"designing a physical model starts with determining the appropriate scales "

River Models: Physical vs. Mathematical*

by

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Abstract

Rigid bed physical and mathematical modelling are each discussed at review level. A brief historical background, an introduction to the theoretical basis for validity, and some sub-classifications and variants are included. The two techniques are then compared, and some criteria for selection of method to use are mentioned. It is suggested that problem definition should govern in the choice of models. Recent trends that are radically changing the criteria are also mentioned.

Introduction

Models in Engineering Practice

The use of models has become at times indispensable in the planning and design of water resources projects. Modelling is a broad term that covers mathematical, physical, electrical analogue, and physical analogue simulation systems that are applicable to Fluid Mechanics, Hydraulic Engineering, Hydrology and even Water Resources systems planning and design.

Figure 1.1 shows engineering activities in general. For problems familiar to the investigator, some procedures may have been formulated beforehand; methods that are either analytical, empirical, or semi-empirical and are theoretically sound and have been proven through extensive experience. The planning and design engineer uses these methods with confidence even without the use of models.

Many problems, however, do not have straightforward solutions. For these, we take the detour, as in Figure 1.1, and use models. River models are used in design of structural measures for flood control and river training such as levees, revetments, groynes and cut-offs. Planning of non-structural measures such as flood forecasting and warning systems oftentimes uses river models. In fact, rivers, being nature's route for water that is most accessible to man, are often the focus of attention of water resources projects. Models prove their worth in testing various

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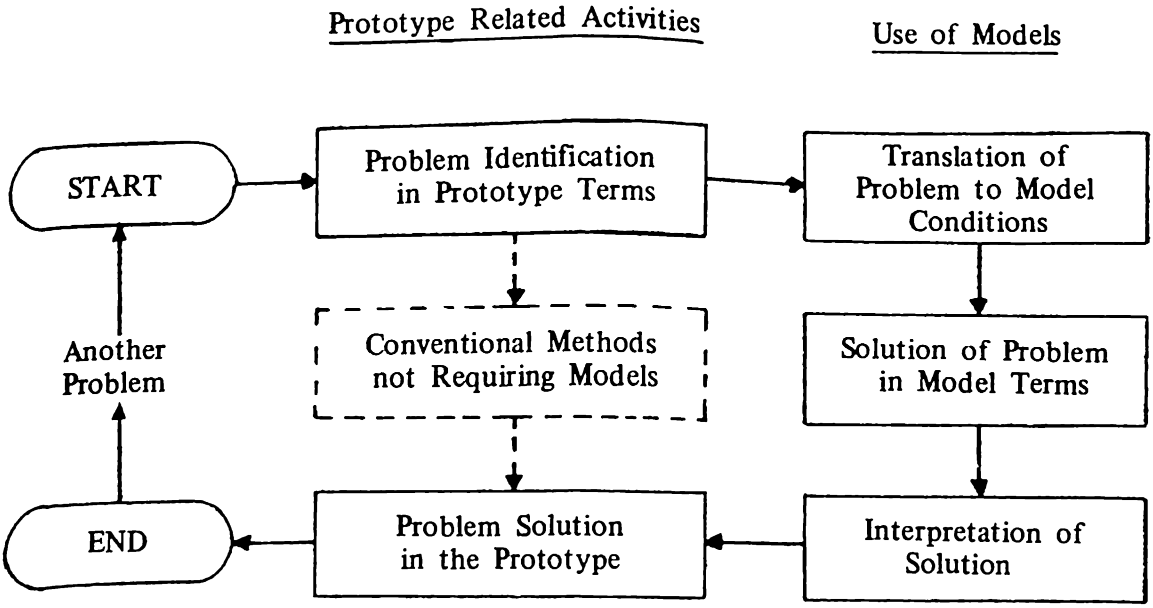


Figure 1.1

alternatives to arrive at optimal solutions before implementation in the prototype. Further assurance is also derived from model studies in projects where lives, property, or large capital investment on structural measures are at stake.

Types of Models

Only mathematical and physical hydraulic models of rivers are discussed in this paper. Physical analogue (e.g., Hele-Shaw) models have applicability limited to groundwater flow studies. In a strict sense, electrical analogue models are mathematical models since its use involves circuit programming done to emulate a selected set of equations representing laws that govern the flow in the prototype. However, special equipment (analog and hybrid computers) are required and these are not readily available in our country.

Two distinct types of physical models are in use, rigid bed and mobile bed. Theoretical treatment for one type is quite different from the other, and that for movable bed alone deserves a separate paper. Thus, only fixed bed models are treated. The discussion on Physical Hydraulic Models describes both distorted and undistorted models at a review level. Most readers will have had previous exposure to physical modelling concepts and are referred to papers or whole books for intensive treatment on hydraulic models in general. CASTRO (1980) gives more introductory material on the subject.

Mathematical models as applicable on digital computers are then introduced including a brief discussion on their theoretical basis. Recent publications giving complete information are also cited. Only one-dimensional models are treated, since applicability of two-dimensional models has been so far limited to lake, coastal and esturial problems.

Physical and mathematical models are then compared in terms of tasks involved and applicability. Trends in their development are presented and the author advances some opinions on future developments.

Throughout the paper, examples of model studies that have been done in this country are cited. The author does not claim completeness, as only those he has come across in the course of his work are mentioned.

Physical Hydraulic Models

Historical Background

The impetus to use hydraulic models in the study of flow behavior dates back to Leonardo da Vinci's statement that it is necessary to conduct experiments to gain theoretical insight on fluid behavior. Da Vinci himself conducted elementary tests on some open channel flow situations (SHEN, 1979).

