and personnel safety; sample criteria for cargo ships:

   5 times/100 pitch oscillations (Ref. 10)
   20 times/hr (Ref. 11)
   7 percent probability of occurrence (Ref. 12)

2. *Slamming.* emergence of the vessel's bow and subsequent impact with the sea as the vessel pitches down at the stem; accompanied by a rise in pressure at the forebody and whipping stresses amidships; discomfort among crew; damage to ship possible; sample criteria for cargo ships:

   4 times/100 pitch oscillations (Ref. 10)
   3 percent probability of occurrence (Ref. 12)

There is a need to define what constitutes a slam. Critical or threshold values of certain parameters such as whipping stresses amidships, whipping acceleration at the bridge and bow deceleration can be used. Analytically, relative velocity at re-entry has proven easiest to handle. Thus, the following proposed value of velocity at re-entry called critical velocity, Vcr:

$$Vcr = 0.093\,(g * L)^{1/2} \quad \text{(Ref. 12)}$$

where g is the gravitational constant and L is the ship's length on its waterline.

3. *Propeller emergence.* propeller coming out of the water as the ship pitches and heaves; decrease in propeller torque; racing of engine possible; sample criteria for cargo ships:

   25 times/100 pitch oscillations (Ref. 10)
   Average interval of 40 seconds between events (Ref. 10)

4. *Vertical bow acceleration.* related mainly to payload safety and human comfort; sample criteria for cargo ships:

   0.9g double amplitude significant value (Ref. 10)
   3 percent probability of the single amplitude significant value exceeding 0.4g (Ref. 12)

5. *Rolling.* concerned with payload safety and human comfort; helicopter and aircraft operations; sample criteria for aircraft carrier:

   9.6 degrees rms for task proficiency (Ref. 13)
   8.0 degrees (personnel-related) (Ref. 14)
   3.5 degrees (support equipment-related) (Ref. 14)

6. *Vertical velocity.* aircraft touchdown; sample criterion:

   6.5 ft/sec for vertical landing (Ref. 14)

The seakeeping indices are related more to seakindliness than to seaworthiness. Operability and habitability, rather than survivability, are the major concerns. For habitability, motion sickness and degradation of motor capabilities are considered. Operability refers to the capability to perform the intended missions in both the commercial and naval sense.

Appendix C

Numerical Example

GIVENS

Ship:	
Type	... General cargo
Route	... NY-Rotterdam
Length between perpendiculars (L, LBP)	... 479.5 ft
Length on design waterline (LWL)	... 485.8 ft
Beam (B)	... 65.9 ft
Draft (d)	... 28.0 ft
Block coefficient (Cb)	... 0.69
Design speed (V)	... 16.5 knots
Wetted surface (S)	... 44,628 sq ft
Displacement (Δ)	... 17,443 tons
Propeller:	
Maximum diameter (D)	... 18.37 ft
Number of blades (Z)	... 4
Expanded area ratio (EAR)	... 0.552

Engine:	
Type	... Diesel
Maximum continuous rating (MCR)	... 9000 BHP
Maximum RPM	... 125
Design RPM	... 120
Maximum fuel oil consumption rate	... 0.3875 kg/sec
Reduction gear:	... NONE (direct drive)
Seakeeping criteria	... 5% probability of deck wetness (for simplicity)

DATA AND CALCULATIONS

Wake and thrust: From Series 60 charts(2):

$w = 0.269$
$t = 0.171$

Ocean wave data: See Table D.1.

Voluntary speed loss curves: See Figure D.1. (Note: Other criteria are shown only for illustration but the seakeeping criterion remains as 5% probability of deck wetness, which is shown boxed in the figure.)

Generalized power diagram: See Figure D.2. (Note: Probabilities of deck wetness other than 5% shown only for illustration of trends.)

Ship operating points. See Figure D.2 and Table D.2.

