"the availability of a linear programming computer package designed for instructional use may reduce the gap between theory and application of linear programming,"

Linear Programming Cases in the Water Resources Engineering Classroom: Formulation and Implementation With The Linear Programming Language (LPL) Package

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Introduction

The Water Resources Graduate Training Program at the College of Engineering, University of the Philippines includes a course entitled Operations Research in Water Resources, the principal objective of which is to introduce graduate students in water resources engineering to the theory and application of linear programming (LP), dynamic programming, and other operations research methodologies in decision problems of water-resource systems design and operations. One course requirement in the topic of linear programming is an individual LP project report which normally consists of a brief description of a water resource project, either factual or close to factual, a statement of the decision problems, an LP formulation of the problems or subproblems, coding and execution with available computer package, analysts and interpretation of results, conclusion, and recommendations.

The first aim of this short paper is to present the salient features of two selected student LP projects which are addressed respectively to two specific water-resource decision areas: (1) Land allocation for different crops under

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constraints of water supply, fertilizer, insecticide, and labor availability; and (2) Pipe-sizing for a rural water supply system. The LP formulations embodied in these classroom projects are modest attempts to solve subsets of more complex water-resource systems decision problems, achieved through the deliberate treatment of medium-sized sub-problems for which sufficient necessary data are available and which are inherently linear or "linearizable" but "non-trivial" nevertheless in so far as the LP solution provides useful decision information for the subproblem or total problem at hand. The crucial choice of subproblem or problem to be solved by linear programming is left to the student with very little influence from the teacher (the author of this paper). Most of the students are engineers on scholarships who are employed by government water resource agencies such as the National Irrigation Administration (NIA), National Power Corporation (NPC). Local Water Utilities Administration (LWUA), and several others. They possess the field experience and the exposure to the numerous aspects of water-resource projects or programs handled by their agencies. Thus, they can wisely choose their LP topic to be relevant to their present engineering office practice as well as to their future involvement in more responsible decison making within their organizations.

The availability of a linear-programming computer package that is designed specifically for instructional use is a necessary prescription for reducing at least within the confines of the classroom, the gap between theory and application of LP which starts to develop in the students' minds around the middle of a one-semester course. One such computer package is the Linear Programming Language (LPL) which was developed by Prof. Evangel P. Quiwa, Associate Professor of Engineering Sciences, U.P. College of Engineering. Several courses teaching LP have availed of LPL for instructional use through a number of semesters already. One major advantage of having LPL as an instructional tool is the facility that it provides for repetitive "recycling" of a student problem following after every iterative parameterization or else revision of formulation in the event of infeasibility, unfoundedness, or plain unacceptability of output of earlier formulation. In this "recycling" activity, the three classical elements—student initiative and learning, teacher's guidance, and an adaptable instructional medium: LPL— unavoidably interacts for the benefit of the three parties concerned. The student acquires skill in the use of LP, the teacher learns likewise from the students' "toils and troubles"; and equally significant, recurrent demands on the capabilities of LPL—in terms of more and better language features to handle growing needs—foster the development of LPL for wider and more effective use. Other intensive uses of LPL are foreseen in thesis and research works.

The second aim of this paper may thus be stated: to illustrate the use of LPL as the language with which students communicate their LP problems to the computer and with the aid of which they derive the rewarding experience of being amazingly able to apply a fairly sophisticated mathematical technique for solving "non-trivial" extracts of real-world decision problems.

The succeeding sections discuss in turn the two selected student LP projects in a manner that is faithful to the original reports. The coding with LPL is clearly presented in each case.

Land Allocation for Different Crops

"Agricultural business is increasingly becoming popular to businessmen in the country today. This may be attributed to the presidential decrees that give tax and other incentives to agricultural business investors and to G.O. 47 (Corporate Farming) which requires all government and private agencies with 500 or more employees to engage in agricultural production to supply their respective employees' rice and corn needs."

There are many factors that are being considered in the agricultural crop production business such as the availability of farm inputs, water and labor supply, products demand, land suitability, etc. Thus, what crop to plant on a newly acquired agricultural land is always the first question the businessman asks. Recognizing the importance of the decision on what crops to plant and how much area to be allocated for each crop, the efficient businessman may employ management techniques in resource allocation, such as linear programming, so that he obtains quantitative guides to a sound final decision.

A project has an area of 2500 hectares located in the province of Pangasinan where the rainfall pattern belongs to Type I (very distinct wet and dry season). Land classification data from the Bureau of Soils reveals that the soil in the project area is mostly of dual land class (fitted for rice and other crops). There is plenty of water supply during the wet season but during the dry season, there is considerable shortage of water available to the project area. Few years after, however, water supply will not be a problem for the dry season since an on-going major irrigation project is expected to be completed in three years time.

