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Independent Model Method of Aerial Triangulation

by

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The Concept

Photogrammetry, which is defined as the science of obtaining reliable measurements and other information through the use of photographs, has proved to be a very effective tool for mapping. One of the most significant developments of photogrammetry is *aerial triangulation*, the method and technique by which the positions of points are determined using a basic network of only a few ground surveyed points. Statistics show that if all the control points necessary for mapping are surveyed on the ground, this phase of the work claims a big portion of the cost of the project. By providing a means of minimizing ground surveys, therefore, aerial triangulation greatly reduces the costs of photogrammetric surveys. It has also been shown that by using the method, there is a savings in time as compared to a full ground survey. The points whose positions are determined usually are the photo control points needed to give each stereomodel a true scale and a correct geographic orientation so that any topographic map drawn from the stereomodel, or any measurement obtained from it for that matter, will be correct.

Aerial triangulation is made possible by the manner in which aerial photography is obtained. It is standard practice that the coverage of successive photographs in a flight strip overlap by about 60% (forward overlap) and that adjoining flight strips overlap by about 30% (lateral overlap). This can be seen in Figure 1 which shows the photographs of two strips. Two consecutive photographs of a strip constitute a stereopair. Each stereopair generates a stereomodel as shown in Figure 2 and the shaded portion of Figure 1. Each stereomodel will require about four photocontrol points for its correct scaling and orientation.

By the nature of photographic overlaps, stereomodels themselves either within a strip or with adjoining strips have common areas. It is from these common areas between stereomodels that the photocontrol points are selected as can be seen also in Figure 1. Points selected are distinct natural features such as intersections of road edges, culverts, lot corner monuments, isolated rocks, etc., that are definitely identifiable in all the stereomodels they occur in. These control points, therefore, also serve as the link between stereomodels. Before any mapping can be done, the positions of these points have to be determined either by ground survey methods or by aerial triangulation.

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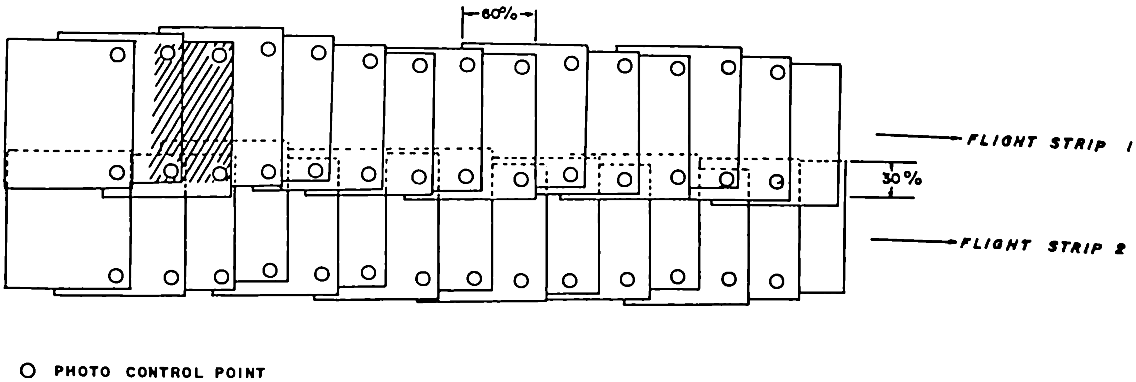

Figure 1. Two Strips of Aerial Photography Showing Forward and Lateral Overlaps. Shaded Portion Constitute a Model.

Methods of Aerial Triangulation

There are three known methods of aerial triangulation, namely the analogue, semi-analytical, and analytical methods. They differ in the manner in which the first two phases of aerial triangulation are achieved. These phases are (1) the recreation of the stereomodels; (2) the model connection; and (3) the adjustment procedure.

In the analogue method, the recreation of the stereomodel and their connection are achieved in universal stereoplotters which are so designed and constructed as to allow continuous connection of models in a strip. All coordinates (X, Y, Z) of photocontrol points observed in the instruments for a strip are in one machine coordinate system, and the coordinates are known as strip coordinates rather than simply model coordinates. In the semi-analytical method, the recreation of stereomodels is done independently of each other in second order stereoplotters, where instrumental model connection is not possible. The coordinate system (X, Y, Z) of each stereomodel is independent of the others. Thus, this method is also more popularly called the Independent Model Method, the main subject of this paper. Model connection is achieved mathematically through appropriate formulation. Besides the fact that universal instruments are now very expensive to produce and manufacturers have opted to manufacture the less expensive second order plotters, the rapid development of the electronic computer at present has made this method more widely used by mapping organizations. In addition, doing part of the work analytically increases the accuracy of the positions of points.

The analytical method is almost purely numerical. What are observed in stereocomparators, the instrumentation used, are simply the x, y, photo coordinates of points. By a more complicated mathematical formulation based on the geometry of the aerial photographs, these photo coordinates are related to the ground positions of points. The recreation of the stereomodels and their connections are done mathematically. Understandably, this method utilizes the computer much more extensibly. A more advanced hardware is the integration of the stereocomparator with the electronic computer in the form of what is called the analytical plotter. This configuration is much more expensive and depends a lot on software support. Definitely, it produces the most accurate results.