Initial estimate of P/D: Using traditional approach, the optimized P/D is 0.893. This is used to determine the initial curve of maximum attainable speeds in Figure D.2.

Result of first iteration: optimized P/D = 0.85195

Result of second iteration: optimized P/D = 0.85195

Third iteration not necessary.

Comparison of fuel consumption: See Table D.3.

Observations:

1. The savings in fuel consumption is 316.8 kg/day as may be determined from Table D.3.
2. The inclusion of motion-limited speeds shows very little influence in the optimized pitch ratio when compared with the case of resistance-limited speeds only as shown in Table D.3. This appears logical because the motion-limited speeds govern in the region of high significant wave heights, which have very low probabilities of occurrence.

STATISTICAL SEA DATA

Table D.1 SIGNIFICANT WAVE HEIGHTS OCCURRING IN NEW YORK-ROTTERDAM ROUTE

$\bar{H}_{1/3}$ (ft)	probability of occurrence
4.76	0.0416
5.48	0.0681
6.92	0.1732
8.37	0.1996
9.81	0.1490
11.25	0.1124
12.70	0.7700
14.14	0.0577
15.58	0.0382
17.03	0.0344
18.47	0.0061
19.81	0.0059
21.36	0.0090
22.80	0.0089
24.25	0.0037
25.69	0.0044
27.20	0.0034
28.58	0.0022
30.02	0.0017
31.46	0.0034