The area is under lease contract for five (5) years and the company wants to maximize profit within this lease period. Recognizing the scarcity of irrigation water during the dry season, the company plans to grow profitable crops requiring less irrigation water than rice such as corn, sorghum and peanuts. During the wet season, the company has no other option but to grow only rice because the other crops could not be grown profitably during such periods.

A study of farm input supply in the area shows that there will be limited amounts of fertilizer, insecticides and labor supply available for the next five years. Market demand for peanuts is also limited to 1500 tons per year, increasing at the rate of 10% per year. Peanut yield is 3 tons/hectare. Table I presents a summary of the supply of farm inputs and crop requirements in the next five years of dry seasons.

The average incomes per hectare planted to rice, sorghum, peanuts, and corn are known to be ₱5,000, ₱5,500, ₱5,000, and ₱4,000, respectively.

TABLE 1
Supply and Requirement of Farm Inputs (Dry Season)

Input	*Supply*	*Requirement*
Water Supply	First three years =2500 liters/sec (lps) No water shortage for 4th and 5th year	Rice = 1.5 lps/ha. Sorghum = O.7 lps/ha. Peanut = 0.6 lps/ha. Corn = 0.8 lps/ha.
Fertilizer	16,000 bags for the first year, and increasing at the rate of 10%/year	Rice = 8 bags/ha. Sorghum = 6 bags/ha. Peanut = 6 bags/ha. Corn = 6 bags/ha.
Insecticides	4,000 liters for the first year, and increasing at the rate of 10%/year	Rice = 1 liter/ha. Sorghum = 2 liters/ha. Peanut = 1.5 liters/ha. Corn = 1.5 liters/ha.
Labor	5,100 man-days for the first year, and increasing at the rate of 3%/year	Rice = 2 man-days/ha. Sorghum = 3 man-days/ha. Peanut = 3 man-days/ha. Corn = 2 man-days/ha.

The LP formulation that seeks to maximize the total profit for the dry seasons of the five-year period runs as follows:

Let t = year number = 1, 2, 3, 4, 5
x_{1t} = hectares planted to rice in year t
x_{2t} = hectares planted to sorghum in year t
x_{3t} = hectares planted to peanut in year t
x_{4t} = hectares planted to corn in year t

$$\text{maximize} \sum_{t=1}^{5} \quad 5000\,x_{1t} + 5500x_{2t} + 5000\,x_{3t} + 4000\,x_{4t}$$

subject to:

water supply constraints (for t = 1 — 3):

(WS1 — WS3) $\quad 1.5\,x_{1t} + 0.7\,x_{2t} + 0.6\,x_{3t} + 0.8\,x_{4t} \leqslant 2500$ lps

area constraints (for t = 1 — 5):

(A1 — A5) $\quad x_{1t} + \quad x_{2t} + \quad x_{3t} + \quad x_{4t} = 2500$ hectares

fertilizer constraints (for t = 1 — 5):

(F1 — F5) $\quad 8\,x_{1t} + \quad 6\,x_{2t} + \quad 6\,x_{3t} + \quad 6\,x_{4t} \leqslant 16000\,(1.)^{t-1}$ bags

insecticide constraints (for t = 1 — 5):

(IS1 — IS5) $\quad x_{1t} + \quad 2\,x_{2t} + 1.5\,x_{3t} + 1.5\,x_{4t} \leqslant 4000\,(1.1)^{t-1}$ liters

labor constraints (for t = 1—5):

(LS1—LS5) $\quad 2\,x_{1t} + 3\,x_{2t} + \quad 3\,x_{3t} + + 2\,x_{4t} \qquad 5100\,(1.03)^{t-1}$ man-days

peanut demand constraints (for t = 1 — 5):

