"the same limitations of the flood frequency methods are inherited by the statistical flood envelopes."

Philippine Flood Envelope Curves Derived From Probability Distributions of Record Maxima*

by

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Abstract

The statistical properties of the largest or maximum observation in a series of annual extreme events are described and utilized in order to develop a statistical method for deriving flood envelope curves. The statistical flood envelope method allows for the upward trend of the envelopes due to increasing record length, for regionalization and maximization of statistical parameters, and for transposition of exceptionally high observation. The study aims to reconcile and unify the flood envelope method with the flood frequency method; as a result, both methods are governed by the same limitations. Finally, statistical flood envelope curves for the Philippines are derived based upon the concepts and approaches developed.

Aims

The aims of this paper are (a) to describe the statistical properties of the largest or maximum observation in a record of annual extreme events for a given length of record and an underlying probability distribution of annual extremes; (b) to apply these statistical concepts to the problem of constructing flood envelope curves in the attempt to reconcile and unify the two flood estimation methods which, heretofore, are recognized as distinct and separate, namely, flood envelopment and flood frequency methods; and (c) to demonstrate the feasibility of deriving flood envelope curves for the Philippines or for some of its regions, based upon the statistical properties of the record maxima of annual extreme floods which, in this exploratory study, are conveniently assigned an initial underlying Extreme-Value type I or Gumbel distribution.

Flood Estimation Methods

The design of dams, spillways, and other flood-control structures and developments (levees, floodways, detention basins, etc.) depends critically on the magnitude of the design flood that is both anticipated and estimated. Where failure of the structure would cause loss of human life or extensive economic losses, the

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probable maximum flood (PMF) is adopted (Snyder, 1964; Koelzer and Bitoun, 1964; Riedel, 1977). Where failure of the structure would result in minor economic losses and inconvenience, a design flood of lower magnitude and with an assigned frequency (recurrence interval or return period) is selected on the basis primarily of economic costs/risks and benefits (Ogrosky, 1964).

Four (4) principal methods of flood estimation have been used and recognized (Riedel, 1977; Johnson, et. al., 1982; Brown, 1982):

(a) Hydrometeorological Methods

These are methods that adopt a design storm, apply losses to produce rainfall excess, and transform rainfall excess into a flood hydrograph (by application of the unit hydrograph and/or hydrologic/hydraulic routing procedures). Modern sophisticated techniques under this method include the digital-computer catchment models.

(b) Empirical Formulas

These are methods that calculate a design flood peak discharge from an empirical formula relating peak discharge to measurable topographic or topographic/climatic characteristics of the catchment. This classification includes the Rational Formula.

(c) Envelope Curves

The design flood peak discharge is selected from the envelope curve of a plot of peak discharge against catchment area for recorded floods in a defined region.

(d) Flood Frequency Methods

These are methods that are based upon statistical analysis of available flood records to produce a design flood peak discharge of specified return period.

The two most dominant methods of flood estimation are the hydrometeorological and flood frequency methods. The hydrometeorological method, in the case of PMF determination, suffers however from the criticism that it is difficult to assign and justify an upper physical limit to rainfall, the probable maximum precipitation (PMP). (Gupta, 1972; WMO, 1973; Fahlbusch, 1979; Brown, 1982). The flood frequency methods, on the other hand, are subject to statistical unreliability in flood estimates associated with high return periods which exceed the lengths of record, or even with low return periods where records are short (Myers, 1967; Riedel, 1982).

For the estimation of design floods with assigned frequencies, maximum use of hydrologic information and higher reliability of estimates may be achieved by applying both hydrometeorological and flood frequency methods. In the hydrometeorological method, design storms with assigned frequencies are derived after statistical analysis of available short-duration rainfall data, space redistribution (isohyetal mapping, transposition, area reduction), and time redistribution (critical temporal pattern). The rainfall excesses obtained after subtraction of losses are converted to flood hydrographs using empirical or synthetic unit hydrographs and/or hydrologic/hydraulic routing procedures. In the flood frequency methods, available annual or partial-duration flood discharge data are statistically analyzed; the statistical parameters are regionalized, if necessary, for greater reliability; and peak flood discharges with assigned return periods are obtained from the fitted probability distributions. This dual process takes advantage of the utilization of both the usually longer rainfall records and the actually measured though shorter flood records. A pitfall to guard against is the tacit acceptance of the equivalence between rainfall return period and flood-discharge return period, without judicious consideration of the variable watershed conditions that transform rains to floods. (Davis,et. al. 1974)

The empirical formulas and the flood envelope curves, though still in use, have now largely been abandoned for important structures. The envelope curve method, together with the empirical formulas, has the following disadvantages (Brown, 1982):

(a) "Very often the records available are short and thus do not include the really exceptional floods that occur infrequently."

(b) "No allowance is made for variation in catchment shape, topography, soil cover, vegetation, etc.–all factors which influence the magnitude of flood discharges."

One modest aim of this paper is to reconcile the envelope curve method with the flood frequency method. Whatever limitations and criticisms apply to the latter, would therefore also apply to the former once the two methods are reconciled. In any case, it is also another goal of the statistical reformulation of the envelope curve method to provide means by which to overcome the disadvantages stated above.

Flood Envelope Curves

An early form of the envelope curve was the Modified Myer's Formula, developed by Jarvis (1926) for sections of the United States:

$$Q = C\sqrt{A} \qquad (1a)$$

or

$$q = C/\sqrt{A} \qquad (1b)$$

where Q = the flood discharge in cfs

q = Q/A = the unit discharge in cfs per sq. mi.

A = the drainage area in sq. mi.

C = 100 (minimum) to 10,000 (maximum)

An envelope curve (q versus A) based on eqn (1b) when plotted to logarithmic scale exhibits a constant slope of -0.5.

In consideration of the property that envelope curves (q versus A) "should be flatter that the average for the smaller areas and steeper for the larger areas when plotted to logarithmic scale," Creager, et.al. (1946) proposed the now well-known Creager's Formula for coverage of both U.S. and foreign river flood data:

$$Q = 46\,C\,A^{(0.894A^{-0.048})} \tag{2a}$$

or $$q = 46\,C\,A^{(0.894A^{-0.048}-1)} \tag{2b}$$

with the same units as in eqns (la) and (lb) and where C, the Creager's constant, varies from 30 to 200.

The WMO (1967) developed envelope curves for maximum floods in the monsoon areas of the ECAFE (now ESCAP) Region. Both straight envelopes based on the Modified Myer's Formula, and curved envelopes similar to Creager's Formula, were derived, with the latter providing a better fit to flood data. The regional envelope curves obtained are as follows:

(a) Sub-region A (Burma, Ceylon, Pakistan, India)

Straight:

$$Q = 206\sqrt{A} \tag{3a}$$

or $$q = 206/\sqrt{A}$$

Curved:

$$Q = 5.37\ A^{1.44A^{-0.05}} \tag{3b}$$

$$q = 5.37A^{(1.44A^{-0.05}-1)}$$

(b) Sub-region B (Cambodia, Mainland China, Indonesia, Laos, Malaysia, Thailand)

Straight:

$$Q = 97\sqrt{A} \tag{3c}$$

or $$q = 97/\sqrt{A}$$

Curved:

$$Q = 0.35A^{1.8A^{-0.05}} \tag{3d}$$

or $$q = 0.35A^{(1.8A^{-0.05}-1)}$$

(c) Sub-region C (Taiwan, Japan, Korea, Philippines, Vietnam – typhoon subregion)

Straight:

$$Q = 364\sqrt{A} \qquad (3e)$$

or $q = 364/\sqrt{A}$

Curved:

$$Q = 87A^{A^{-0.05}} \qquad (3f)$$

or $q = 87A^{(A^{-0.05}-1)}$ for $A < 24{,}000$ sq. km.

where Q = the flood discharge in cms

q = Q/A = the unit discharge in cms per sq. km.

A = the drainage area in sq. km.

The curved envelopes eqns (3b), (3d), and (3f) are more convenient than Creager's Formula, which, when converted to metric units, has the form

$$Q = 1.3C\ (0.385A)^{0.935A^{-0.048}} \qquad (2c)$$

Both Creager's Formula and the monsoon region curved envelopes suffer from the mathematical drawback that the q versus A curve has a positive slope for very small areas less than the critical area where the slope is zero or flat.

The curved envelope, eqn (3f), that is applicable to the typhoon subregion, is controlled by the peak flood data of Taiwan, Japan, and Korea. The Philippine data points utilized by WMO undercut the envelope curve by about 35 percent.

More recently, Crippen (1982) introduced a new convenient form of the envelope curve equation that he applied to 17 regions in the conterminous United States:

$$Q = K_1\ A^{K2}\ Z^{K3} \qquad (4)$$

where $Z = A^{0.5} + 5$, with Q in cfs and A in sq. mi., and K_1, K_2, K_3 are regional constants.

Creager(1939) observed the upward trend of the successive flood envelope curves for the United States, obtained as of years 1890, 1913, 1921, 1934 and 1939, due to "an increasing number of gaging stations and an increasing period of record. Therefore, the occurrence of greater floods as time passes must be according to the laws of probability and of chance." The author suggested further research on "the probability of a storm, of greater magnitude than any that had occurred in the past, centering directly over a given drainage area." After 33 years since 1939, Gupta (1972) addressed himself to this problem by "estimating the probability of occurrence of floods in a unit time interval, based on the random characteristics of storms," here defined as three-dimensional random vectors composed of two storm center coordinates and one storm orientation angle.

The present study shall explore the use of the flood frequency method to explain the upward trend of envelope curves due to increasing length of record. Flood frequency analysis shall be done despite recommendation to the contrary made by Creager in 1939, when the inadequacy and pitfalls in the frequency methods were then just being realized.

Statistics of Extremes

Gumbel (1958) devoted an entire treatise, "Statistics of Extremes," exclusively to the theory of extreme events. He introduced the concept of the characteristic largest value $\tilde{x}_n$, defined as

$$F(\tilde{x}_n) = 1 - 1/n \tag{5}$$

where F is the cumulative distribution function (CDF) of the random variable X. In eqn (5), n coincides with our definition of the return period, and $\tilde{x}_n$ is the n-year event with exceedance probability equal to 1/n.

Assuming independence among n observations derived from a common initial underlying distribution F, the probability F_n that the largest among the n values is less than or equal to x_n is

$$F_n(x_n) = F^n(x_n) \tag{6}$$

Eqn (6) defines the cumulative distribution function F_n of the random variable X_n, an order statistic, defined to be the record maximum among n observations randomly and independently derived from the initial distribution F. Following Gumbel,the moments of the extremes X_n can be defined.

Mean Largest Value or expected record maximum:

$$\bar{x}_n = E(X_n) = \int_o^1 x \ d\ [F^n(x)] \tag{7}$$

$$= \int_o^1 nxF^{n-1} \ dF$$

Variance of X_n:

$$S_n^2 = \text{Var}(X_n) = \int_o^1 (x-\bar{x}_n)^2 \ d[F^n(x)] \tag{8}$$

Eqns (7) and (8) give, for n=1, the mean $\bar{x}$ and variance S^2 associated with the initial distribution F.

Gumbel presented an interesting constrained variational formulation that seeks to maximize $\bar{x}_n$ subject to fixed mean and variance of the initial distribution:

$$\text{maximize} \int_o^1 x \; dF^n - \lambda_1 \; x^2 \; dF - \lambda_2 \; x \; dF$$

The variational solution is the maximum (or upper bound) of all mean largest values:

$$\bar{x}_n = \bar{x} + \frac{n-1}{\sqrt{2n-1}} S \tag{9a}$$

In standardized form, the upper bound is

$$\bar{K}_n = \frac{\bar{x}_n - \bar{x}}{S} = \frac{n-1}{\sqrt{2n-1}} \tag{9b}$$

Gumbel computed the moments for the extremes of the exponential distribution F(x) = 1-exp(-x):

$$\bar{x}_n = \sum_{v=1}^{n} 1/v \tag{10}$$

$$S_n^2 = \sum_{v=1}^{n} 1/v^2 \tag{11}$$

$\bar{x}_n$ in eqn (10) is asymptotic to $\ln n + \gamma$ for large n, where γ = Euler's constant = 0.5772 . . ., while S_n^2 in eqn (11) is asymptotic to $\pi^2/6$ for large n. The properties of normal, gamma, and log-normal extremes are likewise presented by Gumbel.

Extreme-value Distribution

The latter part of Gumbel's book developed the theory of asymptotic distributions in which he derived the three (3) extreme value distributions. These asymptotic distributions follow the stability postulate, stating that "the distribution of the largest value in Nn observations will tend to the same distribution of the largest value in samples of size n, provided that such an asymptote exists."

From an initial distribution that is of the "exponential type," the first asymptotic or the extreme-value type I (EV-I) distribution was derived:

$$F(y) = \exp(-\exp(-y)) \tag{12}$$

where y = the reduced variate

$\bar{y}$ = mean of y = γ, Euler's constant = 0.5772. . .

S_y^2 = variance of y = $\pi^2/6$

For purposes of this study, the stability postulate is exploited and EV-I (eqn (12)) is selected as the initial underlying distribution of a reduced variate y, which is expressable in terms of the annual extreme flood discharge Q, according to

$$y = a(Q-b) \tag{13}$$

where a and b are parameters related to the mean $\bar{Q}$ and variance S^2 of the annual extreme flood discharge Q.

The cumulative distribution function and moments of Q are thus

$$F(Q) = \exp[-\exp(-a(Q-b))] \tag{14}$$

$$\bar{Q} = b + \gamma/a \tag{15}$$

$$S^2 = \pi^2/6a^2 \tag{16}$$

Applying eqn (6) to the initial distribution, eqn (14), gives the distribution function of the record maximum Q_n:

$$F_n(Q_n) = \exp[-n \exp(-a(Q_n - b))]$$

$$F_n(Q_n) = \exp[-\exp(-a(Q_n - b) + \ln n)] \tag{17}$$

From eqn (17), it is clear, by virtue of the stability postulate, that the record maximum, Q_n also follows an EV-I distribution in which the new reduced variate y is related to the record maximum Q_n according to

$$y = a(Q_n - b) - \ln n$$

$$\text{or} \quad Q_n = b + (y + \ln n)/a \tag{18}$$

Without having to apply eqns (7) and (8), from eqn (18) alone, the moments of Q_n are easily obtained by taking expectations:

Mean Largest Value or expected record maximum $\bar{Q}_n$:

$$\begin{aligned} \bar{Q}_n &= b + (\bar{y} + \ln n)/a \\ &= b + (\gamma + \ln n)/a \\ &= \bar{Q} + \ln n/a \end{aligned} \tag{19}$$

Variance of Q_n:

$$S_n^2 = \mathrm{Var}(y)/a^2 = \pi^2/6a^2$$

(same as eqn (16))

Thus, eqn (14) becomes eqn (17) after translation of the mean without change of variance.