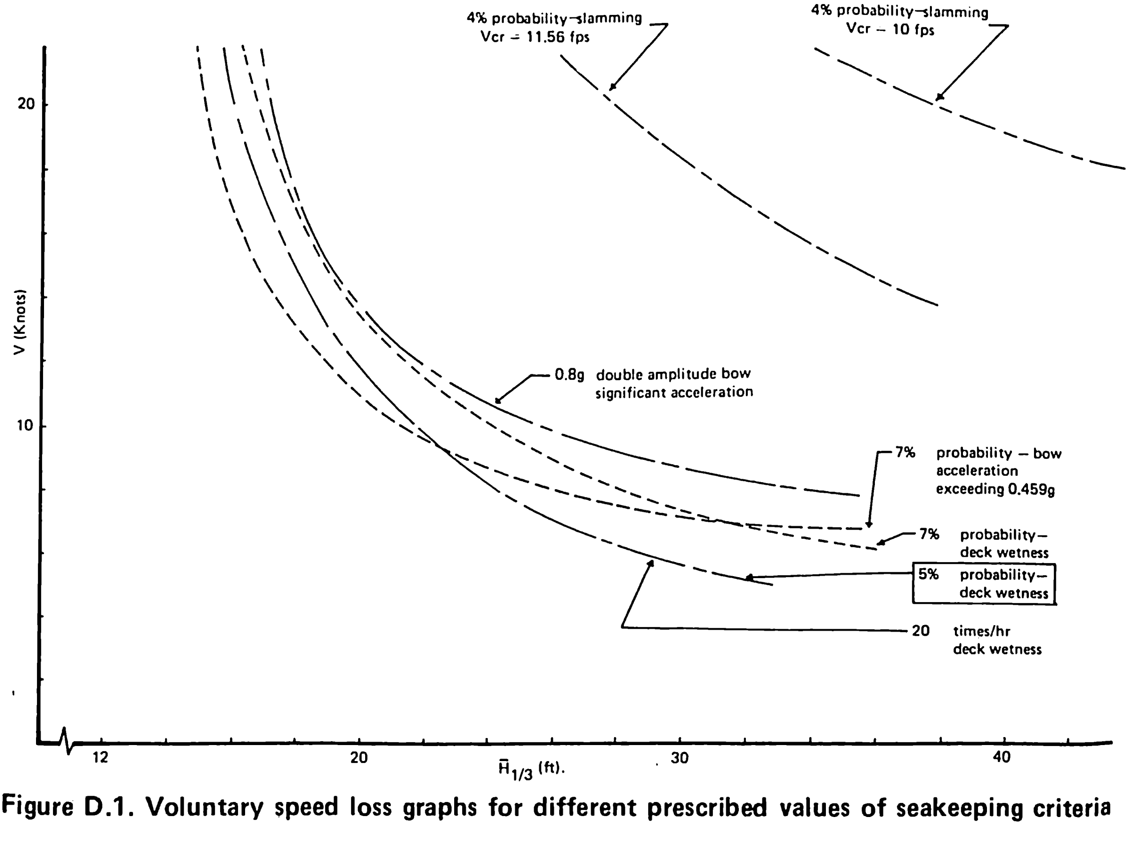

Figure D.1. Voluntary speed loss graphs for different prescribed values of seakeeping criteria

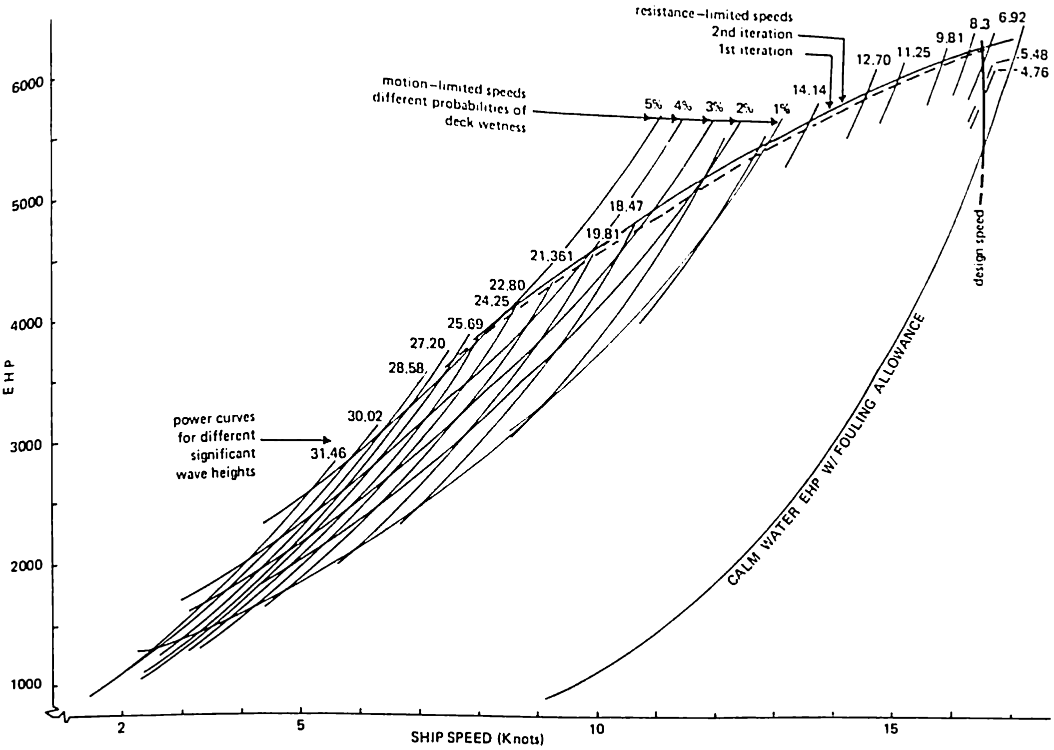

Figure D.2 Generalized power diagram with curve of maximum attainable speed and voluntary speed loss curve

Table D. 2 OPTIMIZATION RESULTS
RESISTANCE-LIMITED SPEEDS AND 5% PROBABILITY OF DECK WETNESS
IN THE 11 ITERATION, FOR P/D .85 FUEL CONS. .3828 -32.00E-05