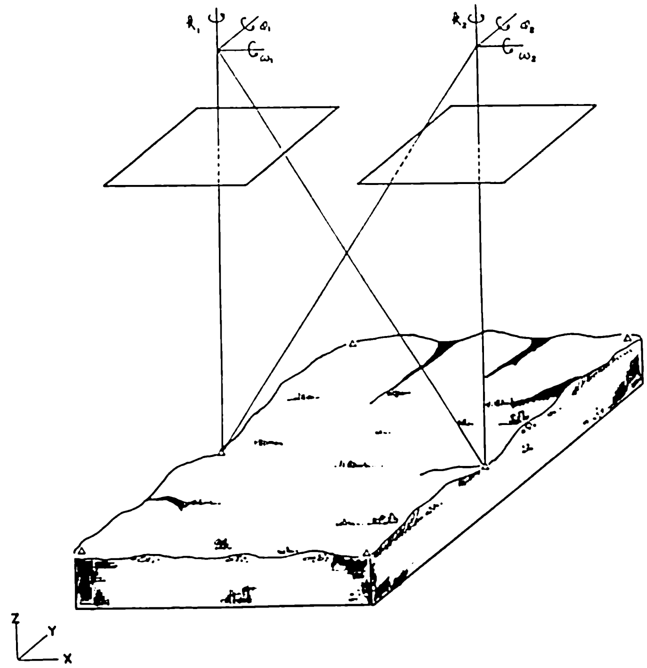

Figure 2. Stereo Model Generated by a Stereopair.

The Independent Model Method

Recreation of Stereomodels

The recreation of stereomodels in second order stereoplotters is done by a process called *relative orientation*. The two photos of a stereopair are introduced in the instrument and their relative positions with respect to each other as during photography are exactly recovered. This generates the stereomodel of the ground covered by the common overlap between the photographs, an exact copy of what is in nature except at a smaller scale. Once this is accomplished the three model coordinates (X, Y, Z) of each point referred to three mutually perpendicular coordinate axes integral to the instrument are observed. If the stereoplotter is equipped with a digitizer, these coordinates together with a point identification number are automatically recorded in magnetic tape or in punched cards. This is done for all the models in each strip. In Figure 3, we have three such stereomodels with their independent coordinate systems. Photocontrol points A, B, and C are common to models 1 and 2 while points D, E, and F are common to models 2 and 3.

Perspective Center Coordinates

If model connection is achieved using only the common model points, such connection would be weak. A further refinement of the method, therefore, requires that the model coordinates of the perspective centers (camera lens positions during photography) 0_1, 0_2, 0_3 and 0_4 in Figure 3 are also obtained. 0_2 becomes a common point between models 1 and 2, 0_3 between models 2 and 3. Since these points can not be observed in the stereomodel directly, some special methods to obtain their coordinates are employed. These are the intersection method or the resection method. With either method, the coordinates (X, Y, Z) of the two perspective centers of each model are determined in the same system as the photocontrol points. These methods of determining the coordinates of the perspective centers can be the subject of another paper, and their discussion will be attempted here. Suffice it to say that the coordinates of the perspective centers are important to achieve stronger and more accurate model connections.

Model Connection for Polynomial Adjustment

Model connection can occur at two different stages of the aerial triangulation procedure depending on what method of adjustment will be employed. If it is desired that the *polynomial adjustment* be used then model connection is done between the models of each strip to produce strip coordinates of points. This method of adjustment is simpler and involves less unknown parameters and therefore, will need less computer time. Or if the *independent model method of adjustment* is chosen then the model connections occur simultaneous with the adjustment itself.

The problem of model connection is one of spatial similarity transformation. It is the objective that all the independent coordinate systems of the models of a strip be transformed into one common coordinate system, usually that of the first model. In Figure 3, for example, this will involve the transformation of model 2 into model 1 using the common points 0_2, A, B and C,i.e., the quadrilateral $0_2{}'A'B'C'$ will be fitted into 0_2ABC. The transformation involves seven parameters: three rotations, three translations, and a scale change. Once these parameters are determined, the other points in model 2 are then transformed. Next, model 3 is transformed into model 2, which at this point is now in the coordinate system of model 1, using the common points 0_3, D, E, F. The rest of the points in model 3, 0_4, G, H, I are similarly transformed so that model 3 is now also in the coordinate system of model 1. The process is continued until the last model of the strip. The coordinates of all photocontrol points in the strip will now be referred to one strip coordinate system.

Since for every transformation only two models are involved at a time, we can generalize by specifying a left model and a right model, the right to be transformed into the system of the left. Let us also assume that the coordinates of n

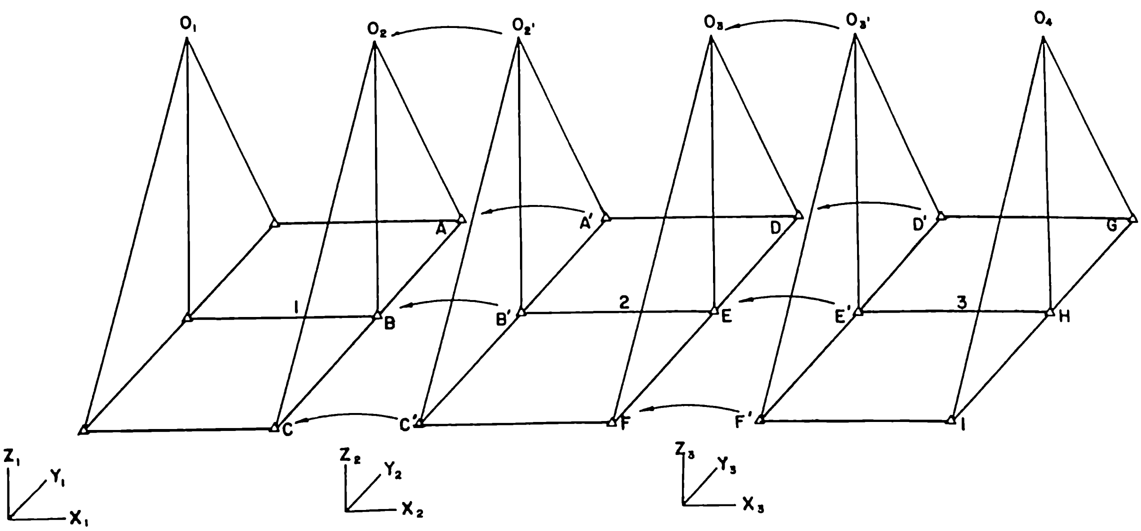

Figure 3. Three Independent Stereomodels Preparatory to Model Connection.