The mean largest value $\bar{Q}_n$, standardized with respect to the mean and variance of the initial distribution, becomes

$$\bar{K}_n = \frac{\bar{Q}_n - \bar{Q}}{S} = \frac{\sqrt{6}}{\pi} \ln n \qquad (20)$$

$$= 0.7797 \ln n = 1.7953 \log_{10} n$$

Eqn (20) exhibits the upward logarithmic trend of the mean largest value due to increasing record length n. The value given by eqn (20) remains below the upper bound set by eqn (9b).

The return period T_n of the mean largest value $\bar{Q}_n$ is obtained by combining eqns (14) and (19) in the expression

$$F(\bar{Q}_n) = 1 - 1/T_n$$

Hence, $\exp\ [-\exp(-\gamma - \ln n)\ ] = 1 - 1/T_n$

$$T_n = \frac{1}{1 - [\exp(-\exp(-\gamma))\ ]^{1/n}}$$

or $$T_n = \frac{1}{1 - 0.57037^{1/n}} \qquad (21)$$

As n increases, T_n asymptotically decreases to $\exp(\gamma)n = 1.7810\ n$. In short, the return period of the mean n-year record maximum is at least equal to 1.7810 n, assuming EV-I distribution.

Plotting Position

The familiar Weibull plotting position

$$F(j) = 1 - \frac{j}{n+1} \qquad (22)$$

that assigns the empirical cumulative probability F(j) to the j-th highest observation in a series of n independednt observations can, in fact, be derived from the theory of extremes by means of a distribution-free argument due to Thomas (1948). F(j) is interpreted as the expected value of the cumulative probability F(Q) itself.

Recognizing that the probability dP of F is

dP = Probability that (n-j) floods $\leq$ Q

<u>and</u> (j-l) floods $\geq$ Q

<u>and</u> one flood = Q ± 1/2 dQ

then $dP = \frac{n!}{(n-j)!\ (j-1)!} F^{n-j}\ (1-F)^{j-1}\ dF$ (23)

Therefore

$$F(j) = E(F) = \int_0^1 F\ dP = 1 - \frac{j}{n+1} \quad (24)$$

Similarly, the variance of F is obtained:

$$Var(F) = \int_0^1 F^2\ dP - E^2(F)$$

$$= \frac{(n-j+1)j}{(n+1)^2(n+2)} \quad (25)$$

For the record maximum (j=1), the plotting position simplifies to

$$E(F) = \int_0^1 F\ dF^n = \frac{n}{n+1} = 1 - \frac{1}{n+1} \quad (26)$$

and the standard deviation of F equals $\frac{1}{n+1}\sqrt{n/(n+2)}$.

Eqn (26) gives a distribution-free parameter in contrast to Eqns (7) and (8) which yield distribution-dependent functions of n. No confusion should arise between the return period T_n of the expected record maximum [eqn (21)] and the "expected" return period (n + 1) of the actual record maximum [eqn. (26)].

The Statistical Flood Envelope

The probabilistic model that emerges to describe the random variable called the n-year record maximum Q_n is

$$Q_n = \bar{Q} + S\ K_n \quad (27)$$

which is strongly reminiscent of V.T. Chow's (1951) flood frequency formula and of Hershfields's (1961) statistical estimation of probable maximum precipitation (PMP).

The random deviate K_n is formed as the sum of the mean $\bar{K}_n$ and the random number of unit standard deviations M from $\bar{K}_n$. Substituting in eqn (27) gives

$$Q_n = \bar{Q} + S(M + \bar{K}_n) = \bar{Q}_n + S\ M \quad (28)$$

Letting $CV = S/\bar{Q}$ = the coefficient of variation of Q, and assuming EV-I distribution, so that $\bar{K}_n = 1.7953 \log_{10} n$, give

$$Q_n = \bar{Q}\ [1 + CV(M + 1.7953\ \log_{10} n\ )] \quad (29)$$

The superficial resemblance of eqn (29), for M=O, to Fuller's (1914) formula:

$$Q_T = \bar{Q}\,[1 + 0.8 \log_{10} T]$$

where Q_T is the T-year flood, is purely coincidental and does not imply that one equation supports the validity of the other.

If the initial underlying distribution of the annual extreme flood is other than EV-I, eqn (29) is replaced by the general formula

$$Q_n = \bar{Q}\,[1 + CV(M + \bar{K}_n)] \tag{30}$$

where $\bar{Q}$ and CV are statistical parameters derived from single-station or regional flood frequency analysis; $\bar{K}_n$ is, in principle, a mathematically defined function of record length n and may be dependent on the higher moments of the underlying distributions; and M retains the character of the random number of unit standard deviations.

To apply the flood frequency formula, eqn (30), to the problem of regional flood envelopment, the following schemes are proposed:

(a) Regionalization and envelopment of the statistical parameters $\bar{Q}$, CV, and the higher moments, if necessary. The envelopment procedure is not susceptible theoretically to the upward trend due to increasing record length, since the parameters are stable under the stationarity assumption. Moreover, the standard errors of parameter estimates favorably decrease with longer records.

(b) Assignment of a maximized value of M based upon the highest experienced deviation from the expected record maximum. This is similar to Hershfield's (1961) approach, except that, in the latter case, the deviation is measured from a simple mean computed by excluding the record maximum from the entire sample.

In scheme (a), the spatial density and distribution of the gaging network over the spectrum of drainage areas and different topographic, climatic, soil/rock, and vegetative factors or conditions, may significantly influence the form of the derived envelope equations for the statistical parameters. For regionalized parameters developed by regression with the causative factors (Kite,1977; Yin, 1979) the envelopment procedure may be interpreted as a maximization over all possible joint occurrences of the said factors, implicitly defining a potential set of "maximized" catchment areas. In the light of observed changing watershed conditions, the upward (or downward) trend of the parameter envelopes is unavoidable since the basic assumption of stationarity is violated.

In scheme (b), which may be applied with or without scheme (a), the assignment of a maximized M, based upon the highest experienced deviation from the expected record maximum, is equivalent to the transposition of the highest M to other stations within the same region defined for the envelope curve. This process may amount to extrapolation to large flood magnitudes to which the assignment of return periods becomes highly unreliable.

Combinations of schemes (a) and (b) may produce estimates that can equal or even exceed PMF estimates, to which the assignment of return periods becomes more highly unreliable.

The same limitations of the flood frequency methods are thus inherited by the statistical flood envelopes.

Application to Philippine Flood Data

The concepts and approaches developed in the previous sections are now applied to the annual flood data of the Philippines. The NWRC (1980) publication "Philippine Water Resources Summary Data, vol. 1," provides the series of annual peak flood discharges in the streamflow gaging stations of the country, inclusive of the period 1946-1970. For stations with continuous recorders, reported annual peaks were instantaneous peaks. For other stations, the reported peaks were based on maximum daily or hourly staff gage readings. In certain cases, area-slope methods were used to estimate high flood discharges. The same publication gives a summary of statistical parameters including the means, standard deviations, and record maxima of annual flood discharges. The means and standard deviations were computed by the method of moments:

$$\bar{Q} = \frac{1}{N} \sum_{i=1}^{N} Q(i) \qquad (31)$$

$$S = \left[\frac{1}{N-1} \sum_{i=1}^{N} (Q(i) - \bar{Q})^2 \right]^{1/2} \qquad (32)$$

The record maxima, here denoted by Q_m, divided by drainage area A, are plotted against drainage area in a series of charts (Figures 1-6), according to regional groupings for the Luzon stations, and in aggregates of regions for the Visayas and Mindanao stations. The maximum controlling points in each chart are connected by broken dashed lines in order to give a rough impression of the empirical envelope shapes. The controlling points are identified by the river name, region number/station number (Rx – xx), and length of record N in years.

The first step of the analysis is the examination of the behavior of the standardized deviates, $K_n = \max_n K = (Q_m - \bar{Q})/S$ of the record maxima when plotted against the length of record for all stations. Figures 7-14 exhibit the plots of K_n against the length of record n. The solid curves on the charts, labelled as E ($\max_n K$), represent the standardized deviate of the expected record maximum, $\bar{K}_n = 1.7953 \log_{10} n$, assuming an EV-I distribution. The dashed curves represent $\bar{K}_n + M$ for values of M = -2, -1, +1, and +2. The points are grouped according to regions or aggregates of regions. The points are identified by station numbers.

There is a discernable upward trend of the random variable K_n against the length of record in all regions. The central tendency for the points to group around the solid curve is also apparent. For n higher than 10, the scatter of the points from the mean $\bar{K}_n$ exceeds $M = \pm 1$. The highest experienced M equals 2, as shown in Fig. 9, for the case of Peñaranda R, station 39, Region 3. For n much less than 10, the sampling variation of K_n is very much suppressed. For the extreme case n = 2, which is not present in the charts, $K_2 = 1/\sqrt{2} = 0.707$, theoretically, a constant.

The second step is the envelopment of the statistical parameters $\bar{Q}$ and CV = $S/\bar{Q}$. Figures 15-17 depict $\bar{Q}/A$ versus A. A suitable form for the upper and lower envelopes of $\bar{Q}/A$ is provided by the monsoon sub-region C curved envelope (eqn. (3f)). The envelopes obtained are

upper: $\bar{Q}/A = 20\ A^{(A^{-0.05}-1)}$ (Regions 1-6,8) (33a)

$= 10\ A^{(A^{-0.05}-1)}$ (Region 7) (33b)

$= 15\ A^{(A^{-0.5}-1)}$ (Regions 9-12) (33c)

lower: $\bar{Q}/A = A^{(A^{-0.05}-1)}$ (Regions 1-6,8) (34a)

$= 0.5\ A^{(A^{-0.05}-1)}$ (Region 7) (34b)

$= 0.75\ A^{(A^{-0.05}-1)}$ (Regions 9-12) (34c)

The upper envelope curves given above cover a wide range of areas, 1 sq. km. $<A<$ 10,000 sq.km., and allow for undercutting of some outliers which lie above the continuous band of points in the charts.

Figures 18-20 present CV versus A. The coefficients of variation behave very stably with respect to drainage area, the points being confined within one log cycle. Although there is a slight apparent downward trend with area, here exhibited by the envelope:

$$CV = 2\ A^{-0.1003} \qquad (35)$$

a constant envelope, given by

$$CV = 1.335 \qquad (36)$$

is selected in order to preserve the mathematical form of eqn (3f) in the final statistical envelope curve.

Finally, a series of flood envelope curves for the entire Philippines is derived, all shown in Figure 21.

Adopting the highest upper envelope for $\bar{Q}/A$ (eqn (33a)) and the constant enveloping CV = 1.335, the following "maximized" envelope curve, parameterized by record length N and number of unit deviates M, is obtained after substitution in eqn. (29):

$$Q_m/A = 20\ [\ 1 + 1.335\ (M + 1.7953\ \log_{10}N)\ ]\ A^{(A^{-0.05}-1)} \qquad (37a)$$

For M = 0, N = 10, eqn (37a) reduces to

$$Q_m/A = 67.9\ A^{(A^{-0.05}-1)} \qquad (37b)$$

Eqn. (37b), as shown in Figure 21, envelopes all regional controlling points except three higher points – Oco R., Laoag R., and Cagayan R.

For M = 0, N = 25 (corresponding to period 1946-1970), eqn (37a) becomes

$$Q_m/A = 87\ A^{(A^{-0.05}-1)} \qquad (37c)$$

Eqn (37c) is identical to the Monsoon sub-region C envelope, eqn (3f). Thus the latter can be interpreted as the expected flood envelope curve for generalized potential "maximized" Philippine catchment areas for the period of record 1946-1970 or N = 25. This envelope curve exceeds all 1946-1970 peak records.

For M = 1, N = 25 or M = 0, N = 90, eqn (37a) simplifies to

$$Q_m/A = 113.7\ A^{(A^{-0.05}-1)} \qquad (37d)$$

This envelope curve is higher than the points corresponding to the PMF estimates for the existing major dams of the country (Figure 21), with the exception of the Magat Dam PMF.

An envelope curve associated with generalized but "unmaximized" catchment areas can be obtained by adopting middle values of $\bar{Q}/A$ and CV:

$$\bar{Q}/A = 10\ A^{(A^{-0.05}-1)} \qquad (38)$$

$$CV = 0.70 \qquad (39)$$

Using eqns. (38) and (39) together with N = 25 and the highest observed M = 2, in eqn (29), gives the flood envelope curve for the "unmaximized" case that cuts through the middle of the regional controlling points in Figure 21:

$$Q_m/A = 41.6\ A^{(A^{-0.05}-1)} \qquad (40)$$

Similar other envelope curves may be obtained for particular regions of the country by substitution of locally valid relations for $\bar{Q}$ and CV.

As a parting example, the record maximum floods obtained from the publication, "Surface Water Supply of the Philippine Islands, 1908-1922" (BPW 1923), are plotted as Q_m/A versus A in Figure 22. It can be seen that the "maximized" envelope curve (M = 0, N = 10) or eqn. (37b), obtained from the 1946-1970 records provides an acceptable envelope for the pre-war controlling points (with N less than 10), with the single exception of one outlier, Agno Chico R. (N = 5), whose M value is presumably in the order of two or more.

Conclusions and Recommendations

It can be observed that the statistical envelope curves were constructed not by the simple selection and connection of controlling points but rather by the application of systematic steps which we recapitulate here:

(a) Examination of highest observed M from K_n versus n charts (Figures 7-14).

(b) Envelopment of $\bar{Q}$ and CV (Figures 15-20).

This particular step may be extended to take into account all the relevant causative factors which jointly produce maximized $\bar{Q}$ and CV. This procedure may easily be imbedded in the parameter regionalization of most regional frequency analyses (Kite 1977).

(c) Substitution of highest observed M and record length N.

The dependence on N accounts for the temporal upward trend of envelope curves. Transposition of highest observed M and extrapolation of N beyond the length of record offer the advantage of predicting higher future flood discharges, but raise problems of uncertainty in the return periods to be assigned.

Steps (b) and (c) constitute the means by which to overcome the disadvantages cited by Brown (1982).