Isaac Newton's theorems on mechanical similarity formed the basis for model laws. In 1875, a Frenchman, L.J. Fargue built a river model to demonstrate his proposed regulation schemes. In 1885, Osborne Reynolds built two models of the same prototype at different scales to study river regulation and tidal effects (IVICSICS, 1975).

The first permanent river hydraulics laboratory was founded by Hubert Engels in Dresden, Germany, where in 1913 he built the first large-scale river model (KOBUS, 1980).

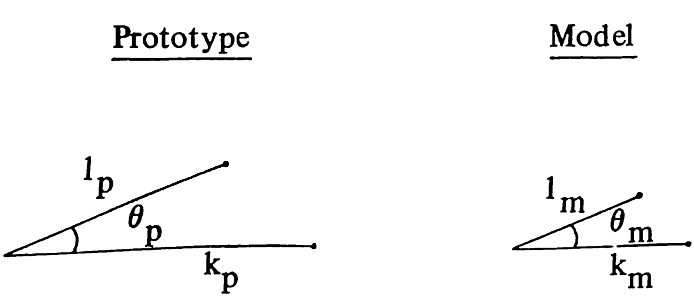
The hydraulic laboratory of the University of the Philippines was built in 1954. In addition to its academic function, the laboratory was capable of conducting model investigations. Much of the work involved mainly modelling of hydraulic structures, but under the direction of Prof. A.A. Alejandrino, it built and tested two models to study the bifurcation at the Agus and Linamon rivers.

Local experience was also gained when the then Bureau of Public Works built a model of the Pampanga river system to test flood control alternatives.

The National Hydraulic Research Center, since its conversion from the U.P. Hydraulic Laboratory in 1973, has conducted a number of studies on river training and control. The Pasig river cut-off study was done in 1974, the Napindan hydraulic control structure was tested in 1977 and the Libuganon river control study was undertaken in 1980. Tests on the Mangahan Floodway have just recently been completed this year, and the Allah River model is presently being tested.

Basis of Validity of Models

For proper simulation of a prototype river system, flows in the model should be "similar" to that in the prototype. It is not enough that the model be geometrically similar (see Figure 2.1), but also dynamically similar (Figure 2.2).



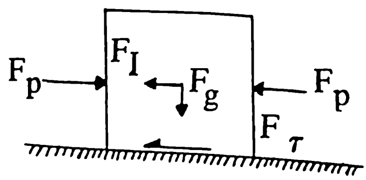
Geometric Similarity

$$\theta_p = \theta_m$$

$$\frac{l_p}{l_m} = \frac{k_p}{k_m}$$

Figure 2.1

Forces on a fluid element



F_p = pressure force

F_g = gravity force

F_I = inertial force

F_τ = shear force

Dynamic Similarity

A ratio of any two forces in the model is equal to that in the prototype.

Figure 2.2

To achieve a *completely* similar model, we need to construct it at a scale of 1:1, which defeats the purpose of modelling. To illustrate, consider the Froude Number, $F = \frac{U}{\sqrt{gl}}$ which can be regarded as the root of the ratio of inertia force over gravity force, and the Reynolds Number, $R = \frac{Ul}{v}$, the ratio of inertia force over shear force. If we try to satisfy $F_m = F_p$ and $R_m = R_p$ simultaneously, we end up with $\frac{v_m}{v_p} = \left(\frac{l_m}{l_p}\right)^{3/2}$ We have to find large quantities of an uncommon or non-existent fluid to satisfy the required ratio of kinematic viscosities, or use water but build our model at $\frac{l_m}{l_p} = 1$.

In practice, Froude similarity is considered sufficient for models of open channels. Investigators simply ensure that shear and surface tension forces in the model are negligible in comparison to gravity or pressure forces.

Model Scales

From the requirement of equality of Froude Numbers, the discharge velocity and time scales follow from the length scale, i.e., that for undistorted models:

$$\frac{Q_p}{Q_m} = \left(\frac{L_p}{L_m}\right)^{5/2}$$

$$\frac{U_p}{U_m} = \left(\frac{L_p}{L_m}\right)^{1/2} \qquad (2.1)$$

$$\frac{t_p}{t_m} = \left(\frac{L_p}{L_m}\right)^{1/2}$$

and for models with a vertical distortion,

$$\frac{Q_p}{Q_m} = \frac{X_p}{X_m} \cdot \left(\frac{Z_p}{Z_m}\right)^{3/2}$$

$$\frac{U_p}{U_m} = \left(\frac{Z_p}{Z_m}\right)^{1/2} \qquad (2.2)$$

$$\frac{t_p}{t_m} = \frac{X_p}{X_m} \cdot \left(\frac{Z_m}{Z_p}\right)^{1/2}$$

where z and x are length quantities in the vertical and horizontal directions, respectively. Equation 2.1 or 2.2 is used to determine model parameters to be used in test runs for simulating a prototype situation, or to convert measured quantities in the model to prototype terms.

Although the determination of scales are straight-forward, the decision as to what length scale to use is influenced by instrument precision and logistical limitations, i.e., availability of funds, space, and facilities. For reasons of economy, the "smallest" possible model is adopted, without unduly violating the similarity criteria. Several recent works have been written on hydraulic models in general, e.g., KOBUS (1980), IVICSICS (1975), and NOVAK and CABELKA (1981). Each of these works contains extensive treatment on finding the lower limit of model size. Distortion of the vertical scale to enable the modelling of a large area is also subject to limitation. The distortion ratio, n, defined as horizontal scale $\frac{x_p}{x_m}$ divided by the vertical scale $\frac{Z_p}{Z_m}$ has a maximum value. KNAUSS (1980) puts the upper limit of n as ten percent of the width to depth ratio of the prototype river, or 5.0, whichever is less.