RESULTS OF OPTIMIZATION FOR 20 LCAD CASES OF A PROPELLER WITH DIAMETER 18.37 FEET, EXPANDED AREA RATIO .552 AND 4 BLADES
OPTIMIZED PITCH RATIO = .851953125

CASE	EHP	SPEED (KNOTS)	THRUST DEDUCTION	WAKE FRACTION	RELATIVE ROT. EFFIC.
1	5875.00	16.500	.1710	.2690	1.0200
2	5950.00	16.500	.1710	.2690	1.0200
3	6200.00	16.500	.1710	.2690	1.0200
4	6320.00	16.300	.1710	.2690	1.0200
5	6230.00	15.850	.1710	.2690	1.0200
6	6070.00	15.100	.1710	.2690	1.0200
7	5950.00	14.590	.1710	.2690	1.0200
8	5705.00	13.620	.1710	.2690	1.0200
9	5470.00	12.710	.1710	.2690	1.0200
10	5170.00	11.720	.1710	.2690	1.0200
11	4840.00	10.720	.1710	.2690	1.0200
12	4520.00	9.840	.1710	.2690	1.0200
13	4270.00	9.190	.1710	.2690	1.0200
14	4070.00	8.640	.1710	.2690	1.0200
15	3850.00	8.090	.1710	.2690	1.0200
16	3600.00	7.600	.1710	.2690	1.0200
17	3260.00	7.000	.1710	.2690	1.0200
18	2990.00	6.400	.1710	.2690	1.0200
19	2790.00	5.900	.1710	.2690	1.0200
20	2675.00	5.450	.1710	.2690	1.0200

Table D.2 (Con't.)

CASE	RELATIVE PROBABILITY	OPEN WATER EFFICIENCY	RPM	SHP	FUEL RATE (KG/SEC)
1	.041600	.63577	115.4	7988.56	3.5974E-01
2	.068100	.63425	115.9	8109.92	3.6383E-01
3	.173200	.62901	117.3	8521.08	3.7847E-01
4	.199600	.62206	117.7	8783.07	3.8699E-01
5	.149000	.61295	116.9	8786.70	3.8699E-01
6	.112400	.59669	115.5	8794.89	3.8733E-01
7	.077000	.58471	114.7	8797.12	3.8784E-01
8	.057700	.56085	112.9	8793.77	3.8733E-01
9	.038200	.53583	111.5	8825.11	3.8822E-01
10	.034400	.50693	109.9	8816.76	3.8822E-01
11	.006100	.47529	108.4	8803.43	3.8860E-01
12	.005900	.44614	106.9	8758.51	3.8699E-01
13	.009000	.42335	105.9	8719.53	3.8631E-01
14	.008900	.40284	105.2	8734.20	3.8699E-01
15	.003700	.38201	104.4	8712.69	3.8631E-01
16	.004400	.36460	103.2	8535.81	3.8120E-01
17	.003400	.34417	101.1	8188.67	3.6928E-01
18	.002200	.31964	100.0	8086.71	3.6655E-01
19	.001700	.29745	99.5	8108.66	3.6758E-01
20	.003400	.27379	100.3	8446.43	3.7950E-01

ENGINE CHARACTERISTICS

DIESEL ENGINE
MAX. SHP = 9000 MAX. RPM = 125
DESIGN FUEL CONSUMPTION = .3875
REDUCTION GEAR RATIO = 1

Table D.3 Results of second iteration

	optimized pitch ratio	expected average speed (knots)	fuel consumption (kg/sec)	fuel consumption for P/D = 0.893359375
100 % probability of calm water	0.89335	16.50		
resistance-limited speeds only	0.85195	15.19	0.3830	0.3839
5 % probability of deck wetness included	0.85195	15.19	0.3828	0.3837
4 % probability of deck wetness included	0.85195	15.17	0.3821	0.3830
3 % probability of deck wetness included	0.85195	15.13	0.3811	0.3820
2 % probability of deck wetness included	0.85117	15.07	0.3799	0.3807