Further studies in the light of the results of this preliminary study should include:

(a) Extension of the method to other initial underlying distributions (log-normal, Pearson III, log-Pearson III) which are generally applicable to annual flood data.

(b) Refinement and extension of the method for the envelopment of statistical parameters, to take into account all relevant causative factors that affect floods.

(c) Application of the statistical method to the problem of envelopment of annual extreme 24-hr. or shorter-duration rainfall data, which have been treated to follow the EV-I distribution (PAGASA, 1981) and which have the advantage of longer records than flood data.

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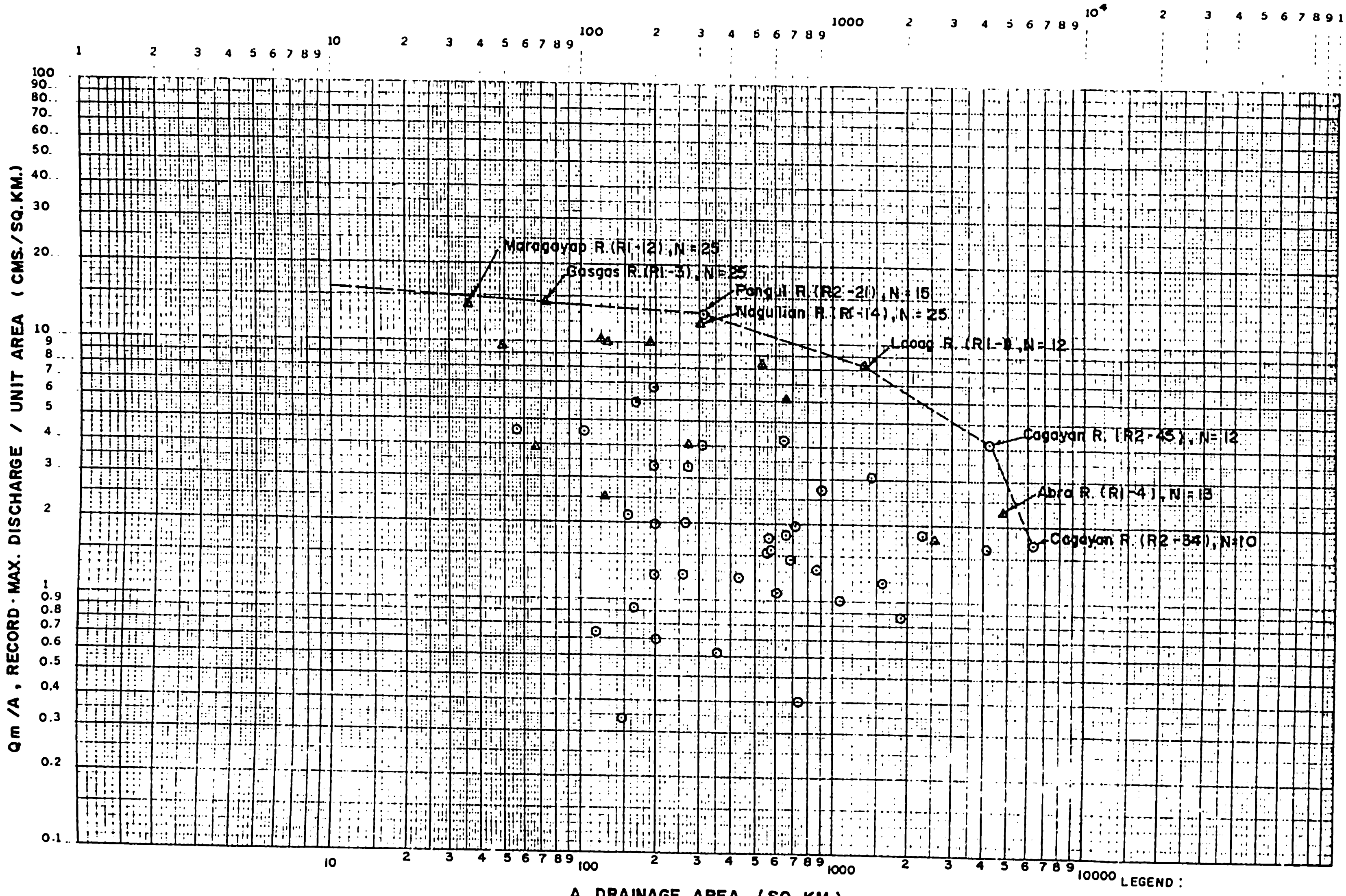