Mathematical models

Historical Background

The earliest attempts to mathematically express the behavior of flowing water were done by P.S. Laplace in 1775-1776 and I.L. Lagrange in 1781. Their studies, however, did not suffice to describe the flow in natural channels. Nobody did until 1871 when Barre de Saint-Venant presented a paper on the theory of unsteady flow at the French Academy of Sciences. The original forms of his equations of continuity and motion as interpreted by YEVJEVICH (1975) are:

$$\frac{\partial w}{\partial t} + \frac{\partial (wU)}{\partial s} = 0 \qquad (3.1)$$

$$\frac{\partial \epsilon}{\partial s} = \frac{1}{g}\frac{\partial U}{\partial t} + \frac{U}{g}\frac{\partial U}{\partial s} + \frac{X}{w}\frac{F}{\rho g} \quad (3.2)$$

where w = cross-sectional area

U = mean velocity

ϵ = position of water surface above a reference level

$\frac{XF}{w\rho g}$ = friction slope

s = length along the rectangular prismatic canal

t = time

Saint-Venant's work was closely related to some of his contemporaries, notably H.L. Partiot, J.S. Russell, H. Bazin, and J. Boussinesq with their studies on the propagation of waves in open channels. Saint-Venant, however, was superior in his mathematical approach to the formulation of the equations.

In the absence of electronic computers, the equations were, in practice, integrated only for regular shaped boundaries. The first attempt to use digital computers to simulate natural channels was done in 1952-1953 by Isaacson, et. al. and applied to portions of the Ohio and Mississippi Rivers. In France, Preissmann, et. al. developed a model in 1959-1962 that became popular and has since been applied extensively (CUNGE, 1975). SOGREAH (1973), used the model in the hydraulic analysis of the Pasig-Marikina-Napindan river system.

One form or another of the Saint-Venant equations endures to this day. Subsequent work served to generalize their application, but with simplifying assumptions they reduce to the original form. A brief derivation is given and various modifications that have been used in their application are presented. The required boundary and initial conditions are also provided and methods of solving the governing equations are introduced.

Governing Equations of River Flow

There are several ways of arriving at the continuity and momentum equations from the basic principles of fluid mechanics. Basic textbooks in free-surface flow give derivations of the equations, e.g., CHOW (1959) and HENDERSON (1966). Several works have been written with computer simulation as the ultimate objective.

YEN (1979) takes the mathematically rigorous approach of integrating the continuity and momentum equations defined at a point. The "point-form" momentum equations are more generally known as the Navier-Stokes equations. The first step involves the time averaging of the turbulent fluctuations of velocity. Next, integration over the depth, or using average velocity through each vertical, results in the two-dimensional form of the equations. Integration over the cross-section results in the generally used one-dimensional flow equations.

CUNGE, et.al. (1980) and LIGGETT (1975) use the control volume approach which is conceptually easier to comprehend. Immediately, the assumptions originally made by Saint-Venant are invoked, namely:

a. The pressure distribution over the vertical remains hydrostatic.

b. Velocity distribution across the wetted cross-section does not affect surface wave propagation.

c. The water surface across any cross-section is horizontal.

d. Friction losses from turbulence and boundary resistance do not depart significantly from that of steady flow.

e. The average longitudinal slope, a, of the channel bed is small so that tan a may be replaced by sin a and cos a may be considered as unity.

For the one-dimensional case, the control volume approach proceeds from the definition sketch in Figure 3.1.