PD1 — PD5) $\qquad 3\,x_{3t} \leqslant \quad 1500\,(1.1)^{t-1}$ tons

The LPL coding for the land allocation problem appears as shown:

```
TASKNAME    LAND ALLOCATION PROBLEM
VARIABLES x11(5000) x12(5000) x13(5000) x14(5000) x15(5000)
* x21(5500) x22(5500) x23(5500) x24(5500) x25(5500)
* x31(5000) x32(5000) x33(5000) x 34(5000) x35(5000)
* x41(4000) x42(4000) x43(4000) x44(4000) x45(4000)
FORMAT FREE MATRIX
CONSTRAINTS WS1<2500 WS2<2500 WS3<2500
* A1=2500 A2=2500 A3=2500 A4=2500 A5=2500
* F1<1600 F2<17600 F3<19360 F4<21296 F5<23325
* IS1<4000 IS2<4400 IS3<4840 IS4<5324 IS5<5856
* LS1<5100 LS2<5253 LS3<5410 LS4<5572 LS5<5739
* PD1<1500 PD2<1650 PD3<1815 PD4<1997 PD5<2197

1.5 0   0   0   0   0.7 0   0   0   0   0.6 0   0   0   0   0.8 0   0   0   0
0   1.5 0   0   0   0   0.7 0   0   0   0   0.6 0   0   0   0   0.8 0   0   0
0   0   1.5 0   0   0   0   0.7 0   0   0   0   0.6 0   0   0   0   0.8 0   0
1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0
0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   ·0  0
0   0   1   0   0   0   0   1   0   0   0   0   1   0   0   0   0   1   0   0
```

```
0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0
0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1
8 0 0 0 0 6 0 0 0 0 6 0 0 0 0 6 0 0 0 0
0 8 0 0 0 0 6 0 0 0 0 6 0 0 0 0 6 0 0 0
0 0 8 0 0 0 0 6 0 0 0 0 6 0 0 0 0 6 0 0
0 0 0 8 0 0 0 0 6 0 0 0 0 6 0 0 0 0 6 0
0 0 0 0 8 0 0 0 0 0 0 0 0 0 6 0 0 0 0 6
1 0 0 0 0 2 0 0 0 0 1.5 0 0 0 0 1.5 0 0 0 0
0 1 0 0 0 0 2 0 0 0 0 1.5 0 0 0 0 1.5 0 0 0
0 0 1 0 0 0 0 2 0 0 0 0 1.5 0 0 0 0 1.5 0 0
0 0 0 1 0 0 0 0 2 0 0 0 0 1.5 0 0 0 0 1.5 0
0 0 0 0 1 0 0 0 0 2 0 0 0 0 1.5 0 0 0 0 1.5
2 0 0 0 0 3 0 0 0 0 3 0 0 0 0 2 0 0 0 0
0 2 0 0 0 0 3 0 0 0 0 3 0 0 0 0 2 0 0 0
0 0 2 0 0 0 0 3 0 0 0 0 3 0 0 0 0 2 0 0
0 0 0 2 0 0 0 0 3 0 0 0 0 3 0 0 0 0 2 0
0 0 0 0 2 0 0 0 0 3 0 0 0 0 3 0 0 0 0 2
0 0 0 0 0 0 0 0 0 0 3 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 3 0 0 0 0 0 0 0 0
6 0 0 0 0 0 0 0 0 0 0 0 3 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 3 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 3 0 0 0 0 0
PRINTFORMAT F
ECHOPRINT PRIMAL
TABLEAUX FIRST LAST *
METHOD TWO-PHASE
MAXIMIZE PRIMAL PROFIT
TASKEND
```

The output of LPL is summarized in Tables 2 and 3. Among the major conclusions which may be drawn from Tables 2 and 3 are the following:

1. Peanuts cannot be grown during the five-year lease period together with the other crops.
2. Corn can be grown most profitably only in the first three years.
3. Rice and sorghum can be grown most profitably throughout the five-year lease period.
4. Labor supply is limiting throughout the five-year lease period.
5. Insecticide supply is not limiting during the five-year lease period.
6. Fertilizer supply is limiting only in the first year.
7. Water supply becomes limiting in the second and third years.

TABLE 2

Areas to be Allocated to Different Crops
(in hectares)

	YEAR				
	1	*2*	*3*	*4*	*5*
Crop					
Rice	500	48	1475	1928	1761
Sorghum	100	253	410	572	739
Peanuts	0	0	0	0	0Z
Corn	1900	2199	615	0	0
T o t a l	2500	2500	2500	2500	2500

TABLE 3

Binding and Slack Constraints

	YEAR				
Constraint	*1*	*2*	*3*	*4*	*5*
Water Supply, lps	S(160)	B	B	—	—
Fertilizer, bags	B	S(2504)	S(1409)	S(2440)	S(4803)
Insecticides, liters	S(450)	S(547)	S(1632)	S(2252)	S(2617)
Labor Supply, man-days	B	B	B	B	B
Peanut Demand, tons	S(1500)	S(1650)	S(1815)	S(1997)	S(2197)

Notes: 1. Area constraints are equalities and therefore are always binding.
2. B—binding
S—slack; numbers in parentheses are values of slack variables.