Figure 1

RECORD MAXIMUM FLOODS (REGIONS 1 & 2)
1946 - 1970

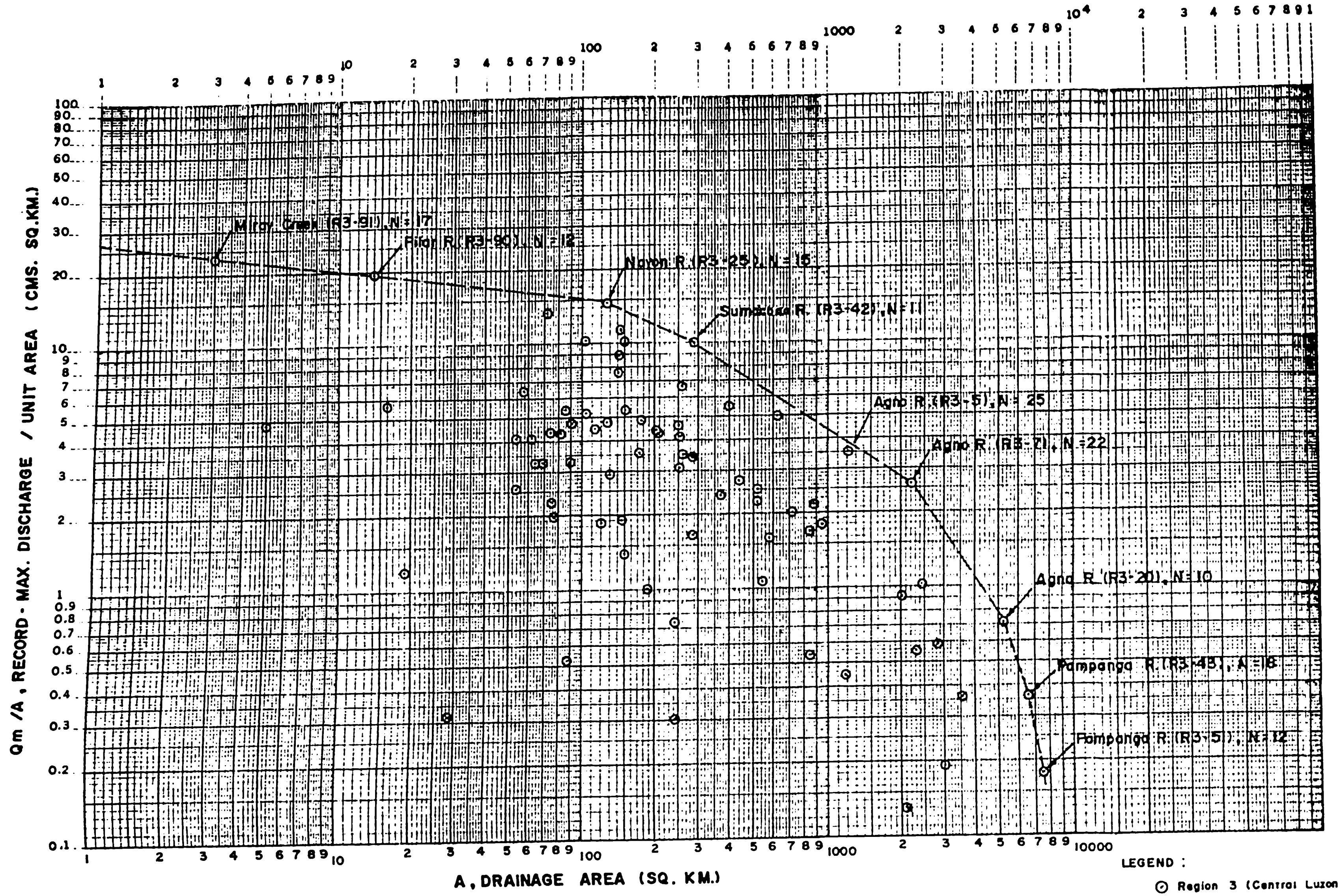


Figure 2

RECORD MAXIMUM FLOODS (REGION 3)
1946-1970

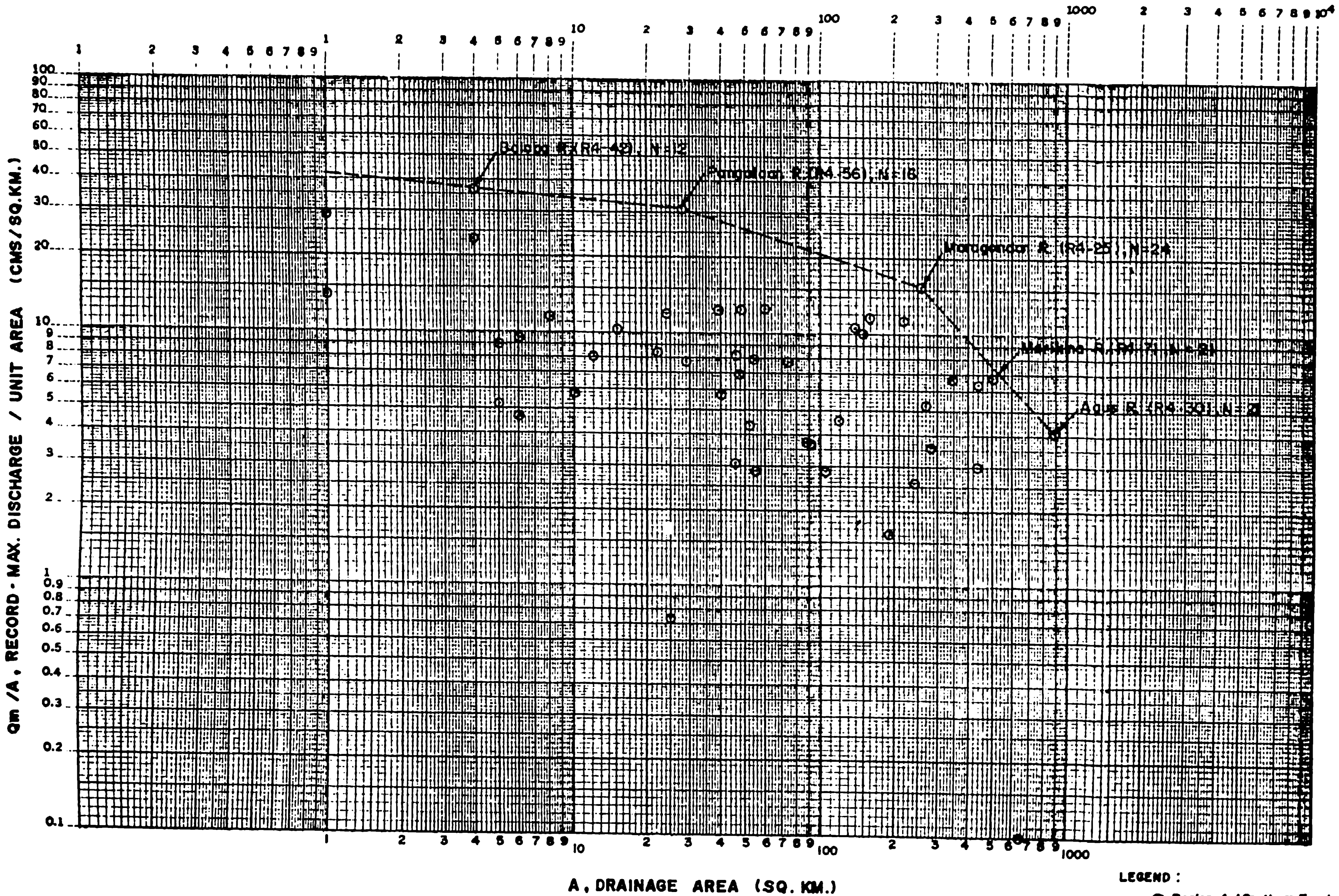


Figure 3

RECORD MAXIMUM FLOODS (REGION 4)
1946-1970

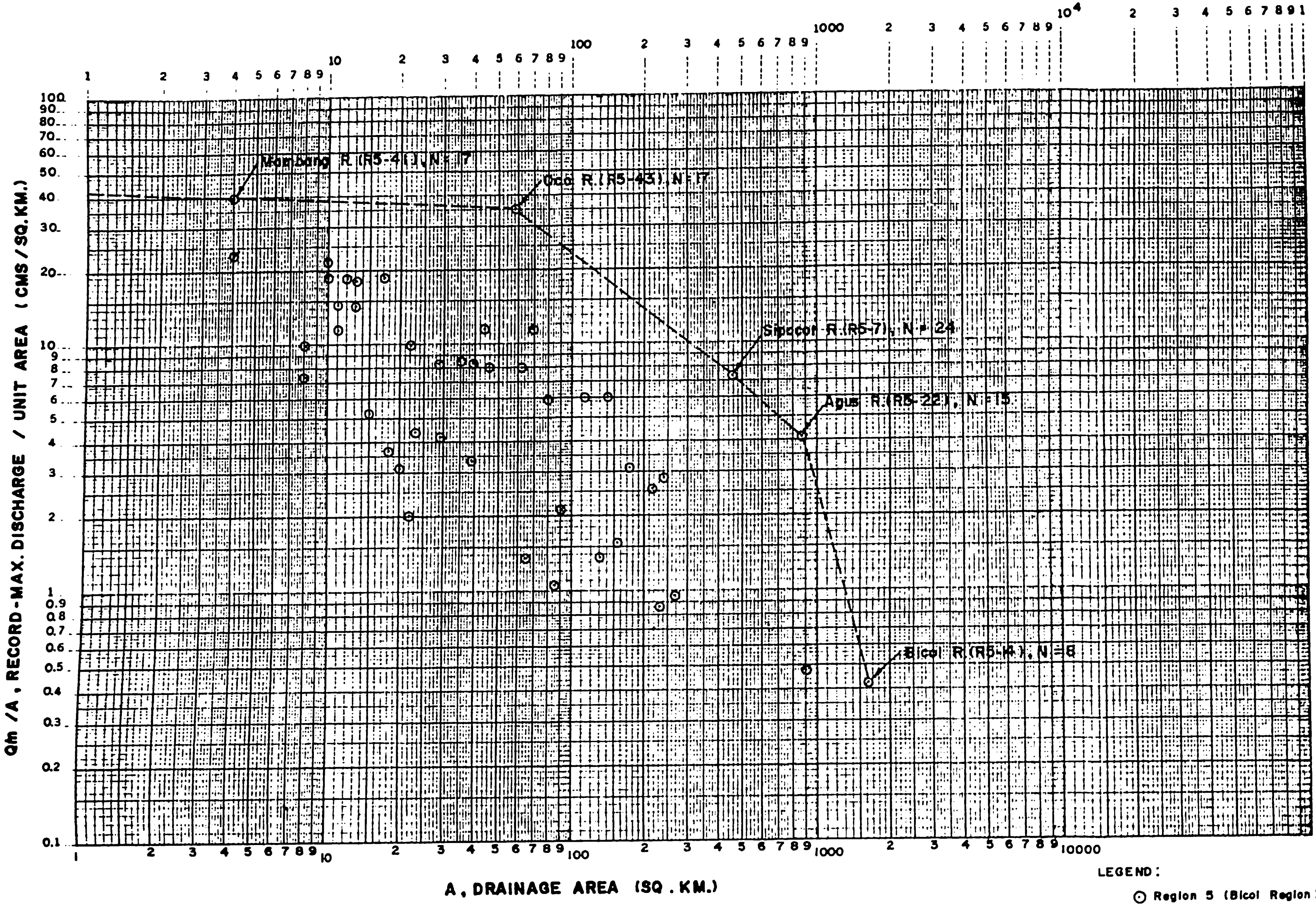


Figure 4 RECORD MAXIMUM FLOODS (REGION 5) 1946-1970

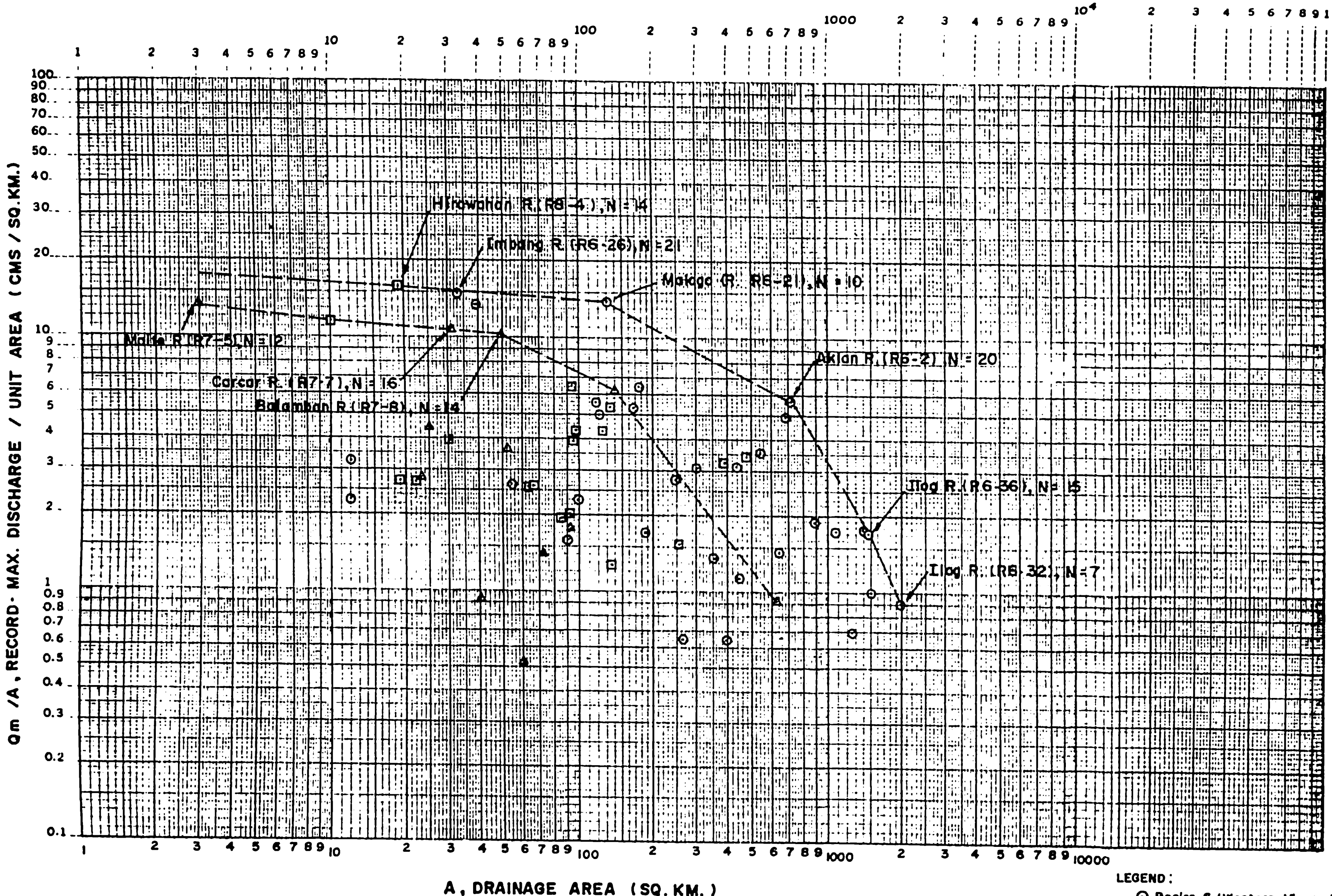


Figure 5 RECORD MAXIMUM FLOODS (REGIONS 6, 7 & 8) 1946-1970

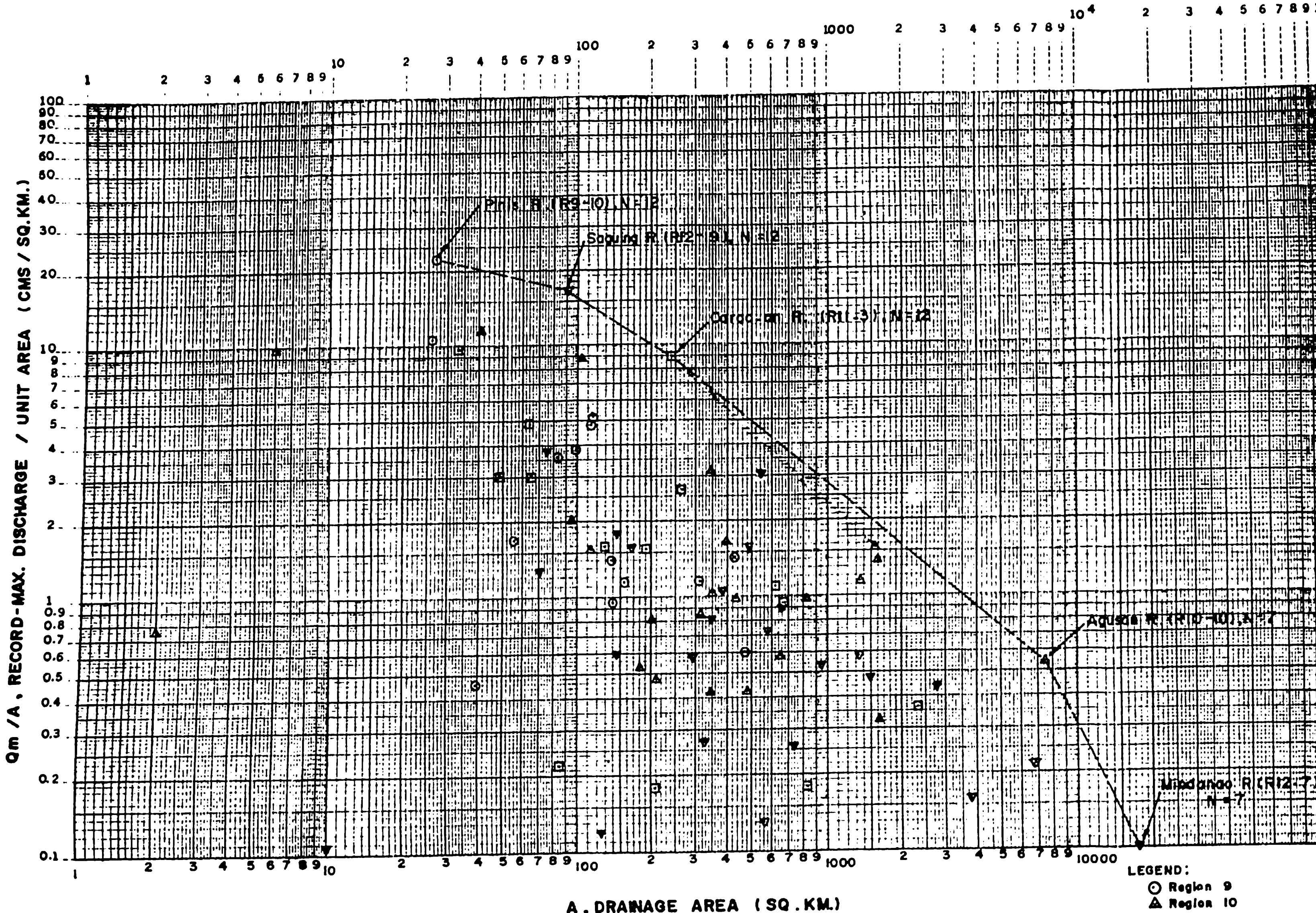


Figure 6 RECORD MAXIMUM FLOODS (REGIONS 9, 10, 11 & 12)
1946-1970

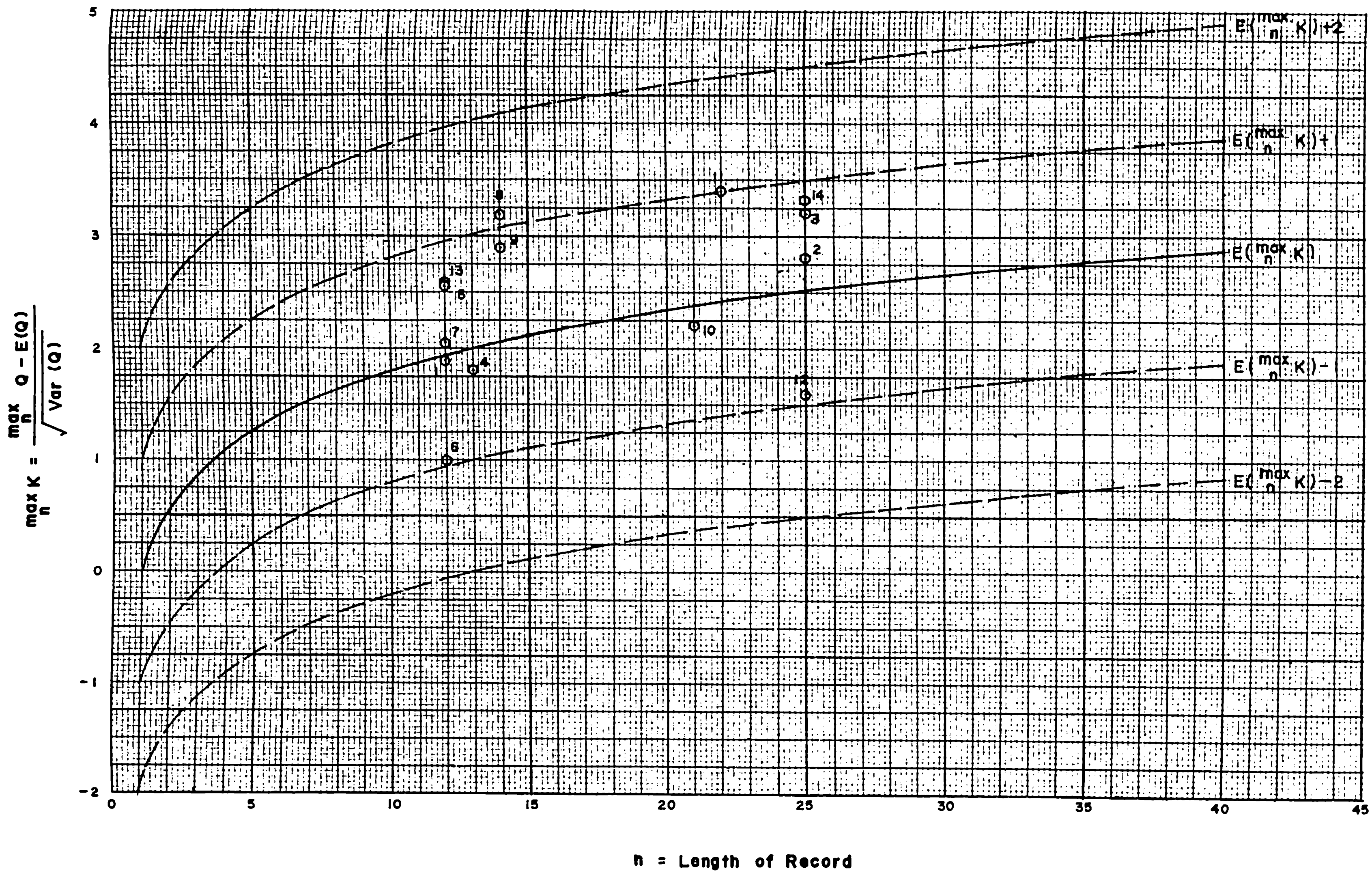


Figure 7 **Standardized deviates of record maxima of annual extreme floods plotted against length of record (Region I)**