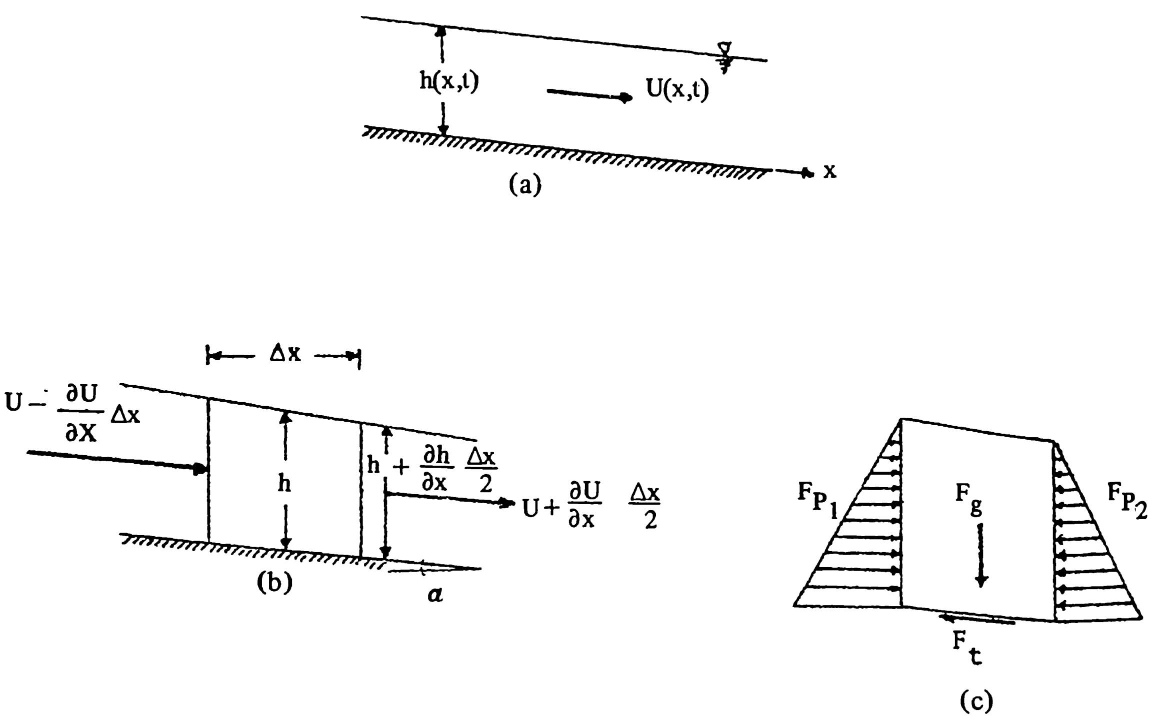


Figure 3.1

The law of conservation of mass is applied to get the continuity equation, i.e.

$$\left(U - \frac{\partial U}{\partial x}\frac{\Delta x}{2}\right)\left(h - \frac{\partial h}{\partial x}\frac{\Delta x}{2}\right) - \left(U + \frac{\partial U}{\partial x}\frac{\Delta x}{2}\right)\left(h + \frac{\partial h}{\partial x}\frac{\Delta x}{2}\right) = \frac{\partial h}{\partial t}\Delta x$$

inflow − (outflow) = (change in storage)

(Strictly, mass flow should be expressed as ρU, but since water is incompressible through the range of events being considered, the density term is cancelled). Simplifying and taking the limit as Δx approaches zero,

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(Uh) = 0 \tag{3.3}$$

The law of conservation of momentum can be applied as follows:

$$\rho\,[U^2h - \frac{\partial}{\partial x}(U^2h)\,\frac{\Delta x}{2}] - \rho\,[U^2h + \frac{\partial}{\partial x}(U^2h)\,\frac{\Delta x}{2}]$$

(momentum influx) — (momentum efflux)

$$-\frac{\partial}{\partial t}(\rho\,Uh)\,\Delta x = \Sigma F = F_g + F_f + F_{P_1} - F_{P_2}$$

– (rate of momentum accumulation) = (gravity component along x) + (friction) + (pressure, hydrostatic)

where: $F_{P_1} - F_{P_2} = \frac{\rho g}{2}\,[\,(h^2 - \frac{\partial h^2}{\partial x}\,\frac{\Delta x}{2}) - (h^2 + \frac{\partial h^2}{\partial x}\,\frac{\Delta x}{2})\,]$

$F_g = \rho g h\,\Delta x\ \tan a = \rho g h\,\Delta x\ \sin a = \rho g h\,\Delta x\ S_o$

$F_f = \rho g h\,\Delta x\ S_f$; S_f = friction slope

Combining terms and simplifying, we get

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = g\,\frac{\partial h}{\partial x} = g(S_o - S_f) \tag{3.4}$$

Equations 3.3 and 3.4 are the familiar basic forms of the continuity and momentum equations. Many variants of the equation are in use, depending on the physical system to be simulated, the degree of accuracy and precision of results and available data, and the equipment and other resources available to the modeller.

Simplified Equations

The simplest way of numerically simulating river flow is the sole use of the continuity equation. This technique is oftentimes called "hydrologic routing" or "storage routing." Accuracy is limited but sufficient as a first approximation of river flow. It is seldom used singly but as a part of the the modelling of a larger hydrologic system, e.g., in water balance studies. Many projects at pre-feasibility and feasibilty level undertaken locally have used the method, e.g., TAHAL (1978) on the Pampanga river basin.

When one form or another of the momentum equation is used in combination with the continuity equation, a "hydraulic routing" model is developed. The simplest form of the momentum equation is arrived at if we retain only the bed and friction slope terms, i.e., from Equation 3.4,

$$S_o = S_f \tag{3.5}$$

which is called the "kinematic wave equation." Use of Equation 3.5 with the continuity equation is called "kinematic flood routing," and is equivalent to making use of a rating curve for every section of the model, with flow variations through each time step defined by the continuity equation. This method may only be used for systems where Froude Numbers are much less than 1 and backwater effects are negligible.

The "diffusion wave equation", evolves from Equation 3.5 through the addition of the diffusion term, sometimes called the "pressure term":

$$\frac{\partial h}{\partial x} = (S_o - S_f) \tag{3.6}$$

The inclusion of the slope of the water surface (on the left side of Equation 3.6) enables the model to simulate the translation of a hydrograph along the stretch of the channel represented, as well as permits the upstream propagation of backwater effects.

Two other terms from Equation 3.4 are not included in Equation 3.6. The term $\frac{\partial U}{\partial t}$ is called the "local acceleration" term as it reflects changes in the velocity at a point through time, while $U \frac{\partial U}{\partial x}$ is called the "convective acceleration" term and expresses the variation of velocities through space. Neglecting the local acceleration term from Equation 3.4 results in the "quasi-steady dynamic wave" model. According to YEN (1979), it is preferable to use either the full dynamic wave model (Equations 3.3 and 3.4) or the diffusion wave model. Neglecting either local or convective terms gives worse results than neglecting both.

Added Sophistications

Lateral inflow is the easiest add-on to implement as it modifies Equations 3.3 and 3.4 only slightly. It is used to reflect actual lateral inflow from banks, and to approximate the contribution of minor tributaries or losses through percolation.