common points are X_i, Y_i, Z_i (i=1. . .n) in the left model and x_i, y_i, z_i in the right model. We require the transformation parameters X_s, Y_s, Z_s (translations) Ω, Φ, κ (rotations) and λ (scale change). Basically, we can write, in matrix notation, for every common point, the equations:

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}$$

right model coor- dinates	scale change	rotation matrix involving Ω, Φ, κ	left model coor- dinates	translation

For n common points, we can write 3n equations involving the seven unknown parameters. It will require a least squares solution. Also, the equations are non-linear and will need linearization, estimation of approximate values of unknowns, and an iterative solution.

A simpler computational algorithm is as follows:

1. Computation of mean coordinates.

$$X_s = \frac{[X_i]}{n} \qquad Y_s = \frac{[Y_i]}{n} \qquad Z_s = \frac{[Z_i]}{n} \qquad (i = 1. . .n)$$

$$x_s = \frac{[x_i]}{n} \qquad y_s = \frac{[y_i]}{n} \qquad z_s = \frac{[z_i]}{n}$$

2. Computation of reduced coordinates.

$$\bar{X}_i = X_i - X_s \qquad \bar{Y}_i = Y_i - Y_s \qquad \bar{Z}_i = Z_i - Z_s$$

$$\bar{x}_i = x_i - x_s \qquad \bar{y}_i = y_i - y_s \qquad \bar{z}_i = z_i - z_s$$

3. Computation of approximate values.

$$\Omega = \Phi = \kappa = 0$$

$$\lambda = \frac{\left[\sqrt{\bar{X}_i^2 + \bar{Y}_i^2 + \bar{Z}_i^2}\right]}{\left[\sqrt{\bar{x}_i^2 + \bar{y}_i^2 + \bar{z}_i^2}\right]}$$

4. Computation of elements of rotation matrix.

$$a_{11} = \cos\Phi \cos\kappa$$

$$a_{12} = -\cos\Phi \sin\kappa$$

$$a_{13} = \sin\Phi$$

$$a_{21} = \cos\Omega \sin\kappa + \sin\Omega \sin\Phi \cos\kappa$$

$$a_{22} = \cos\Omega \cos\kappa - \sin\Omega \sin\Phi \sin\kappa$$

$$a_{23} = -\sin\Omega \cos\kappa$$

$$a_{31} = \sin\Omega \sin\kappa - \cos\Omega \sin\Phi \cos\kappa$$

$$a_{32} = \sin\Omega \cos\kappa + \text{cost}\,\Omega \sin\Phi \sin\kappa$$

$$a_{33} = \cos\Omega \cos\kappa$$

5. Computation of rotated coordinates.

$$\bar{\bar{X}}_i = \lambda\,(a_{11}\,\bar{x}_i + a_{12}\,\bar{y}_i + a_{13}\bar{z}_i)$$

$$\bar{\bar{Y}}_i = \lambda\,(a_{21}\,\bar{x}_i + a_{22}\bar{y}_i + a_{23}\bar{z}_i)$$

$$\bar{\bar{Z}}_i = \lambda\,(a_{31}\bar{x}_i + a_{32}\bar{y}_i + a_{33}\bar{z}_i)$$

6. Formation of Normal Equations involving corrections $d\kappa$, $d\Phi$, $d\Omega$, $d\lambda$ to approximate values.

$$[\bar{\bar{Z}}_i^{\,2} + \bar{\bar{Y}}_i^{\,2}].d\Omega - [\bar{\bar{X}}_i\,\bar{\bar{Y}}_i].d\Phi - [\bar{\bar{X}}_i\,\bar{\bar{Z}}_i].d\kappa + [\bar{Y}_i\,\bar{\bar{Z}}_i - \bar{Z}_i\,\bar{\bar{Y}}_i] = 0$$

$$-[\bar{\bar{X}}_i\,\bar{\bar{Y}}_i].d\Omega + [\bar{\bar{X}}_i^{\,2} + \bar{\bar{Z}}_i^{\,2}].d\Phi - [\bar{\bar{Y}}_i\,\bar{\bar{Z}}_i].d\kappa + [\bar{Z}_i\,\bar{\bar{X}}_i - \bar{X}_i\,\bar{\bar{Z}}_i] = 0$$

$$-[\bar{\bar{X}}_i\,\bar{\bar{Z}}_i].d\Omega - [\bar{\bar{Y}}_i\,\bar{\bar{Z}}_i].d\Phi + [\bar{\bar{X}}_i^{\,2} + \bar{\bar{Y}}_i^{\,2}].d\kappa + [\bar{X}_i\,\bar{\bar{Y}}_i - \bar{Y}_i\,\bar{\bar{X}}_i] = 0$$

$$d\lambda = \frac{[\bar{\bar{X}}_i(\bar{X}_i - \bar{\bar{X}}_i) + \bar{\bar{Y}}_i(\bar{Y}_i - \bar{\bar{Y}}_i) + \bar{\bar{Z}}_i(\bar{Z}_i - \bar{\bar{Z}}_i)]}{[\bar{\bar{X}}_i^{\,2} + \bar{\bar{Y}}_i^{\,2} + \bar{\bar{Z}}_i^{\,2}]}$$

7. Computation of new approximate values.

$$\Omega_{k+1} = \Omega_1 + d\Omega$$

$$\Phi_{k+1} = \Phi_1 + d\Phi$$

$$\kappa_{k+1} = \kappa_1 + d\kappa$$

$$\lambda_{k+1} = \lambda_1\,(1 + d\lambda)$$

8. Repetition of steps 4-7 until corrections reduce to insignificance, i.e., values of the parameters stabilize.

9. Computation of mean square error of transformation.

$$V_{xi} = X_i - X_i' \quad V_{yi} = Y_i - Y_i' \quad V_{zi} = Z_i - Z_i'$$

$$m_0 = \sqrt{\frac{\left[V_{xi}^2 + V_{yi}^2 + V_{zi}^2\right]}{3n - 7}} = m_x = m_y = m_z$$

X_i', Y_i', Z_i' are computed using formulas in step 10.