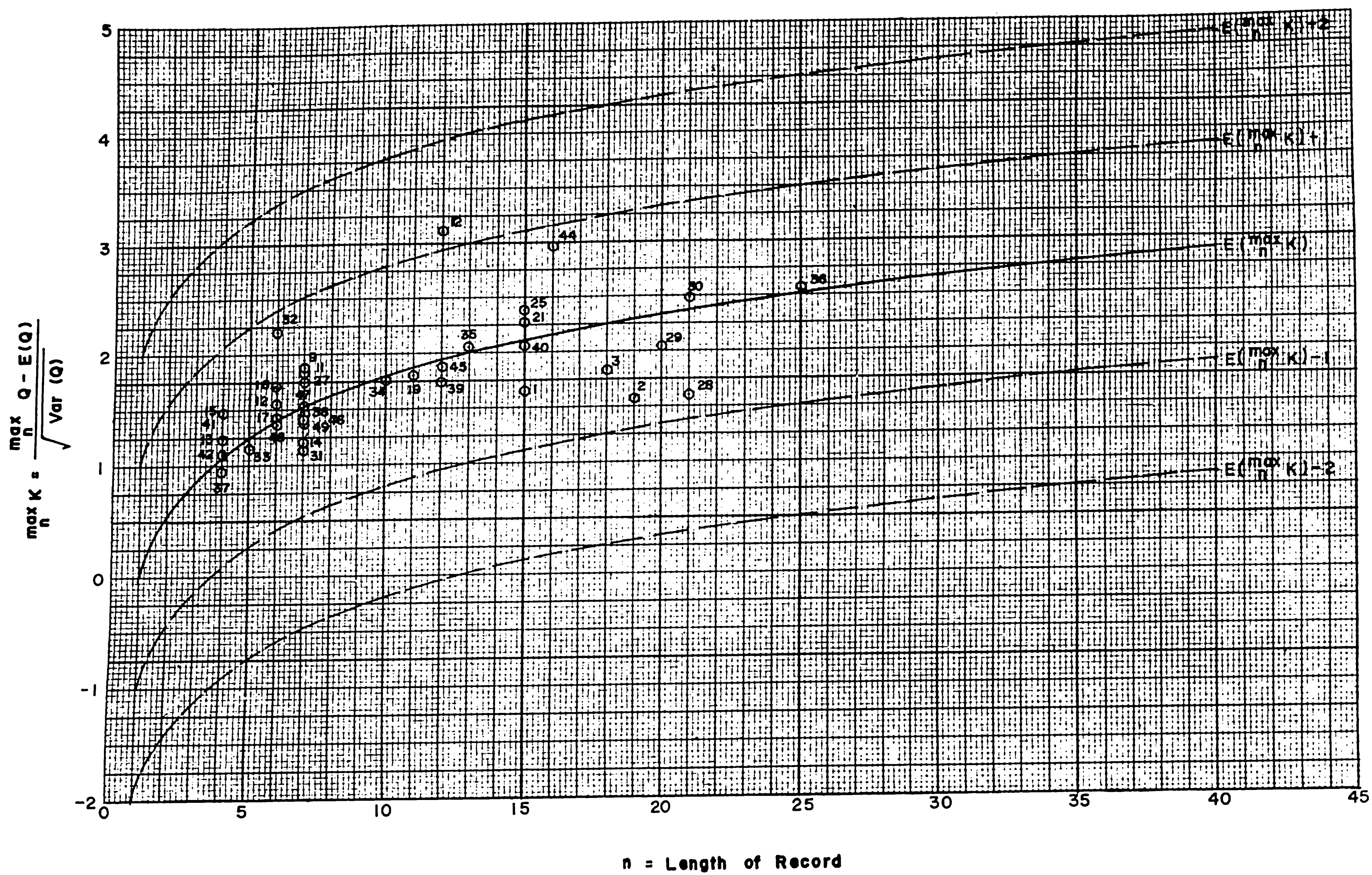


Figure 8 Standardized deviates of record maxima of annual extreme floods plotted against length of record (Region 2)

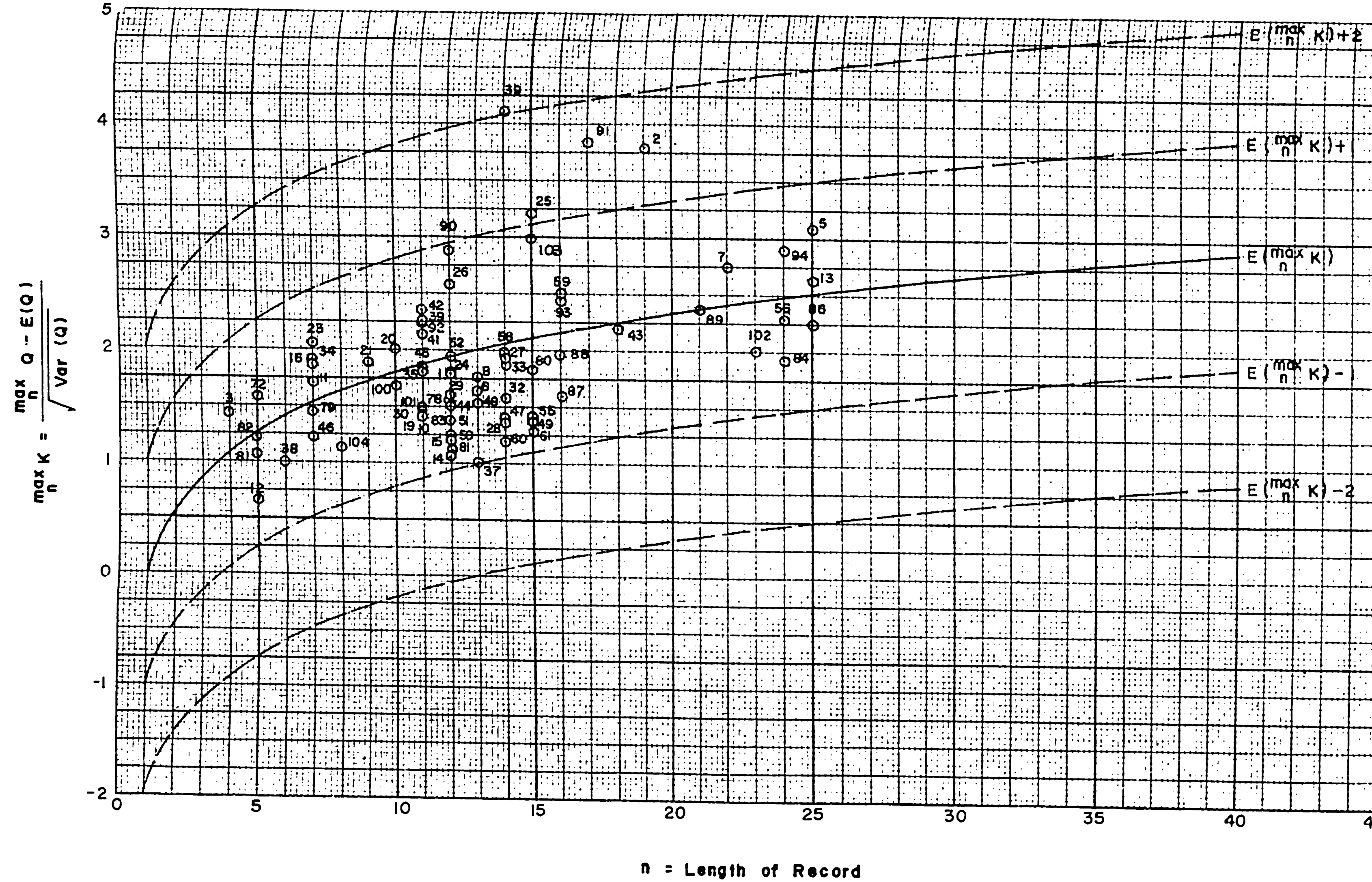


Figure 9 Standardized deviates of record maxima of annual extreme floods plotted against length of record (Region 3)

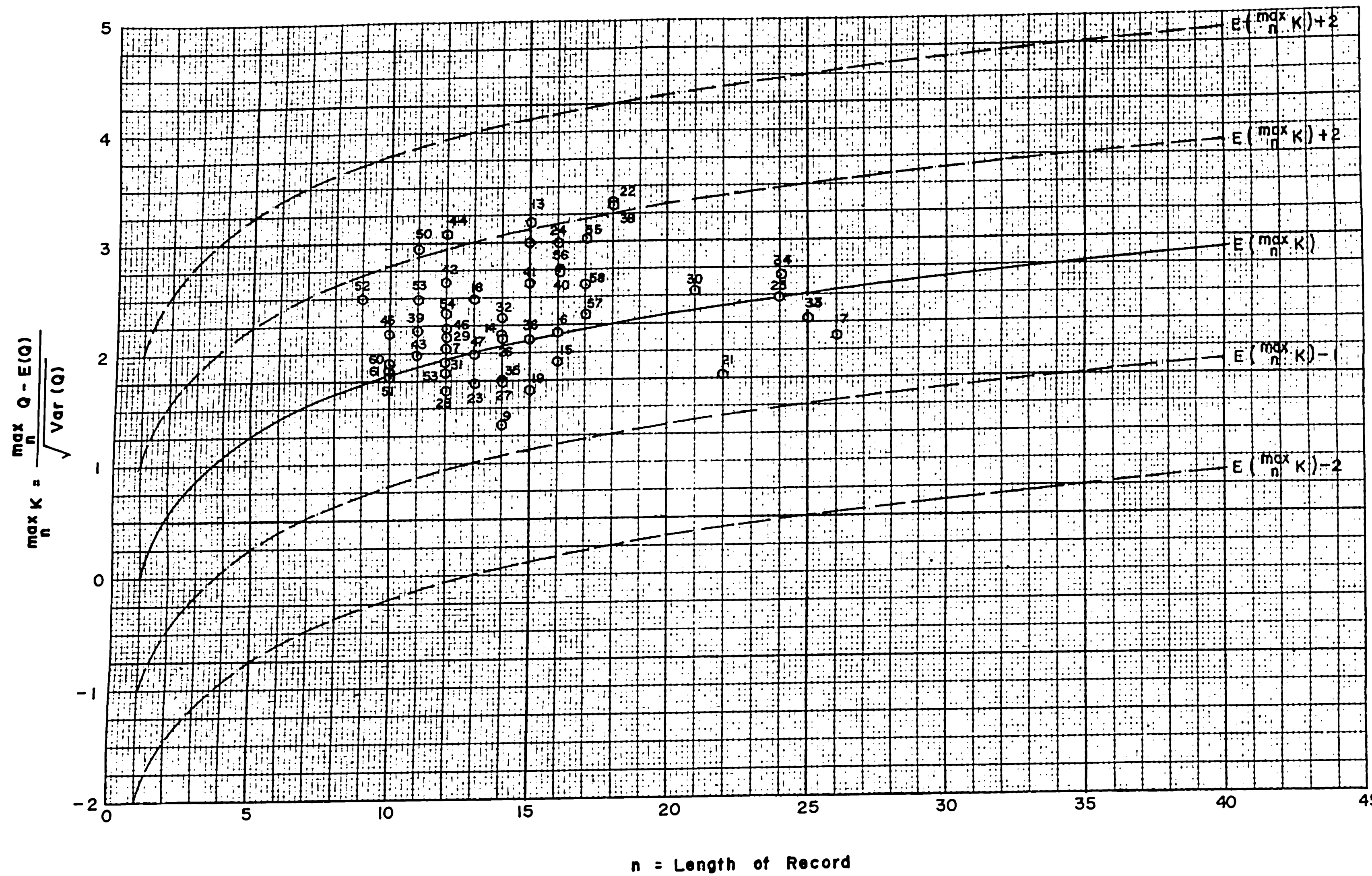


Figure 10 Standardized deviates of record maxima of annual extreme floods plotted against length of record (Region 4)

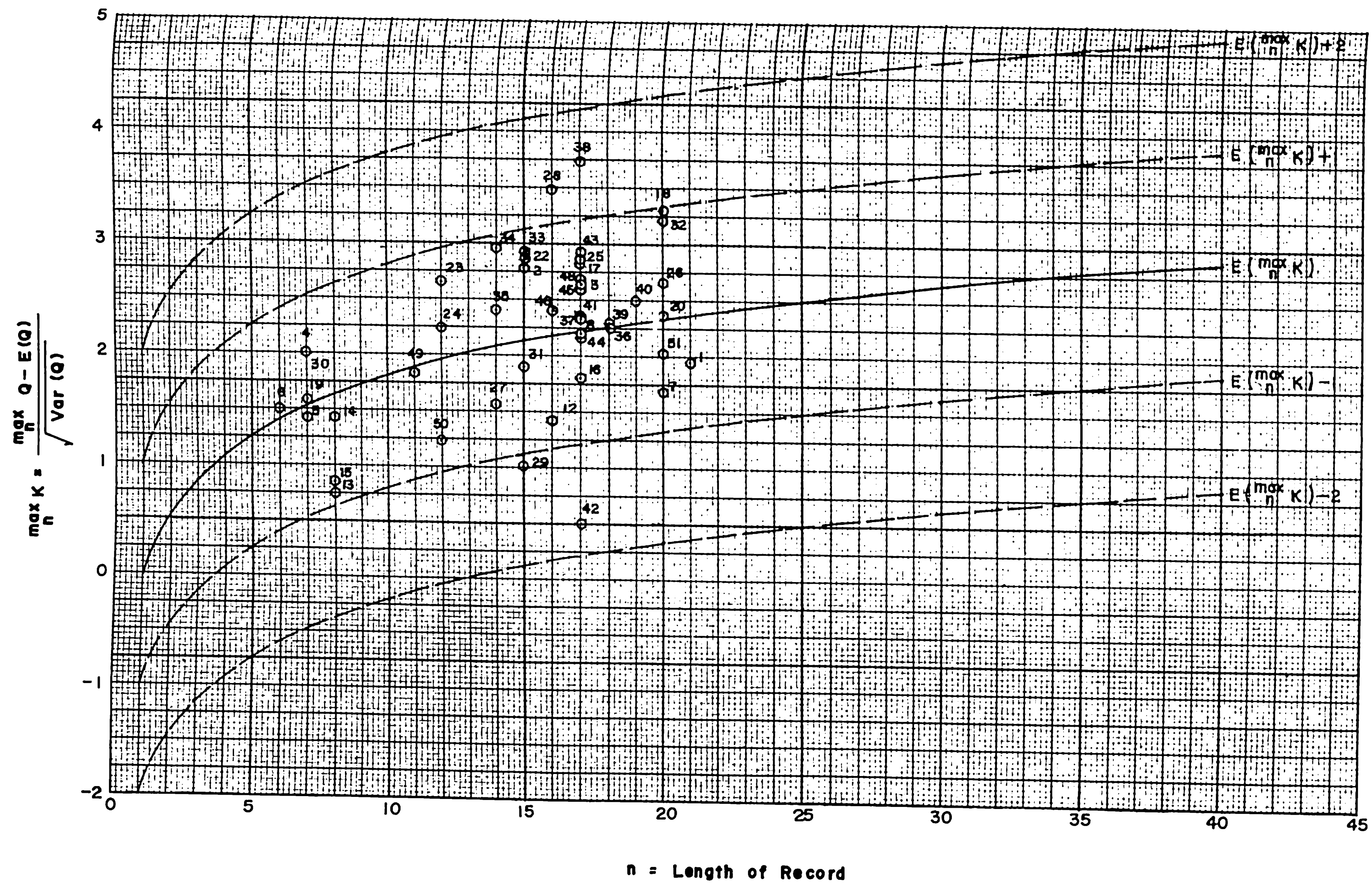


Figure II Standardized deviates of record maxima of annual extreme floods plotted against length of record (Region 5)