Flood plain storage is simulated by modifying the application of the continuity equation alone. However, when the flow through the overbank areas is considerable, the momentum equation is also applied to flood plain discharge.

Momentum and conveyance correction factors are applied to the momentum equation to reflect non-uniformity of the velocity across the flow cross-section. These factors, however, end up modified during the calibration process as modellers attribute some inexplainable behavior of the model to the uncertainties involved in the determination of these correction factors.

All of the above sophistications were adopted by ACKERMANN and SHIIGAI (1976) where they developed a model that was applied to the Bicol river system.

Boundary and Initial Conditions

The application of the full dynamic wave equations to a river reach requires that the velocity or discharge, and the wetted cross-sectional area or depth of flow, be defined throughout the reach at time t = 0. Zero discharge or velocity is not allowed at any point at any time and modellers usually attach provisions for a minimum discharge. Subcritical flow through the reach requires that, at all points in time, the velocity or discharge at the upstream boundary and the depth or wetted cross-sectional area at the downstream boundary be known. Super-critical flow is an upstream control case, thus requiring that both depth or wetted area and discharge or velocity at the upstream boundary be a function of time.

Structures, control sections, junctions, and discontinuities (e.g., sudden contraction or expansions of channel cross-section that effectively violate the Saint-Venant assumptions) have to be treated as internal boundary conditions. A model of a complex river system therefore consists of segments where Equations 3.3 and 3.4 are valid, bounded and/or linked by discontinuities governed and simulated by special equations. As an example, a junction schematically shown in Figure 3.2 is governed by the following equations:

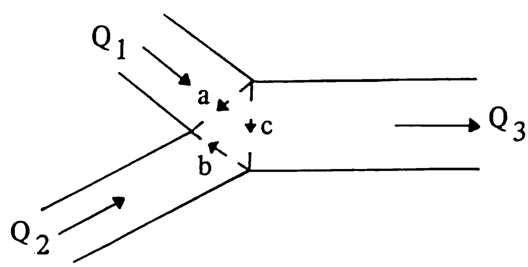


Figure 3.2

$$\left.\begin{aligned} \Sigma Q &= 0 \text{ or } Q_1 + Q_2 = Q_3 \\ h_a &= h_b = h_c \end{aligned}\right\} \qquad (3.7)$$

where Q_1, Q_2, Q_3 are discharges

h_a, h_b, h_c are total energies at corresponding points.

Methods of Solution

Three main classes of numerical solutions are available to integrate Equations 3.3 and 3.4, namely a) the method of characteristics, b) explicit methods, and c) implicit methods, in chronological order of their genesis. The methods involve the discretization of the continuous space and time dimensions of the simulated river system into finite intervals. The methods are called finite difference techniques also because the partial differential equations (3.3 and 3.4) are transformed into a set of algebraic equations expressing discrete changes occurring between the finite space and time intervals.

The first step in using the method of characteristics is to transform the governing equations into the so-called "characteristic equations." These equations define the propagation of small discontinuities in water surface slope or velocity slope ($\frac{\partial h}{\partial x}$ and $\frac{\partial U}{\partial x}$ in Equation 3.4).

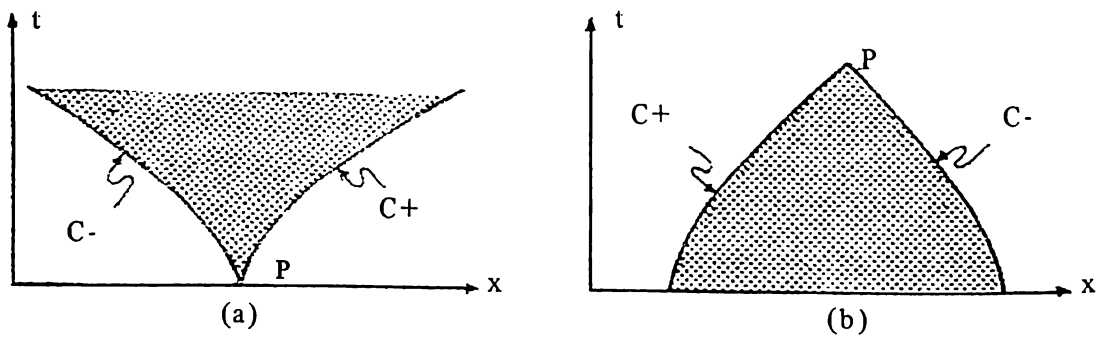


Figure 3.3

Figure 3.3 diagrammatically shows that the disturbance occurring at any point P travels along "characteristic curves" downstream (C+) and upstream (C–), and forward in time (Figure 3.3 (a)) or backward (Figure 3.3 (b)). The shaded regions define the "domain of influence" of the disturbance, i.e., that outside of the regions the disturbance does not have any effect. Equations 3.3 and 3.4 are each transformed into a pair of ordinary differential equations which are mathematically easier to handle than the original partial differential equations. The four characteristic equations are then solved by finite differences. The method, being the oldest, is treated extensively in the literature (e.g., ABBOTT, 1975). LIONGSON (1973) tested it on the Agus River.

Both explicit and implicit methods use a fixed grid to approximate the physical system.