10. Transformation of other points.

$$X_i' = Xs + \lambda(a_{11}\overline{x}_i + a_{12}\overline{y}_i + a_{13}\overline{z}_i)$$

$$Y_i' = Ys + \lambda(a_{21}\overline{x}_i + a_{22}\overline{y}_i + a_{23}\overline{z}_i)$$

$$Zi' = Zs + \lambda(a_{31}\overline{x}_i + a_{32}\overline{y}_i + a_{33}\overline{z}_i)$$

Once the model connections of all strips are done, the adjustment follows.

Adjustment Procedures

Adjustment is accomplished for two basic reasons:

1. compensation of the errors in aerial triangulation – in relative orientation of models and in model connection.
2. determination of the adjusted ground coordinates of the photocontrol points.

Polynomial Adjustment

The main problem considered in this method of adjustment is to find a suitable "mathematical function" to adjust what is called strip deformation caused by errors in aerial triangulation. Practice has produced two conclusions regarding error propagation in a strip, namely:

1. systematic errors cause second order coordinate errors ΔX, ΔY, ΔZ along the strip; and
2 random errors cause smooth coordinate error curves of unknown order (unpredictable).

In strips less than twenty (20) models, it was found that the coordinate error curves due to both systematic and random errors are "smooth curves rather close to a 2nd or a 3rd order curve. The problem, therefore, is to find a polynomial surface of 2nd or 3rd order that will serve as the interpolation function to determine corrections to computed Z strip coordinates of points or as direct transformation equations to solve for the adjusted X, Y ground coordinates. In practice, the following 2nd order polynomials have been found to be sufficient.

$$X = a_0 + a_1 x - b_1 y + c_1 z + a_2 x^2 - 2b_2 xy + 2c_2 xz$$

$$Y = b_0 + b_1 x + a_1 y - d_1 z + b_2 x^2 + 2a_2 xy - 2d_2 xz \qquad \text{(A)}$$

$$\Delta Z = c_0 + c_1 x + d_1 y + a_1 z + c_2 x^2 + 2d_2 xy - 2a_2 xz$$

where

X,Y = adjusted horizontal coordinates of points

ΔZ = correction to the strip elevation

x,y,z = computed strip coordinates of a point

$a_0 \ldots d_2$ = unknown parameters

Equations (A) provide for a simultaneous adjustment of the three coordinates. However, for flat terrain (range in terrain elevations compared to flying height during photography is not more than 10%), the planimetric and height adjustments may be done separately. In this case the polynomials used are:

$$X = a_0 + a_1 x - b_1 y + a_2 x^2 - 2b_2 xy$$

$$Y = b_0 + b_1 x + a_1 y + b_2 x^2 + 2a_2 xy \qquad \textit{plan} \qquad \text{(B)}$$

$$\Delta Z = c_0 + c_1 x + c_2 x^2 + c_3 y + c_4 xy \qquad \textit{height}$$

For every strip, therefore, there will be six unknown parameters for planimetric adjustment and five unknown parameters for height adjustment. If the two adjustments are separate, for the six planimetric parameters three ground control points are sufficient for a unique solution. One each should be located in the end models and one in the middle model. More than 3 points will provide better determination of parameters. See for example Figure 4. For the height adjustment five control points will be needed to solve the five parameters. Again these should be distributed in the end and middle models. Each strip will therefore have eleven parameters.

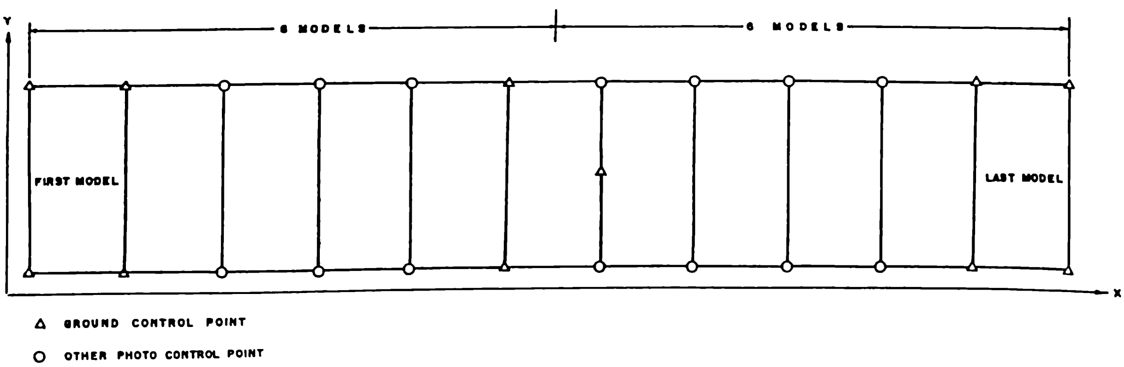

Figure 4. A Strip of Stereo Models Showing Ground Control Points and other Photo Control Points.

When there are several strips, these should be adjusted simultaneously in order that the discrepancies in the coordinates of points common to adjoining strips (tie points) will be minimized. In the adjustment, therefore, the tie points will also be used. Several strips form a block and the procedure is called *block adjustment.* The coordinates of the tie points will also be unknowns.