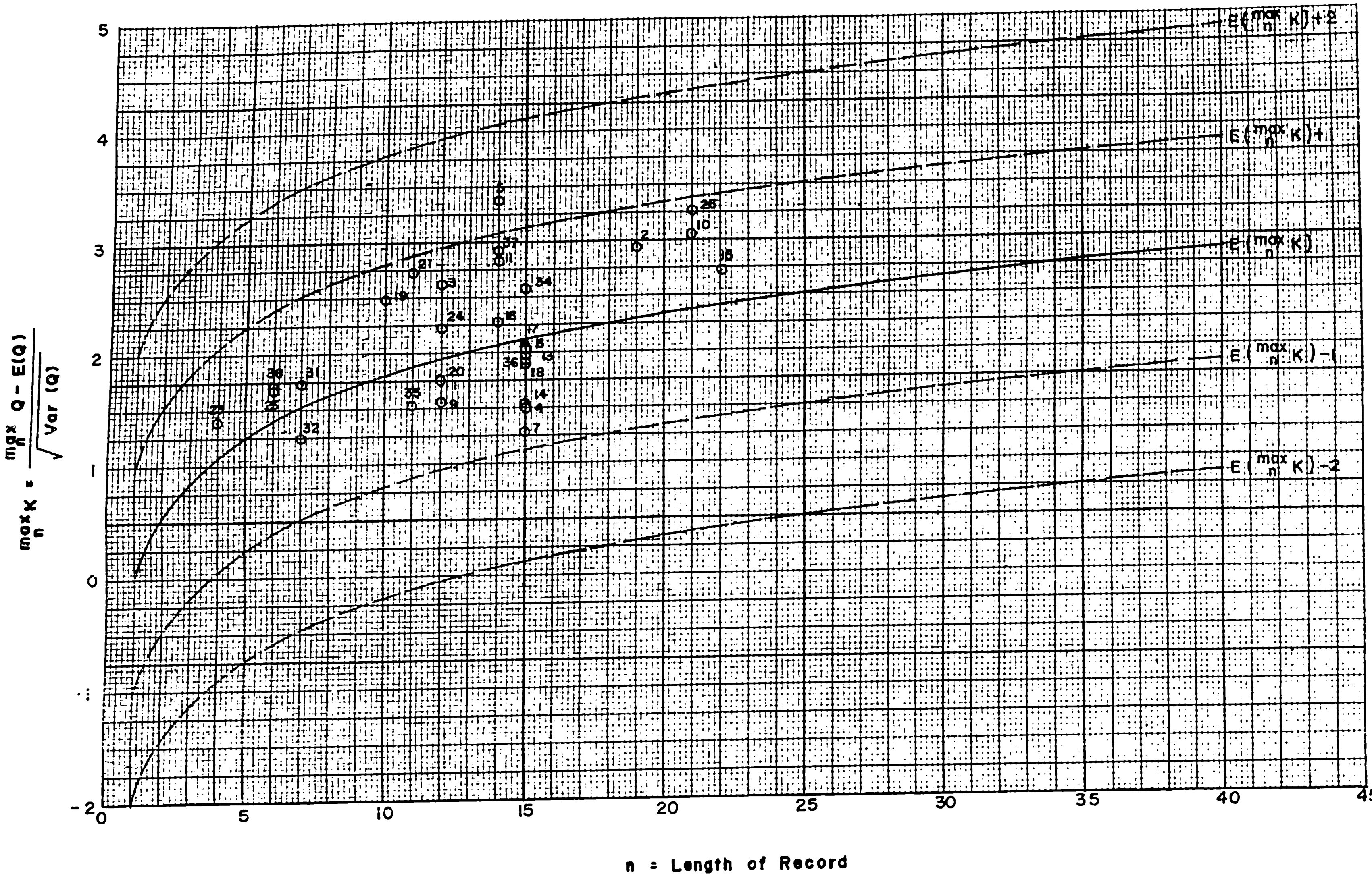


Figure 12 Standardized deviates of record maxima of annual extreme floods plotted against length of record (Region 6)

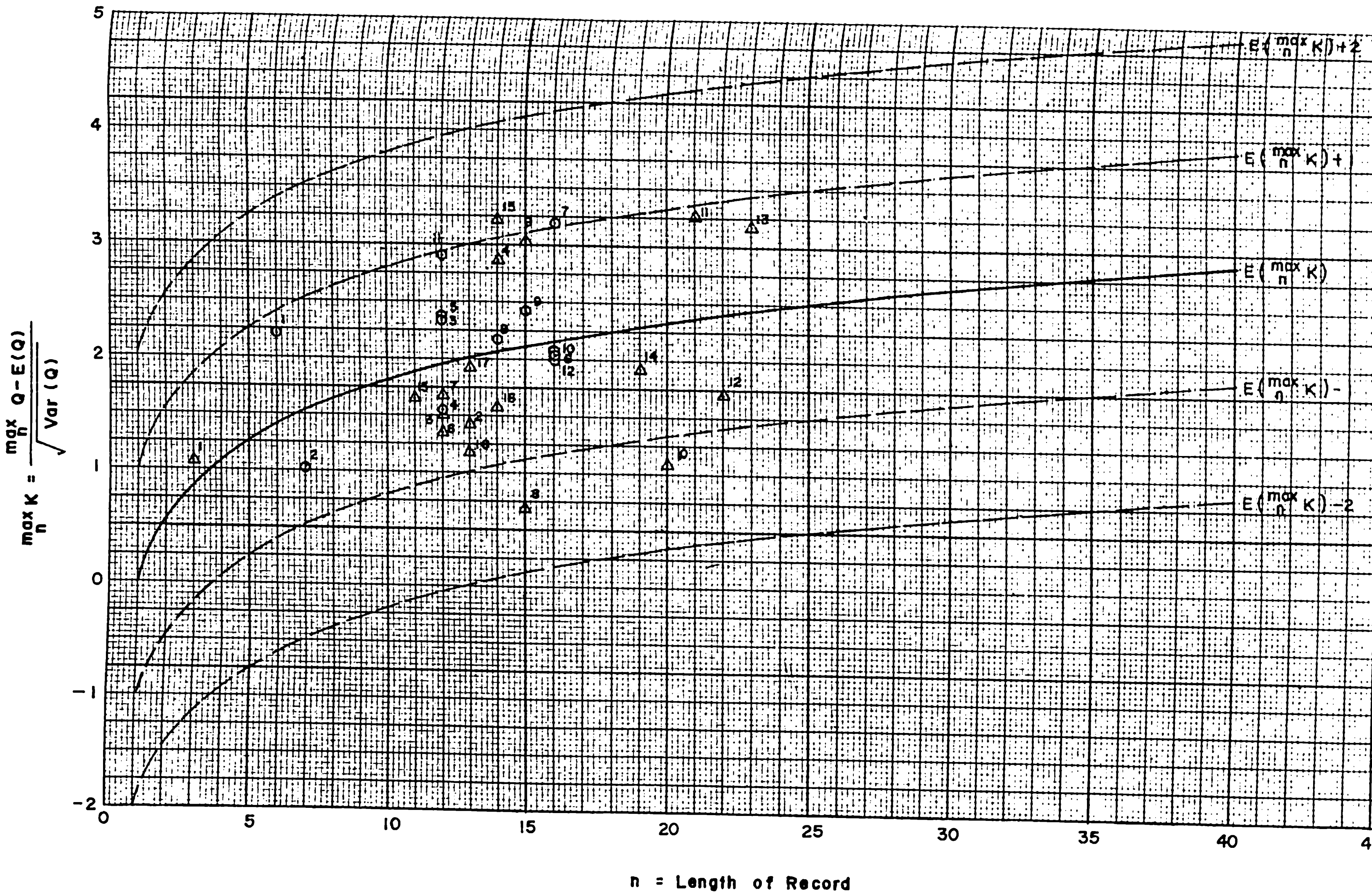


Figure 13 Standardized deviates of record maxima of annual extreme floods plotted against length of record (⊙ Region 7 ; △ Region 8)

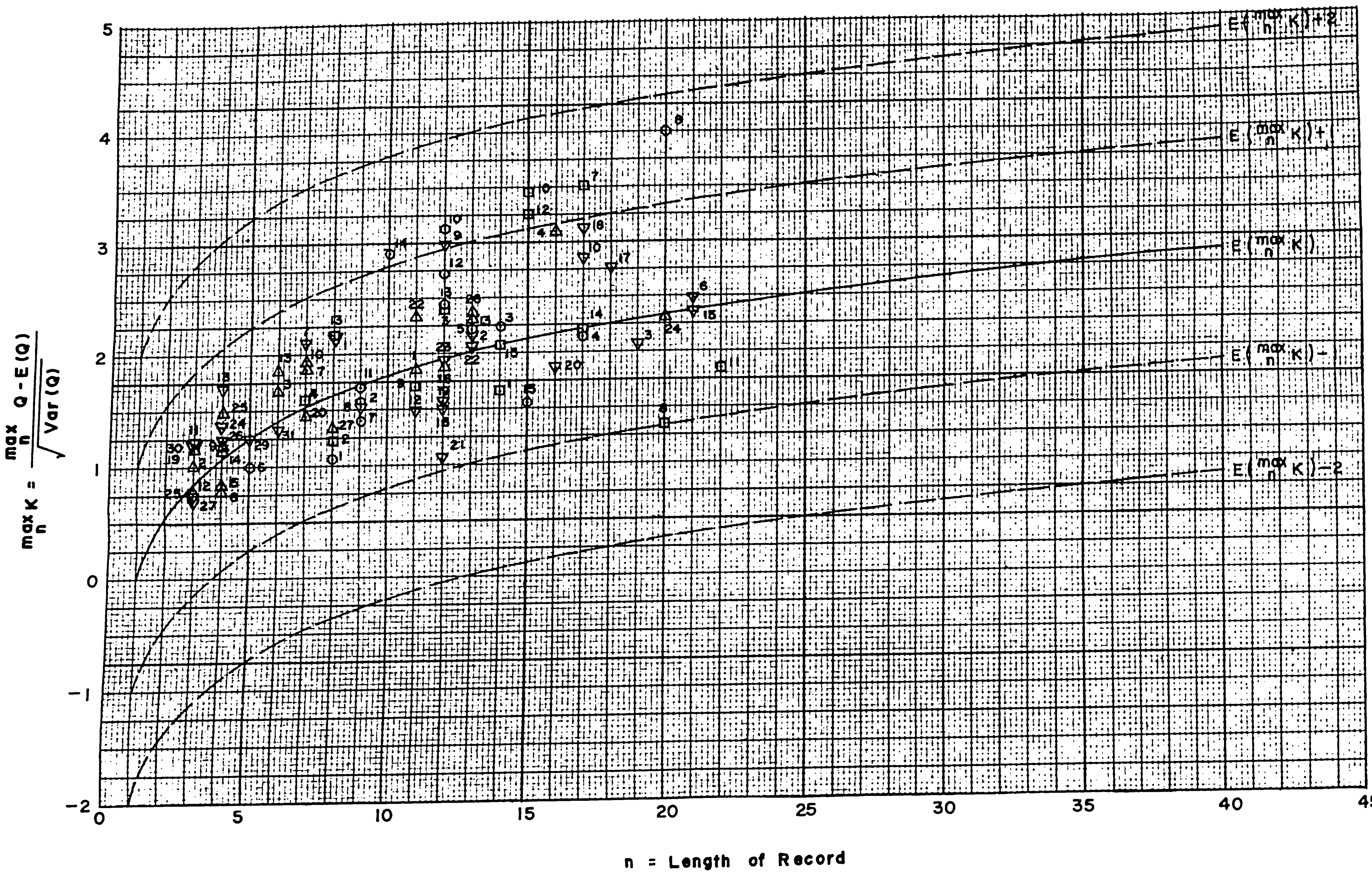


Figure 14 **Standardized deviates of record maxima of annual extreme floods (Mindanao Is.) ploted against length of record (⊙ Region 9 ; △ Region 10 ; ⊡ Region 11 ; ▽ Region 12)**

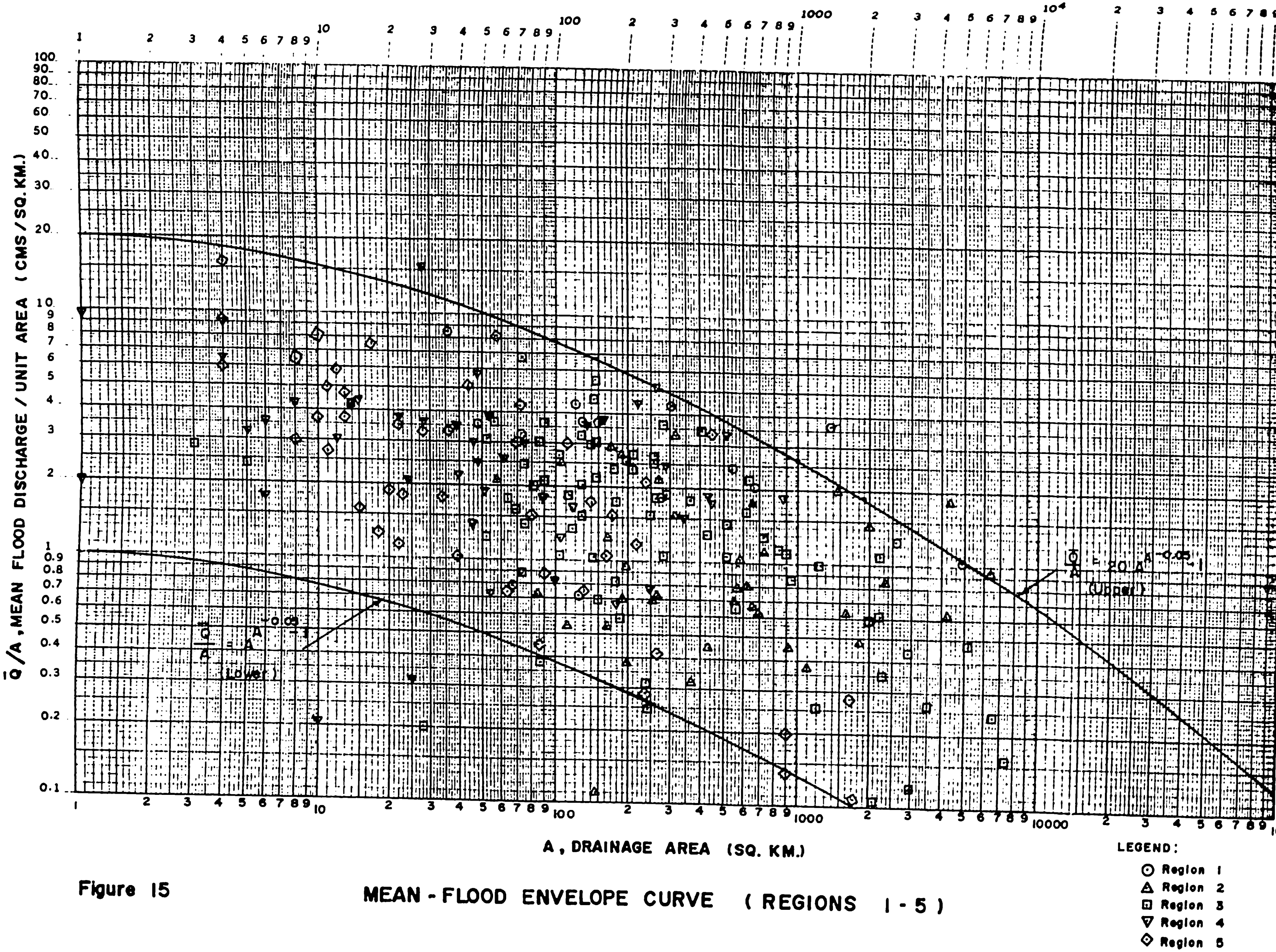


Figure 15

MEAN - FLOOD ENVELOPE CURVE (REGIONS 1 - 5)