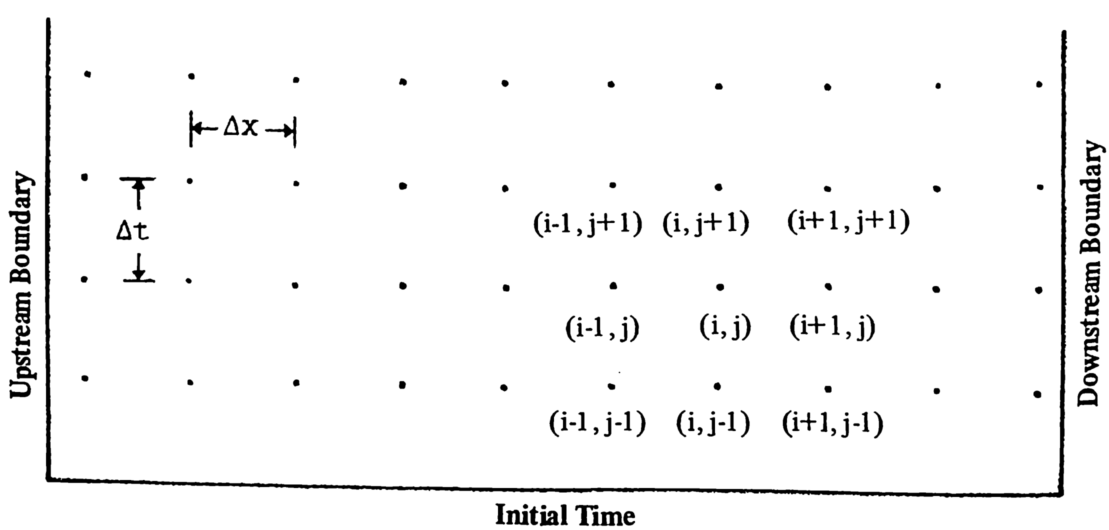


Figure 3.4

Figure 3.4 shows the x-t plane where computation usually proceeds from an initial time $t = t_0$ and progresses through constant time increments Δt, while space increments Δx may or may not be constant. We shall now consider the general case

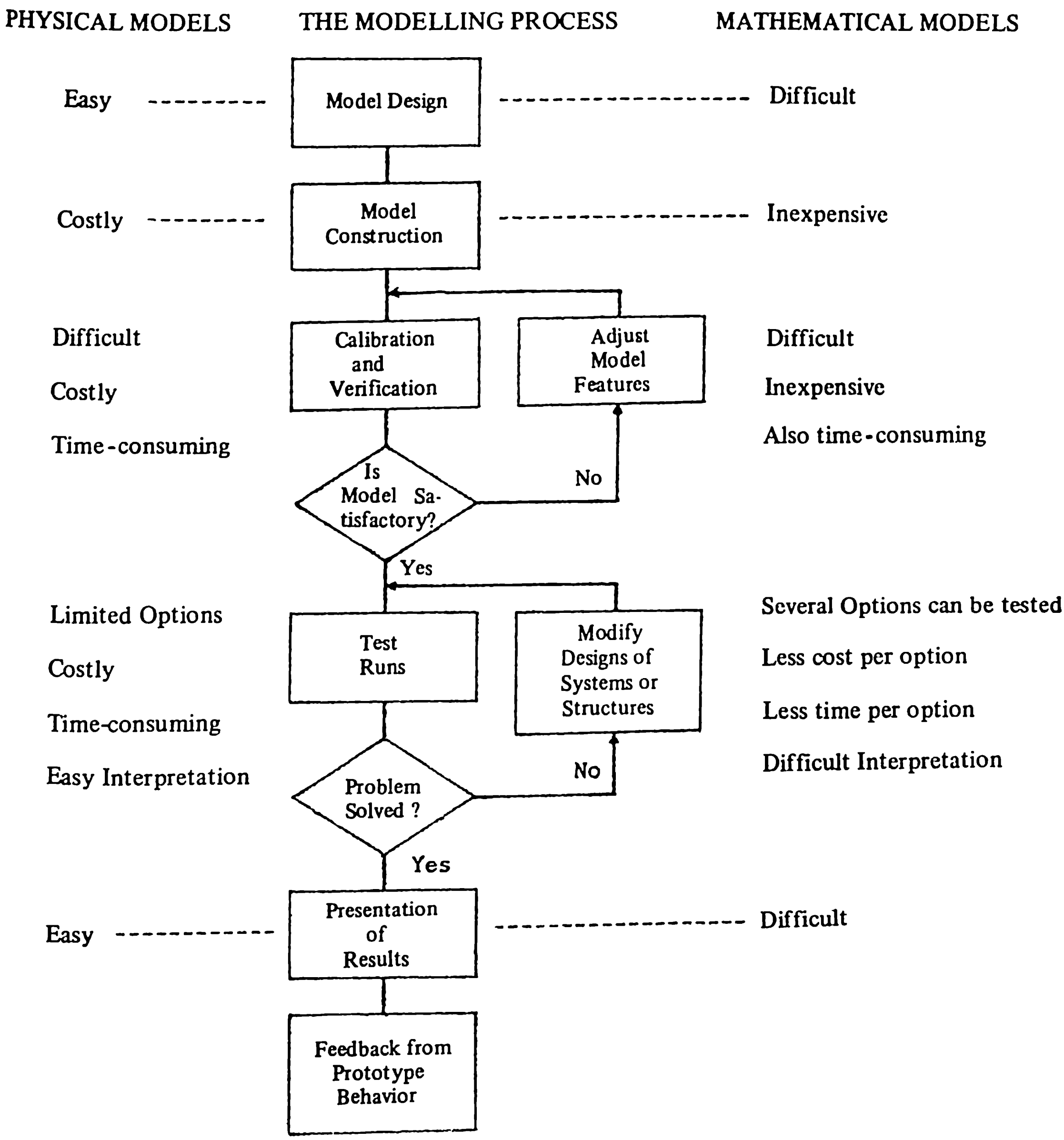


Figure 4.1

where computation has reached the row where time is at t_j. The methods are based on the discrete approximation of the partial derivatives of any function ($f(x, t)$ as:

$$\frac{\partial f}{\partial x} = \frac{f^j_{i+1} - f^j_i}{\Delta x} \quad \frac{f^j_i - f^j_{i-1}}{\Delta x} \tag{3.8}$$

or

$$\frac{\partial f}{\partial x} = \frac{f^j_{i+1} - f^j_{i-1}}{2\Delta x} \tag{3.9}$$

and

$$\frac{\partial f}{\partial t} = \frac{f^{j+1}_i - f^j_i}{\Delta t} = \frac{f^j_i - f^{j-1}_i}{\Delta t} \tag{3.10}$$

or $$\frac{\partial f}{\partial t} = \frac{f_i^{j+1} - f_i^{j-1}}{2\Delta t} \qquad (3.11)$$

where f_i^j is the value of the function, say depth or velocity at point x_i and time t_j.

These equations are derived from truncation of the Taylor series resulting in a first order approximation (Equations 3.8 and 3.10) or second order (Equations 3.9 and 3.11).

In the explicit method, the equations are arranged so that unknown values are expressed explicitly in terms of known values and are computed point-by-point, in contrast with the implicit method where unknown values are implicitly expressed and simultaneous equations are solved for a group of points, usually all points in a row at time t_j. LIGGETT and CUNGE (1975) and CUNGE, et.al. (1980) are excellent references for the derivation of the difference equations and their application. BALLOFFET and SAHAGUN (1976) describe an explicit finite difference model they used for flood studies in the Bicol river system. The model used by SOGREAH (1973) for the Pasig-Marikina-Napindan river system was an implicit one.

Physical vs. Mathematical Models

Comparison of Procedures

Figure 4.1 shows the procedure involved in the execution of a model investigation project, presented in a manner that all activities in either physical or mathematical modelling are grouped into analogous steps and then compared.

Designing a physical model starts with determining the appropriate scales. Besides this, numerous parts of the model and its appurtenant structures have to be conceptualized and designed. Figure 4.2 shows schematically the typical parts of a model. Not shown are the various measuring instruments and their mountings, each of which must be properly selected and located. Although this may seem to be a tedious task, an experienced modeller should not find much difficulty in adopting his methods from one model to another.

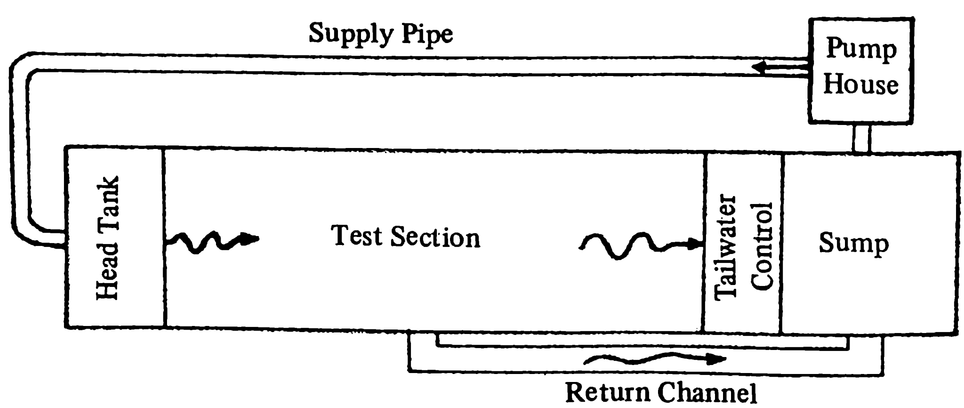


Figure 4.2

Designing a numerical model entails the selection of the proper forms of the governing equations with foresight on possible methods of solution, and hindsight on mastery of the prototype river system. Governing equations for special boundary conditions shall also be determined at this stage. Next, selection of the method of solution appropriate to the requirements of the problem to be solved, be it flood control, river regulation, or otherwise. Throughout the design stage, a targeted host machine or family of computers should be borne in mind.

After a thorough design, the construction of a physical model is quite straight forward but expensive. Technicians and craftsmen involved view it as the fabrication of a precision instrument, which a model should really be. On the other hand, the construction side of mathematical modelling is more aptly termed prototype schematization. Topographic, hydrographic, and hydrologic information are translated to discrete numbers and converted to machine readable form. Recent advances in computer peripheral equipment such as digital tablets, light pens and other forms of digitizers make this task even easier.

Calibration and verification of both physical and mathematical models are more of an educated trial and error process. The ease of achieving success depends on the accuracy of the roughness coefficients assumed in the design stage, as well as on the degree of tolerance acceptable for the problem. In physical models, calibration is a tedious task of adding or substracting model roughness elements until the stage-discharge relationship specified by prototype data is achieved. Interactive processing in time-sharing or single-user computer installations facilitates the task of calibrating a mathematical model. Calibrated coefficients can sometimes lack physical significance, as in the so-called "black box" models. This may be caused not only by poor design or schematization, but also by deficient or inaccurate prototype data. Nevertheless, black-box models can oftentimes be satisfactory.