As an example, we have a block in Figure 5, with the ground control points and tie points indicated. We assume that the control points have all the coordinates X, Y, Z. Here we have three strips, twelve control points and sixteen tie points, with six control points also tie points.

For planimetric adjustment:

A control point i in strip j will provide the observation equations (in matrix notation):

$$\begin{bmatrix} 1 & x & x^2 & 0 & -y & -2xy \\ 0 & y & 2xy & 1 & x & x^2 \end{bmatrix}_{ij} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_j = \begin{bmatrix} X \\ Y \end{bmatrix}_i$$

or $\qquad A_{ij} \cdot P_j = C_i$

where A_{ij} = coefficient matrix (2x6) computed from the strip coordinates of point i in strip j.

P_j = vector of transformation parameters to adjust strip j.

C_i = vector of given ground coordinates of control point i (X_i, Y_i).

A tie point i in strip j will give equations:

$$\begin{bmatrix} 1 & x & x_2 & 0 & -y & -2_{xy} \\ 0 & y & 2_{xy} & 1 & x & x^2 \end{bmatrix}_{ij} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_j - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ y \end{bmatrix}_i = 0$$

or $\qquad A_{ij} \cdot P_j - I \cdot C_i = 0$

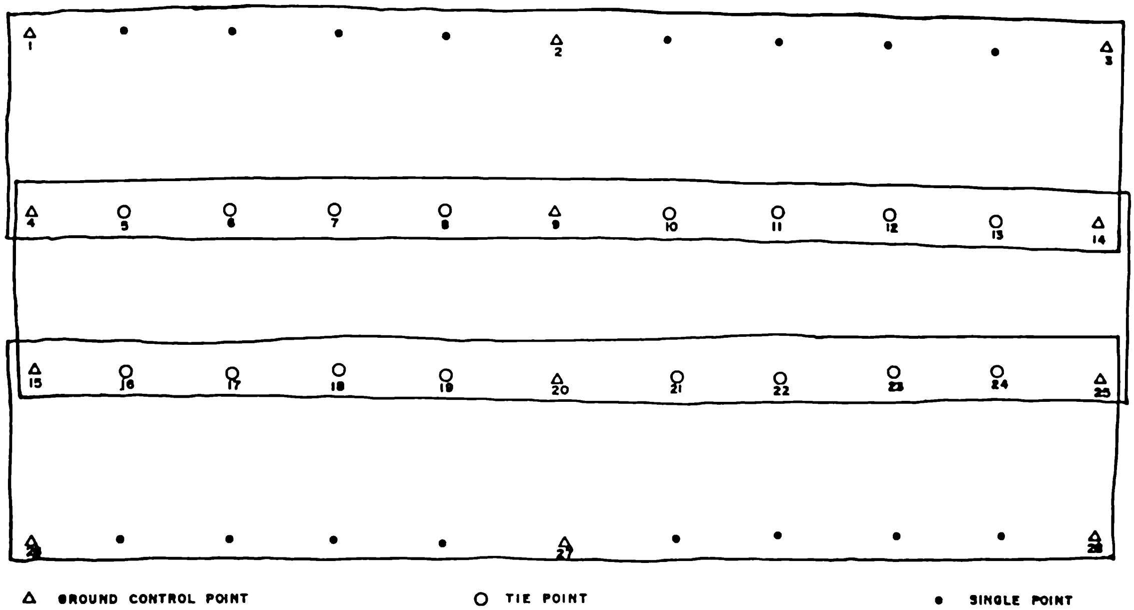

Figure 5. Three Strips for Polynomial Adjustment

where A_{ij} & P_j = similar to control point equations

C_i = vector of unknown ground coordinates of the point i.

There will be 36 control point equations and 64 tie point equations to solve for the 18 unknown parameters and 32 unknown tie point ground coordinates, a total of 100 equations involving 50 unknowns. All the observation equations can be generalized as

$$A_{(100,50)} \cdot U_{(50,1)} = E_{(100,1)}$$

For which the normal equations are

$$A^T A \cdot U = A^T E$$

or

$$N \cdot U = F$$

These normal equations are functions of the two sets of unknowns, the parameters P and the tie point ground coordinates C. Thus, the normal equations can be partitioned into

$$\begin{bmatrix} N_{11} & N^T_{21} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} P \\ C \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \end{bmatrix}$$

or

$$N_{11} \cdot P + N^T_{21} \cdot C = F_1 \qquad (1)$$

$$N_{21} \cdot P + N_{22} \cdot C = 0 \qquad (2)$$

Solving for C in equation (2) we get

$$C = -N_{22}^{-1} N_{21} \cdot P \qquad (3)$$

Substituting (3) in (1) we have

$$[N_{11} - N^T_{21} N_{22}^{-1} \cdot N_{21}] \cdot P = F_1$$

which are the *reduced normal equations* involving only the unknown parameters. The characteristic of these reduced normal equations is that the coefficient matrix is banded which will require less memory and shorter computation time. The solution of the reduced normal equations will give the parameters for all the strips. The adjusted ground coordinates of the tie points and all other points can then be computed per strip. The mean coordinates of the tie points are then computed together with absolute and relative discrepancies, and standard deviations.