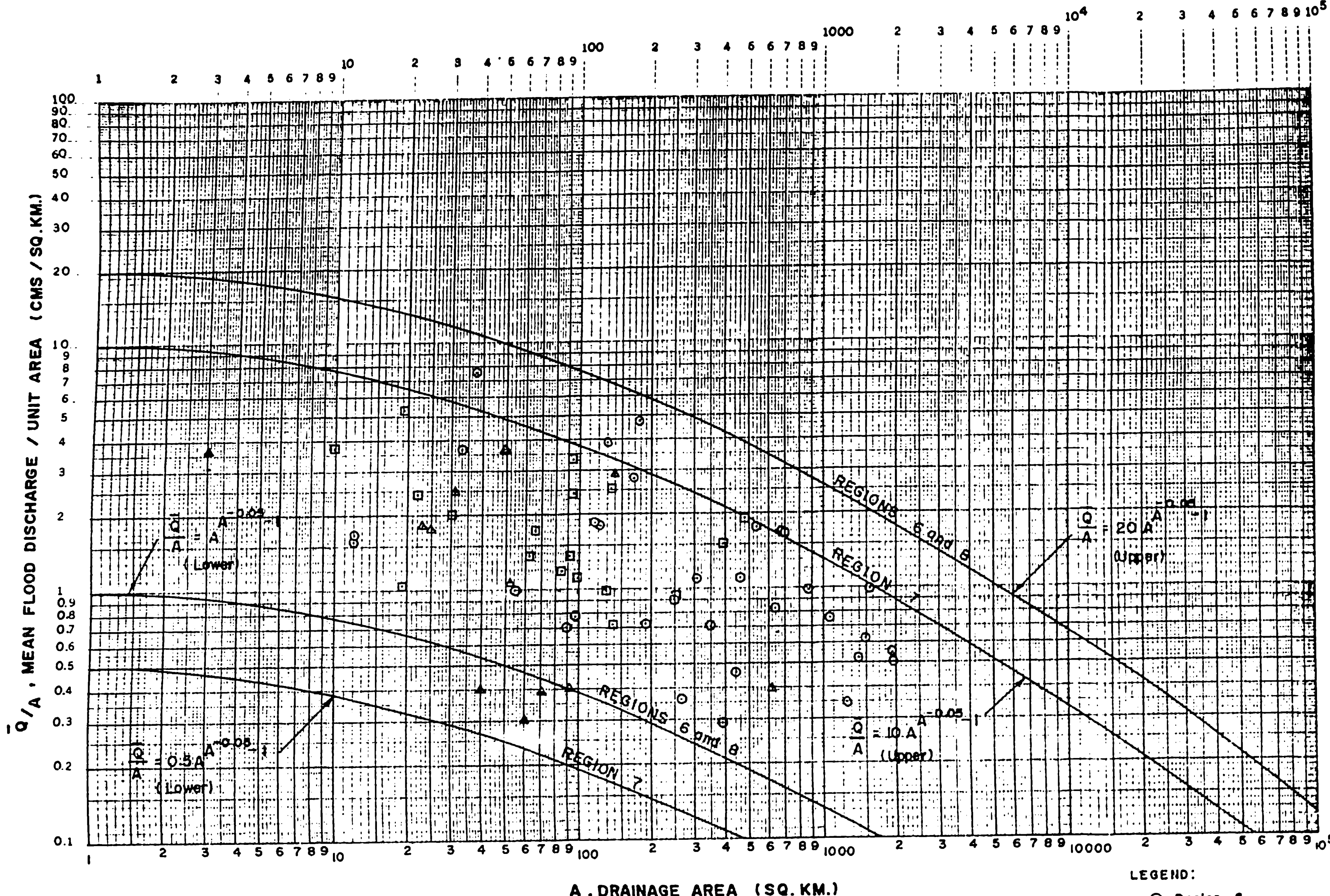


Figure 16 MEAN-FLOOD ENVELOPE CURVES (REGIONS 6 - 8)

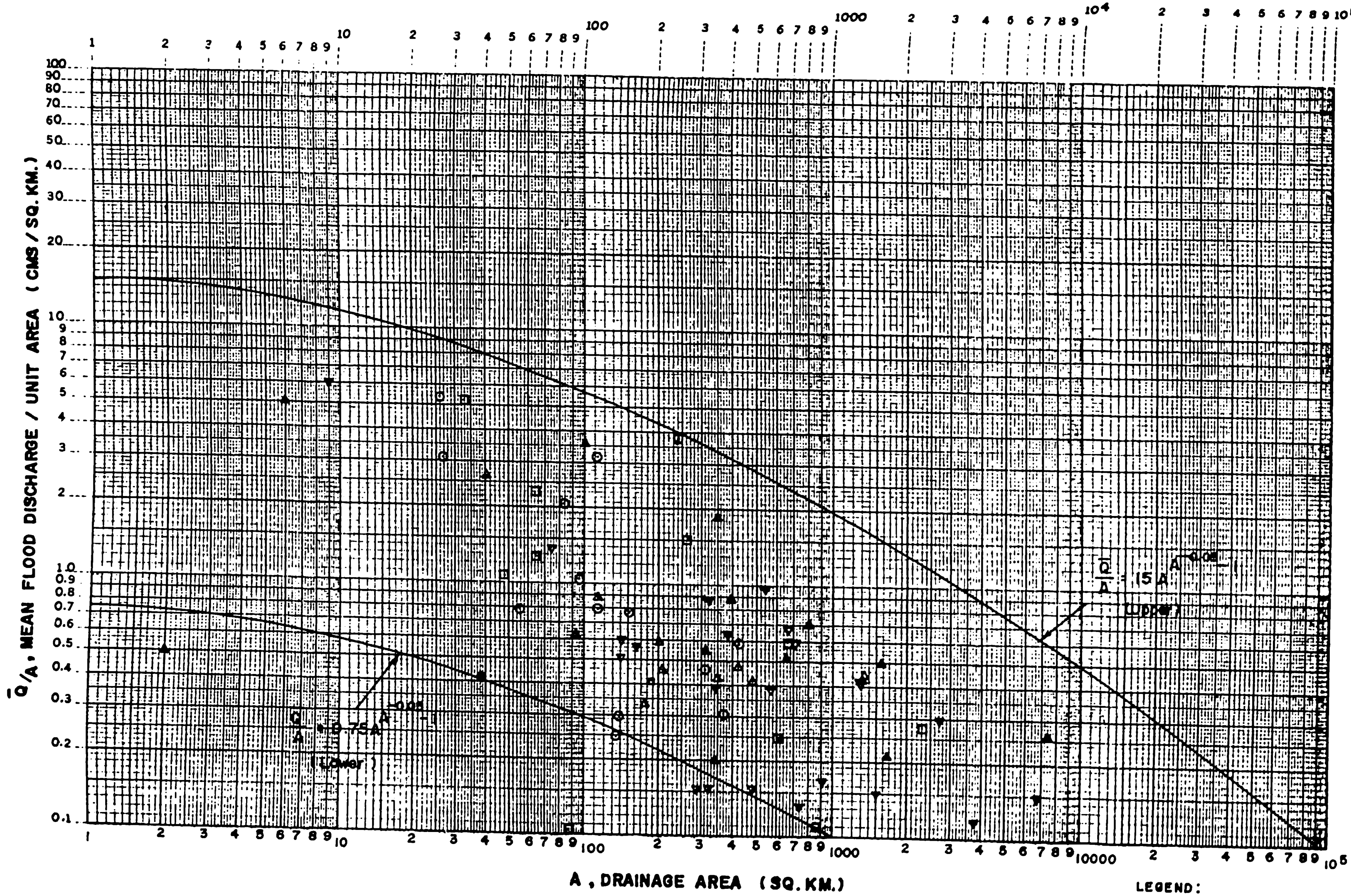


Figure 17

MEAN - FLOOD ENVELOPE CURVE (REGIONS 9 - 12)

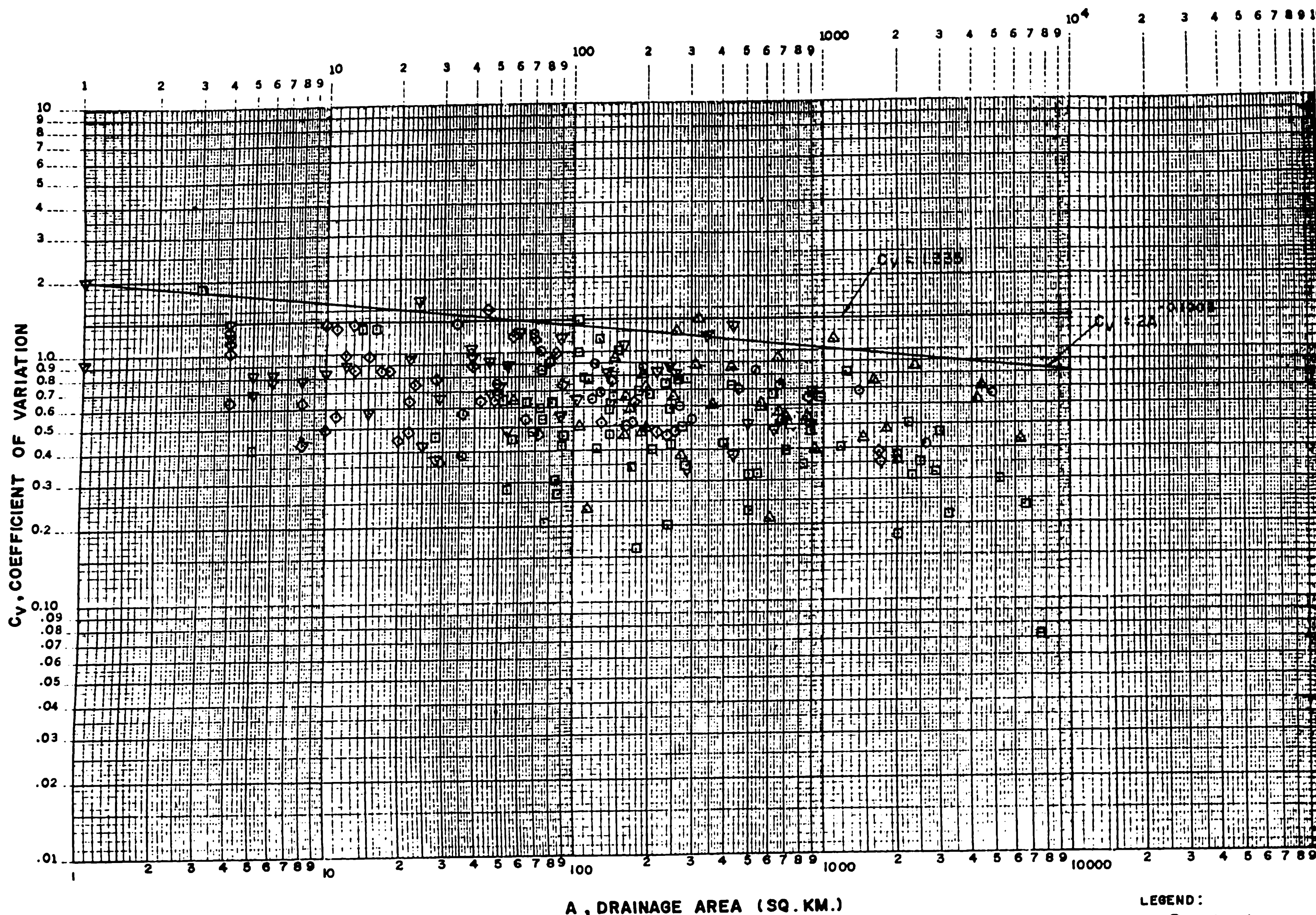


Figure 18

COEFFICIENT-OF-VARIATION ENVELOPE CURVES
(REGIONS 1-5)