Actual test runs of a physical model are expensive in terms of manpower and energy costs. Runs using 50 to 100 kilowatt pumps for four hours each session are not uncommon. For this reason, a well planned test program at the design stage which determines the number of schemes to be tested is called for. Test results are easy to interpret as the investigator sees and feels the water, and a semi-qualitative approach is possible. On the other hand, numerical modelling is a purely quantitative exercise. After getting the "feel" of the numbers, the number of schemes for problem solution that can be tested in the model is limited only by the investigator's time. LIGGETT, et. al. (1978) applied graphical techniques to a lake water quality problem. It looks promising that computer graphics can be applied to reduce the river modeller's effort at number crunching.

Various audio-visual aids are available to document results of physical model tests. Aside from the usual report with accompanying figures and photographs, color slides and video tape may be used. All these are familiar to clients, decision makers, and other interested parties. A bunch of numbers on computer printouts are not similarly palatable, and the mathematical model investigator has to extend his imagination and spend time converting those numbers to comfortable form. Again, computer graphics, whenever equipment availability permits, should be used.

Feedback is included in this discussion merely to emphasize that the best way to improve modelling techniques is to compare model performance with that of the prototype. The model investigator should welcome criticisms on discrepancies observed by others, or solicit prototype performance data, and make site visits occasionally, preferably during important events such as flood or drought.

Comparison of Applicability

Given a problem that requires the use of a river model, the choice as to what type of model to use is governed by: a) parameters to be extracted from the model, or what exactly it is we want to study in the model; b) accuracy and precision required for such parameters; and, c) accuracy and precision of available basic data from the prototype. It is assumed here that facilities are available and affordable. The objective of the selection process is to achieve the required results at the least cost.

A physical model is almost always preferred over mathematical models because of the confidence elicited by the observers' being able to see and feel the water. The larger the size of the model, the more it consumes materials, time, and energy during construction and test runs. The area of the prototype to be represented in the model is fixed before-hand by problem definition. There also exists a minimum depth which has to be maintained in the model, dictated by: a) instrument precision; b) precision of model construction; and, c) minimum Reynolds Number to assure fully turbulent flows throughout the model. The most expensive option, therefore, is the use of an undistorted model. This type should be used only where accurate representation of three-dimensional flows are important, e.g., in detailed bifurcation studies for natural branching or man-made diversions. Problems where measurement of velocities play a central role also require this type of models.

A physical model, however, is inconvenient in applications where unsteady flow has to be simulated, e.g., in the study of combined effects of flood waves and tidal fluctuations. The current practice in simulating the passage of a hydrograph is to discretize it into segments of constant discharge through manageable time periods, say 15 minutes in the model. This is a very rough and expensive approximation and we look to mathematical models to provide us with greater precision and economy. However, state-of-the-art mathematical modelling is still unable to handle three-dimensional river flows.

When a one-or-two-dimensional model suffices for the given application, we either use a distorted physical model or a mathematical model. Again, the physical model is constrained to steady flow situations, while the mathematical model is deficient in terms of flow visualization. Furthermore, two-dimensional mathematical modelling for rivers still has to be proven practical. CHANDRASHEKAR, et, al. (1975) and CODELL (1975) are examples of attempts in its application.

In the gray area where both mathematical and physical models are applicable, the decision is based not only on economics but also on required accuracy and precision, identified during problem definition. It should be remembered, however, that any model cannot be more precise or accurate than the data from which it is based. For physical models, the practical limits of fabrication should be considered. For example, masonry work cannot be more precise than ± 0.002 meters. Some instruments may be extremely precise but mounting and calibrating them introduces other types of errors.

For mathematical models, accuracy is governed by the sophistication of the model, and precision by the fineness of the grid representing the physical system. The simpler the model is, the less resources will be spent in developing and using the model. It would, therefore, be prudent to choose a level of sophistication corresponding only to the requirements of the problem. The selected model is then compared to its counterpart in the preceding paragraph.

Concluding Remarks

After some decision parameters for the selection of the model have been enumerated, it can be seen that problem definition, more than anything else, decides the level of expenditure for the project. For example, if we review a given problem and decide that localized concentration of velocities, evaluated through empirical relationships, would suffice for our design of protection works, then we can opt for a one-or two-dimensional model and greatly reduce the final cost.

A pragmatic approach to selecting methods may be taken in the following manner. For modelling of large areas or lengthy river systems, the mathematical model holds the advantage. Critical areas are identified and the need for a three-dimensional model is decided for each locale. The combined use of multiple models should, in most cases, be a viable alternative. The results of one can be the input to another for a truly interacting system.

The factors influencing decisions are also changing. AMOROCHO, et.al. (1980) describe a physical model they built that was automatically controlled by a computer to permit the simulation of unsteady flow, including tidal action. If applicable here, then this technique eliminates the current disadvantage of physical models with regard to unsteady flow representation. If we are allowed our wildest dreams, the paper also underscores the possibility of automatic inter-action between two or more models of whatever type.

Two-dimensional mathematical modelling of lakes, estuaries, and coastal areas are common and three-dimensional cases have been tried. Modelling of rivers beyond one dimension has been previously constrained by computer capacity and cost. With the advent of very high-speed mainframe computers and multi-megabyte microcomputers, with better ones still to come, this drawback is steadily being erased. We should see the wide-spread use of two-dimensional mathematical models in the near future with three-dimensional models close at its heels.

Better hardware are sure to come as market for them are assured by the general usage of digital computers. As in other disciplines utilizing these machines, software development is lagging way behind. The author hopes that this paper generates discussion, sharing of experiences, and more activity on mathematical modelling in the local setting.

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