For *height adjustment* as stated earlier, the polynomial used is

$$\Delta Z = C_0 + c_1 x + c_2 x^2 + c_3 y + c_4 xy$$

First, the means of ground coordinates, X, Y, Z and strip coordinates x, y, z of all control points in a strip are computed. Thus

$$X_s = \frac{[X_i]}{n} \quad Y_s = \frac{[Y_i]}{n} \quad Z_s = \frac{[Z_i]}{n}$$

$$x_s = \frac{[x_i]}{n} \quad y_s = \frac{[y_i]}{n} \quad z_s = \frac{[z_i]}{n} \qquad (i = 1 \ldots\ldots n)$$

Then, the reduced coordinates are obtained.

$$\overline{X}_i = X_i - X_s \quad \overline{Y}_i = Y_i - Y_s \quad \overline{Z}_i = Z_i - Z_s$$

$$\overline{x}_i = x_i - x_s \quad \overline{y}_i = y_i - y_s \quad \overline{z}_i = z_i - z_s$$

These reduced coordinates are used to compute a scale factor for each strip j:

$$\lambda_j = \frac{[\sqrt{X_i^2 + Y_i^2 + Z_i^2}]}{[\sqrt{x_i^2 + y_i^2 + z_i^2}]}$$

The unadjusted elevation of any point i in strip j is therefore

$$H_{ij}' = \lambda_j z_{ij}$$

For every control point, the error in elevation is

$$\Delta Z_{ij} = H_i - H_{ij}'$$

where H_i = given elevation.

For every other point i the adjusted elevation in strip j will be

$$H_i = H_{ij}' + \Delta Z_{ij}$$

Thus, the observation equations in the adjustment are as follows: For a control point i contained in strip j, we have

$$(c_o + c_1 x + c_2 x^2 + c_3 y + c_4 xy)_{ij} = \Delta Z_{ij}$$

For a tie point i in strip j, we have

$$(c_0 + c_1 x + c_2 x^2 + c_3 y + c_4 xy)_{ij} - \Delta Z_{ij} = 0$$

The corrections ΔZ_{ij} to the unadjusted elevation H_{ij}' of every tie point are also treated as unknowns.

In the example of Figure 5, assuming that all the ground control points have given elevations, there will be 18 control point equations and 32 tie point equations, to solve for the 15 strip parameters and 32 unknown elevation corrections for the tie points. Again, a least squares solution is needed.

The formation and solution of the normal equations follow the same procedure as in the planimetric adjustment. Once the adjustment parameters are obtained, the adjusted elevations of all points can be computed for in each strip. It will still be necessary to obtain the average adjusted elevations of all tie points. Finally, absolute discrepancies in control points and relative discrepancies in tie points will indicate standard deviations of elevations of points.

Because of the fewer parameters that are solved for in the polynomial adjustment method, it is a much simpler procedure and requires less computation time. However, it can be seen that it is not a very rigid method. A more rigid adjustment is attained in the Independent Model Adjustment Method.

Independent Model Adjustment

What makes the polynomial method less rigid is the fact that the single unit being adjusted is the strip that had resulted from model connection of each strip separately. In the Independent Model Method, the single unit is the individual models themselves. Here, each stereomodel undergoes a separate *spatial similarity transformation* together with the rest of the models in a block. The basic transformation equations are, considering a point i in model j:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \lambda_j \cdot R_j \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j$$

where:

X_i, Y_i, Z_i = ground coordinates of point i.

x_{ij}, y_{ij}, z_{ij} = model coordinates of point i in model j.

λ_j = scale factor for model j

R_j = rotation matrix of model j in which the three rotations Ω_j, Φ_j, κ_j are implicit

X_{oj}, Y_{oj}, Z_{oj} = three translations for model j.

These three equations are non-linear in terms of the four parameters λ_j, Ω_j, Φ_j, κ_j. To solve for these parameters there is need to linearize the observation equations and for the estimation of approximate values. Usually λ_j and κ_j are very large compared to Ω_j and Φ_j, so that one disadvantage of this simultaneous solution for the seven parameters for each model in the block is the need for very good approximate values. With the large number of unknowns, large computer storage and computer time are required. In practice, therefore, the iterative procedure is separated into an alternate planimetric adjustment to obtain the four parameters λ, κ, X_0, Y_0 and height adjustment for the remaining three parameters Ω, Φ and Z_0.

A computational algorithm for the procedure is as follows:

A. Planimetric Adjustment (See Figure 6)

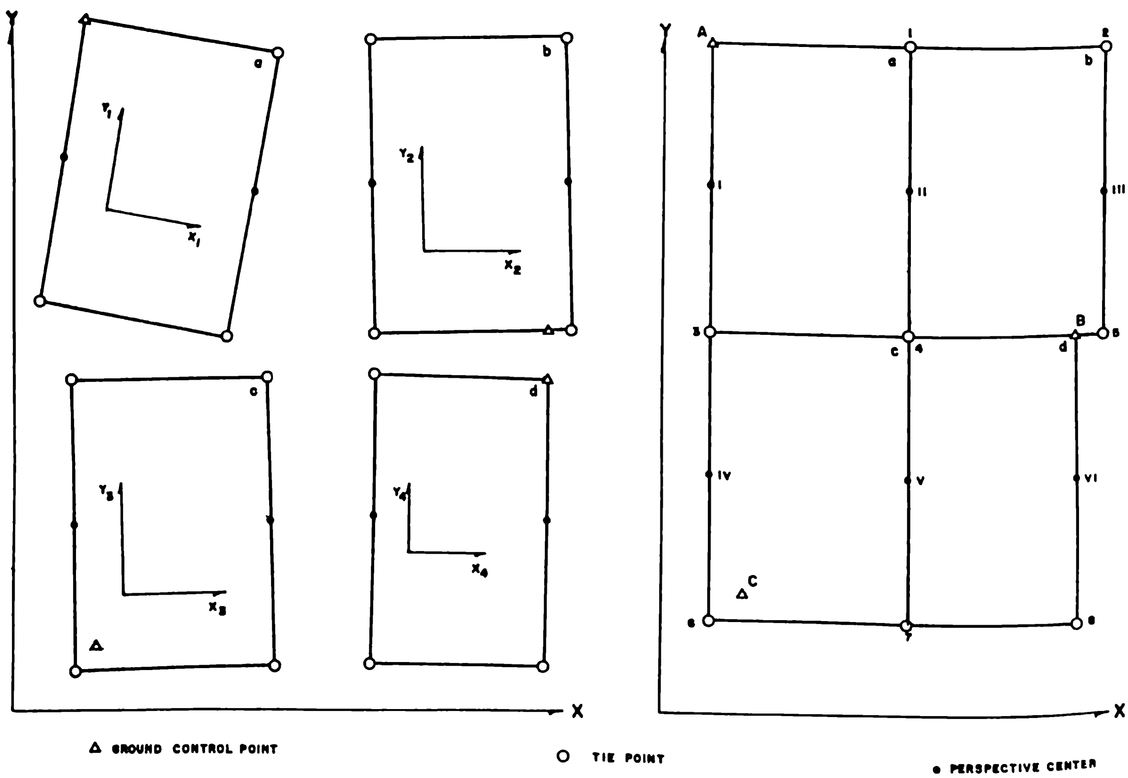

Figure 6. Block Adjustment of Independent Models.

1. Formation of the observation equations.

For *control point* i in model j

$$\begin{bmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{bmatrix}_{ij} \cdot \begin{bmatrix} a \\ b \\ X_0 \\ Y_0 \end{bmatrix}_j = \begin{bmatrix} X \\ Y \end{bmatrix}_i$$

$$a = \lambda . \cos \kappa \qquad b = \lambda . \sin \kappa$$

or

$$A_{ij} \bullet P_j = C_i$$

For *tie point* i in model j

$$\begin{bmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{bmatrix}_{ij} \cdot \begin{bmatrix} a \\ b \\ X_0 \\ Y_0 \end{bmatrix}_j - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix}_i = \begin{matrix} 0 \\ 0 \end{matrix}$$

or

$$A_{ij} \bullet P_j - I \bullet C_i = 0$$

Perspective centers are not included.

All observation equations combined will have the form

$$A \cdot U = E$$

2. Formation and solution of the normal equations.

Assuming all strip coordinates are of equal weight and not correlated, the normal equations will be

$$A^T A \cdot U = A^T E$$

or

$$N \cdot U = F$$

which can be partitioned into

$$N_{11} \bullet P + N_{21}{}^T \bullet C = F_1$$

$$N_{21} \bullet P + N_{22} \bullet C = 0$$

Eliminating C, the matrix of unknown coordinates of tie points, the reduced normal equations are

$$[N_{11} - N_{21}{}^T . N_{22}{}^{-T} N_{21}] . P = F_1$$

The solution of these normal equations will give P the parameters a_j, b_j, X_{oj}, Y_{oj} for all models

By definition $\lambda_j = \sqrt{a_j^2 + b_j^2}$

3. Transformation of all points (control points, tie points, perspective centers) in the model using the parameters obtained in step 2.

$$\begin{bmatrix} \overline{X} \\ \overline{Y} \\ \overline{Z} \end{bmatrix}_{ij} = \begin{bmatrix} a & -b & o \\ b & a & o \\ o & o & \end{bmatrix}_j \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} + \begin{bmatrix} X_o \\ Y_o \\ 0 \end{bmatrix}_j$$

B. Height Adjustment

4. Formation of the linearized observation equations.

For *height control point* i in model j

$$[\overline{Y}_{ij} \quad -\overline{X}_{ij} \quad 1] \cdot \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_o \end{bmatrix}_j = [Z_i - \overline{Z}_{ij}] \qquad (Z_i = \text{given elevation})$$

For *tie point* i in model j

$$[\overline{Y}_{ij} \quad -\overline{X}_{ij} \quad 1] \cdot \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_o \end{bmatrix}_j - [Z_i] = [-\overline{Z}_{ij}] \qquad (Z_i = \text{unknown elevation})$$

For perspective center p in model j

$$\begin{bmatrix} \overline{Z}_{pj} & o & o \\ o & -\overline{Z}_{pj} & o \\ \overline{Y}_{pj} & -\overline{X}_{pj} & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_o \end{bmatrix}_j - \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix} = \begin{bmatrix} -\overline{X}_{pj} \\ -\overline{Y}_{pj} \\ -\overline{Z}_{pj} \end{bmatrix}$$

Again, when all observation equations are combined together, they will take the form

$$A \cdot U - E$$

5. Formation and solution of normal equations.

The normal equations will be

$$A^T A \cdot U = A^T E$$

or

$$N \cdot U = F$$

The technique of reduced normal equations is employed in order to eliminate the unknown elevation coordinates Z_i of tie points, and the unknown coordinates X_p, Y_p, Z_p of perspective centers.

The solution of the reduced normal equations will then give the parameters $\Delta\Omega_j$, $\Delta\Phi_j$, Z_{oj} for all models.

6. Transformation of all model points.

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}_{ij} = \begin{bmatrix} 1 & 0 & +\Delta\Phi \\ 0 & 1 & -\Delta\Omega \\ -\Delta\Phi & +\Delta\Omega & 1 \end{bmatrix}_j \cdot \begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix}_{ij} + \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}_j$$

7. Using these transformed coordinates of points a new planimetric, then a height, adjustment (steps 1-6) is performed until the final values of the transformed coordinates do not appreciably change. Usually two or three iterations will be sufficient.

8. Computation of mean coordinates of tie points. Computation of absolute and relative discrepancies, and standard deviations of coordinates.

It is in the nature of the reduced normal equations that only the unknown parameters of transformation remain to be solved for so that if m is the number of models in the block, the number of unknowns for the planimetric adjustment is 4m, and for the height adjustment, 3m, the solutions for which occur one after the other. Computation of unknown coordinates of points uses the original normal equations.