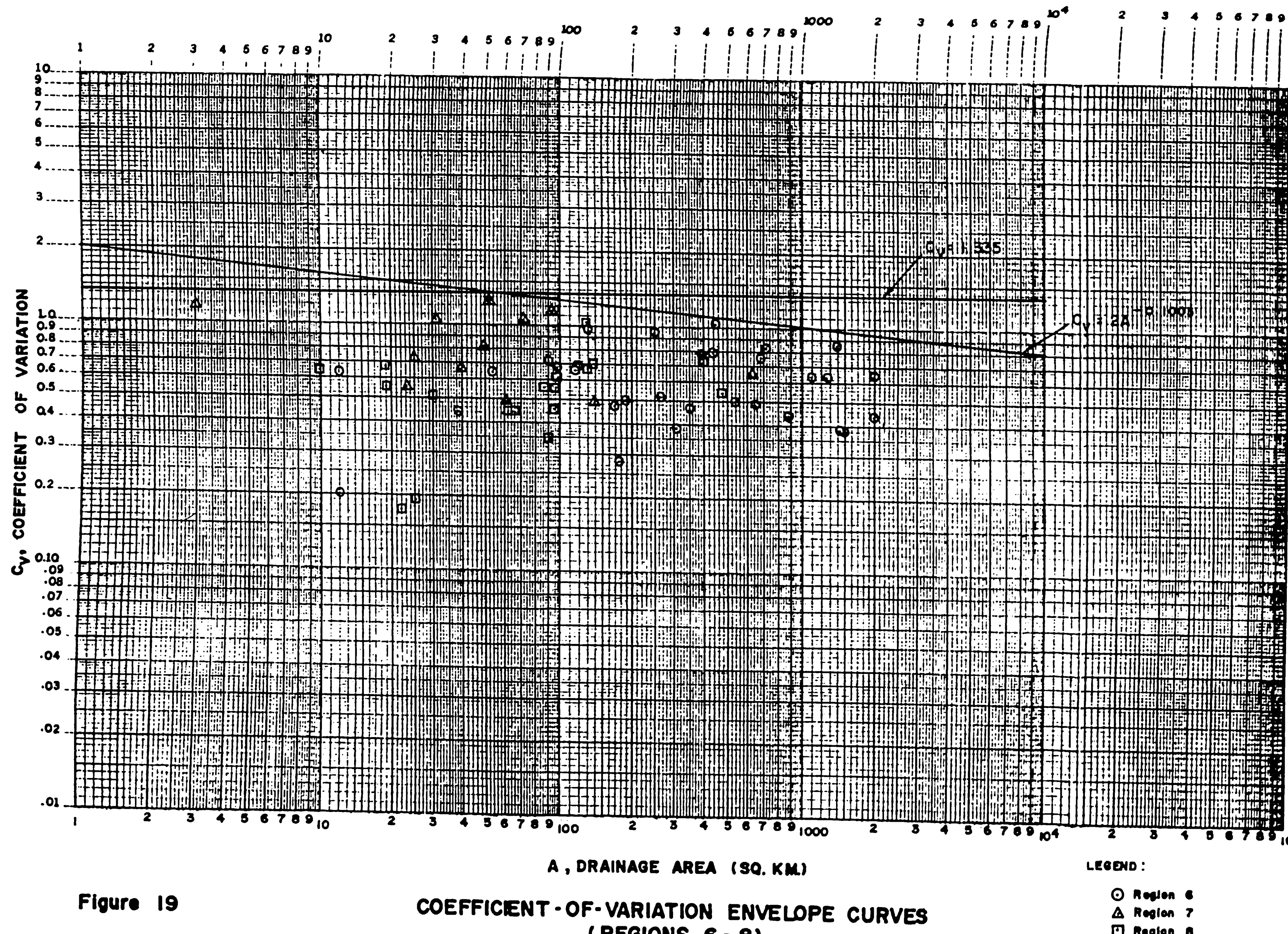
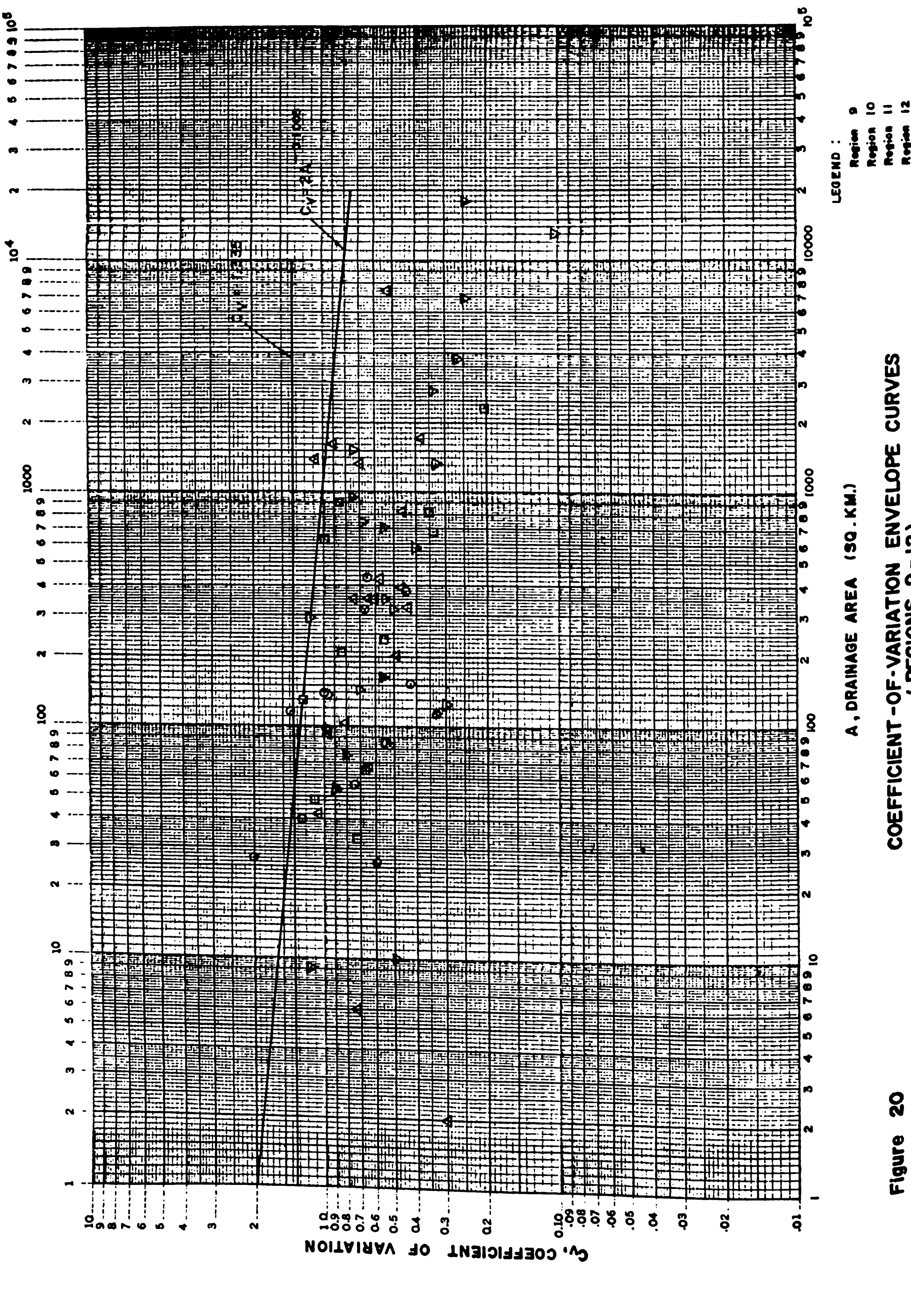


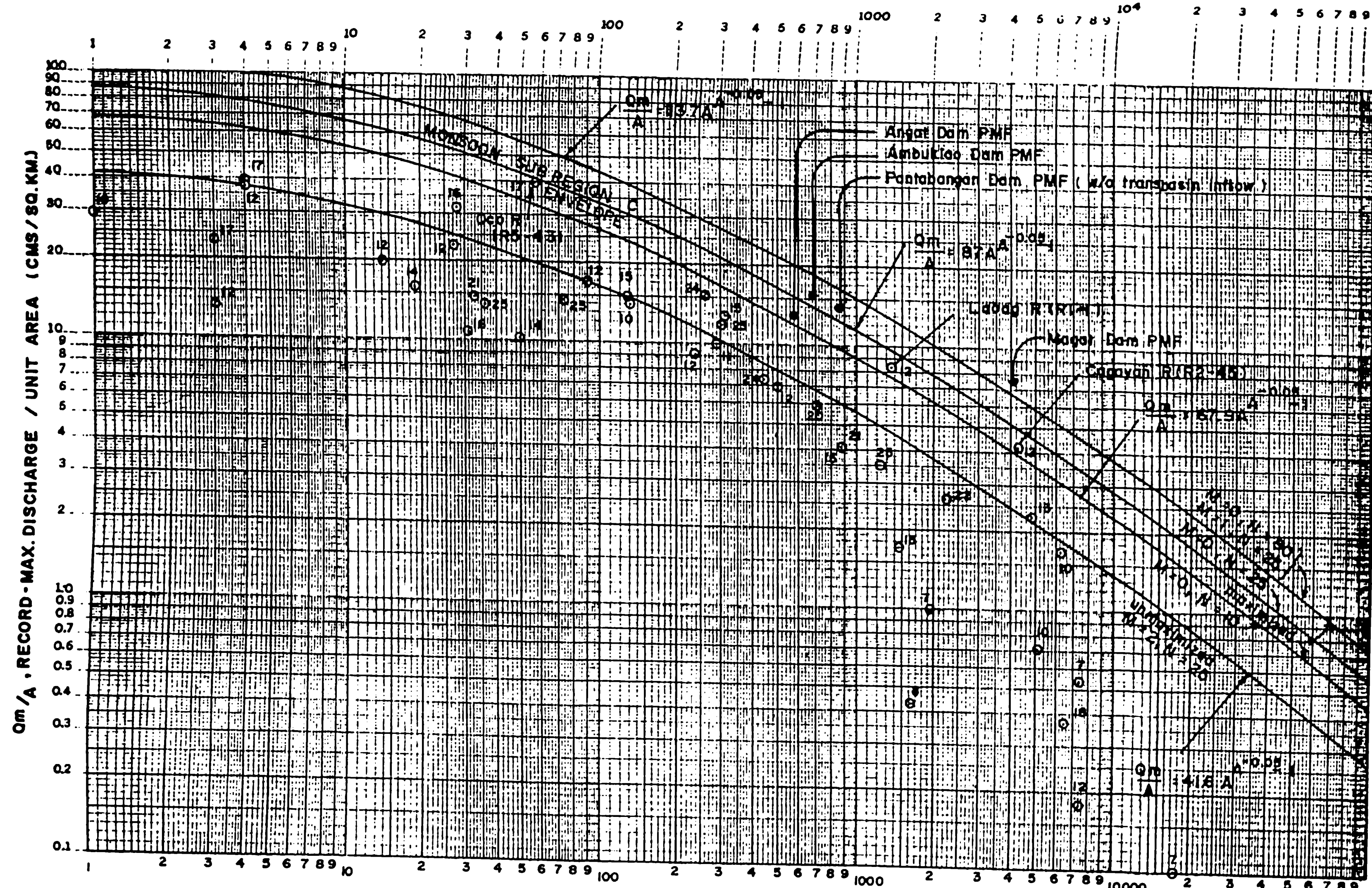
Figure 19

COEFFICIENT-OF-VARIATION ENVELOPE CURVES
(REGIONS 6-8)



COEFFICIENT-OF-VARIATION ENVELOPE CURVES (REGIONS 9-12)

Figure 20



LEGEND:

⊙ Regional controlling points

XX Numbers beside circles are record lengths in years

Figure 21

FLOOD ENVELOPE CURVES

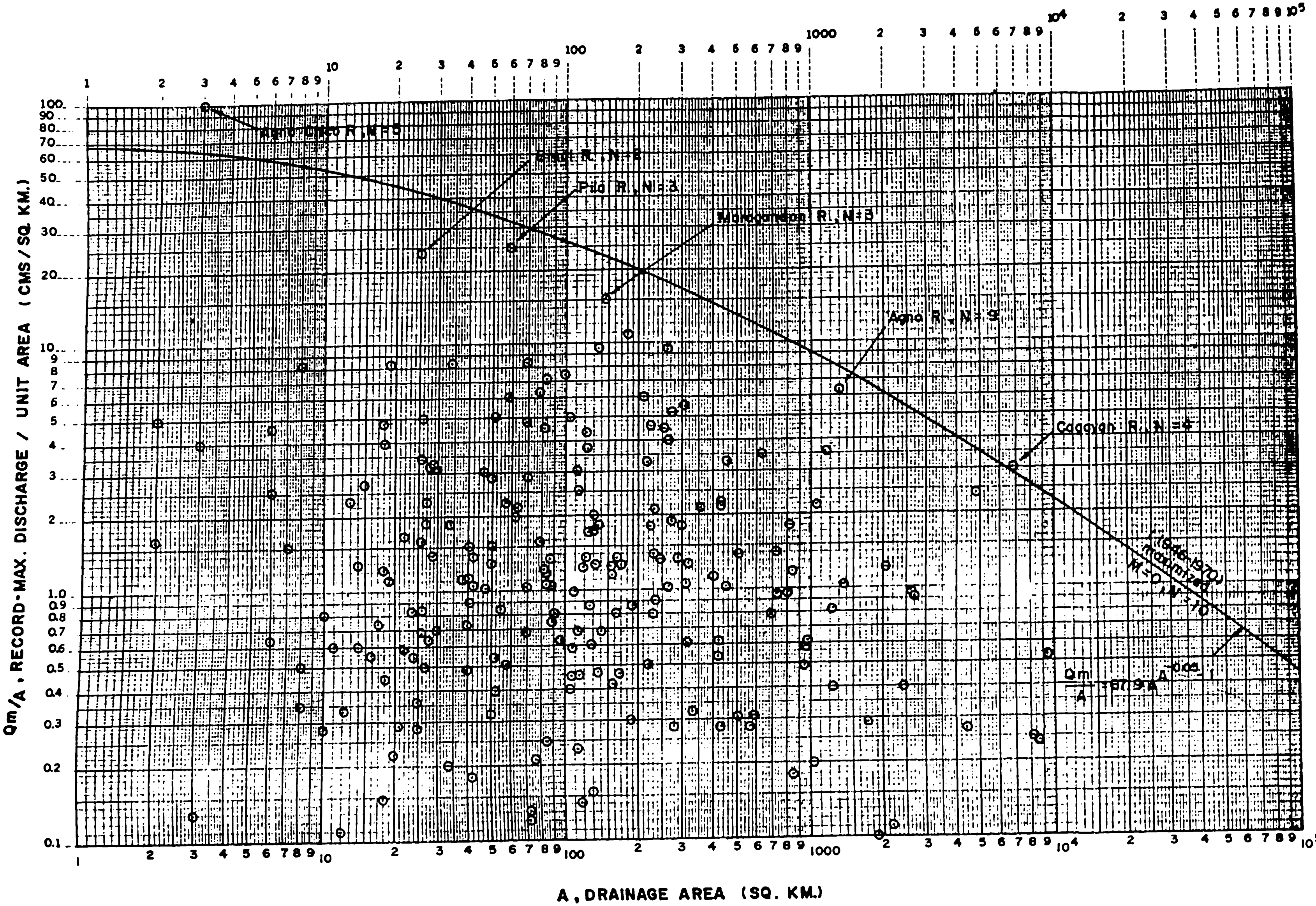


Figure 22 RECORD MAXIMUM FLOODS (1908-1922 RECORDS)