Requirement for Ground Control Points

In the polynomial adjustment, since the polynomials used are of the 2nd or 3rd degree, at least three bands of ground control points, usually at the beginning, middle and end models of the strips are needed. For the block in Figure 5, therefore, 12 ground control points are used to obtain the positions of 32 other photo control points.

In the independent model adjustment, F. Ackermann of Stuttgart University conducted theoretical studies of blocks up to 20,000 models from which the very significant conclusion was made that "blocks with planimetric perimeter control can be used up to virtually any size." Such perimeter control will be one every 4 to 6 models along the axes of the strips and one every 2 strips in the perpendicular direction. He, however, prescribes that height control points be provided inside the block. These are shown in Figure 7. Here, we have a block of 8 strips with 16 models each, a total of 128 models that will require 153 photo control points, of which 16 are horizontal-vertical perimeter control points and 9 additional height control points in the middle of the block. This means that with 25 ground control points, the positions of the remaining 128 control points are obtained by the aerial triangulation.

Accuracy of Aerial Triangulation

Exhaustive studies, and confirmed in practice, show that the independent model method of aerial triangulation with measurements from precision stereoplotters and with perimeter control has consistently reached the 10 μm level at photo scale. The following table shows the accuracies in planimetry attained in some projects abroad.

Photo Scale	Instr.	Number of Models	Cont. Pts.	Unknown Pts.	Tie Pts.	Standard Error of Unit Weight (cm)
1:6000	C8	32	42	4800	892	4.4
1:4300	Planimat	54	65	3178	1548	4.5
1:10000	C8	33	19	3791	408	8.2
1:7500	A7	170	32	1065	950	5.7
1:14000	A8	129	36	442	366	28.0

(Portion of Table from Ackermann, Ebner & Klein)

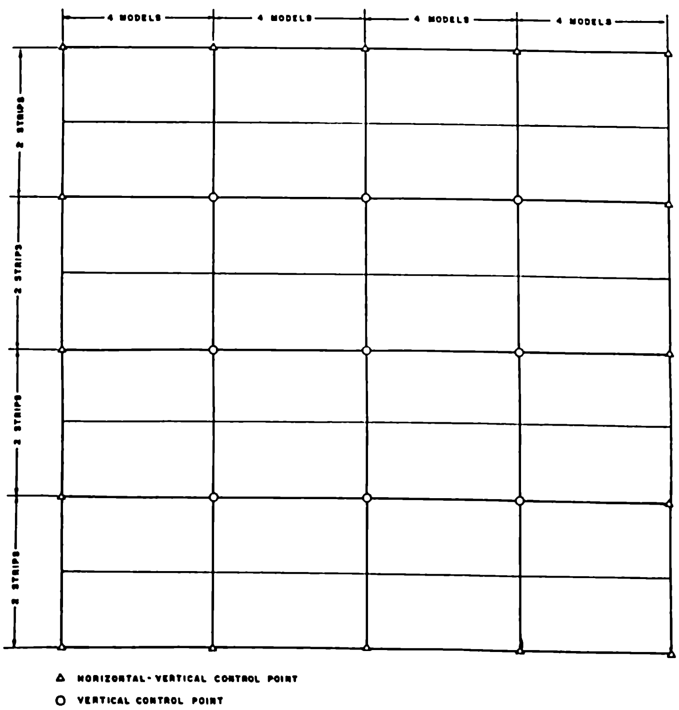

Figure 7. Block of Aerial Photographs Showing Peripheral Locations of Horizontal - Vertical Control Points.

Research Work in NEC

So far, aerial triangulation has not been used in the Philippines in photogrammetric projects. All mapping projects have been done with stereomodels provided with photocontrol points surveyed on the ground. It is significant, therefore, that a research proposal to investigate the application of the Independent Model Method of Aerial Triangulation under Philippine Conditions has been approved by the Committee on Research of the U.P. College of Engineering sponsored by the National Engineering Center. Measurements will be done in the A8 stereoplotter of the Training Center for Applied Geodesy and Photogrammetry. The work will involve a block of six strips with about 70 models. What is significant in the investigation is that all the models are already provided with photocontrol points established by the conventional ground method by a local survey company. Using only the required number for block adjustment, there is the possibility of comparing positions of the other points by aerial triangulation with those obtained by the ground method. One problem that the research will have to hurdle is the fact that the computer programs for independent model adjustment are not yet available in the country. Part of the work may be the development of simpler but adequate programs.

References:

ACKERMANN, F., EBNER, H., and KLEIN, H., "Block Triangulation With Independent Models," Photogrammetric Engineering, 39:9 September, 1973, pp. 967-981.

ALBERTZ, J., and KREILING, W., "Strip and Block Adjustment," Photogrammetric Guide, Karlsruhe, 1975, pp. 203-212.

AMER, F., "Adjustment of Aerial Triangulation," ITC Provisional Notes, Netherlands, March, 1978.

KELLER, M., and TEWINKEL, G.C., "Aerotriangulation Strip Adjustment," Technical Bulletin No. 23, US Coast & Geodetic Survey, 1964.

MARZAN, G.T., "Establishment of Survey Reference Points By Aerotriangulation," Journal of Cartography, Geodesy and Photogrammetry, 1:1 June, 1980.

MORGAN, P., "Rigorons Adjustment of Strips," Photogrammetric Engineering, 37:12, December 1971, pp. 1271-1283.

SCHUT, G.H., "Development of Programs for Strip and Block Adjustment at the NRC Canada," Photogrammetric Engineering, 30:2 February, 1964, pp. 283-291.

SCHUT, G.H., "Similarity Transformation and Least Squares," Photogrammetric Engineering, 39:6 June, 1973, pp. 621-627.

WONG, K.W., and ELPHINGSTONE, G., "Recursive Partitioning By Direct Random Access," Photogrammetric Engineering, 39:3 March, 1973, pp. 267-274.