The interpretation of the output and its sensitivity to the input coefficients can be performed in more depth and detail by consulting the computer printouts of tabular coefficients given by LPL. Hopefully, this brief discussion has been instructive in illustrating a typical LP implementation that is done by students with the aid of LPL.

Pipe-Sizing for a Rural Water Supply System

The government has called for accelerated efforts in rural water supply development in the face of increasing construction costs. The need to evaluate various distribution alternatives and designs rapidly and at a rapid rate has been focused sharply.

Level II water supply systems are characterized generally by a branched distribution system with public faucets and standpipes which also serve as blow-offs for the system. The national goal is that by the year 2000, all households shall be adequately serviced. Level II seems to be the viable design concept if this goal is to be attained.

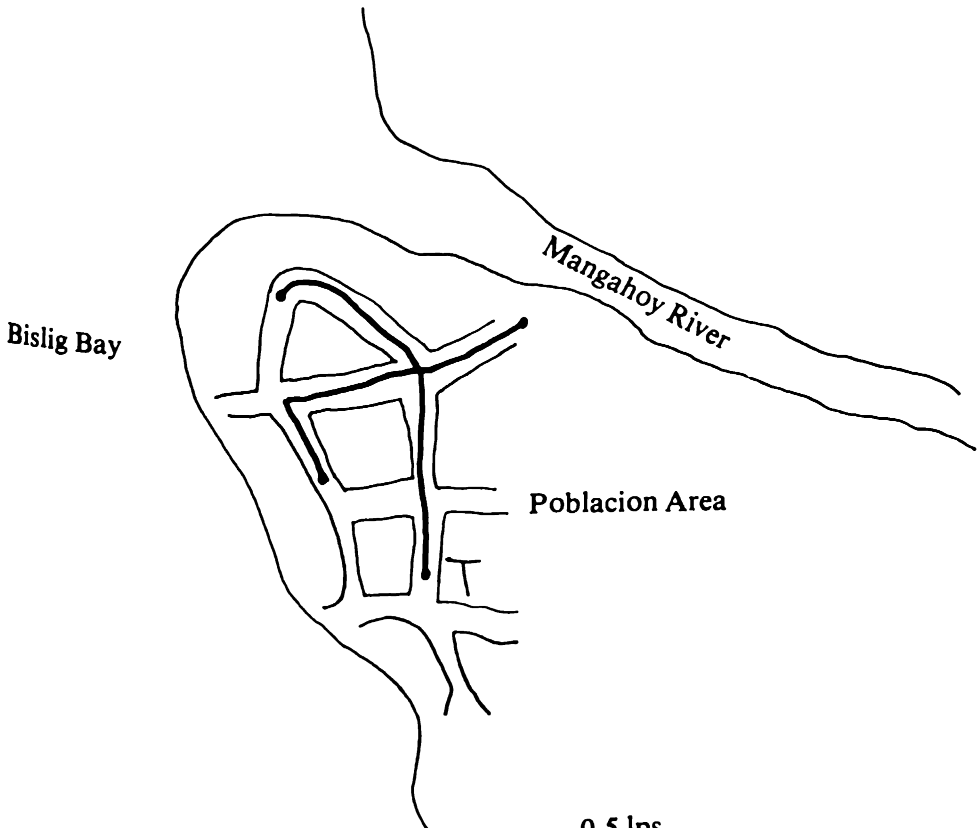
Current estimates show that at the end of 1979, a total of 3,051,000 urban and rural households are not served by either piped water, artesian wells or springs. This represents 56% of the total population whom we may reasonably conclude to have inadequate water service. The figures cited means that approximately 152,550 households have to be put on line annually. At 800 households per system, a staggering number of 190 new systems have to be designed and constructed annually. The price tag for this effort is estimated to cost upwards of ₱6.5 billion.

A linear programming algorithm for rapid assessment and design to minimize project costs may ease this huge task. Experience has shown that about 70% of the total project construction costs go into pipelines and related works. Efforts towards optimizing this project component will certainly yield felt results.

Consider a situation found in the current water supply development project of the Bislig Water District. A population core is found near the Bislig delta. Figure 1 shows a possible pipeline layout scheme. We can simplify the configuration as having five (5) lines connecting the nodes of this system to each other and to the tapping or hock-up point, T.

Given the elevation and the pressure head at T and the elevation, minimum residual pressure and flow take-offs at each of the nodes, the designer should be able to specify a feasible scheme of pipe sizes to meet the flow demands within the head loss margin available.

Our intention is to proceed one step further by making the design optional i.e., determining the least cost mix of commercially available pipe diameters.



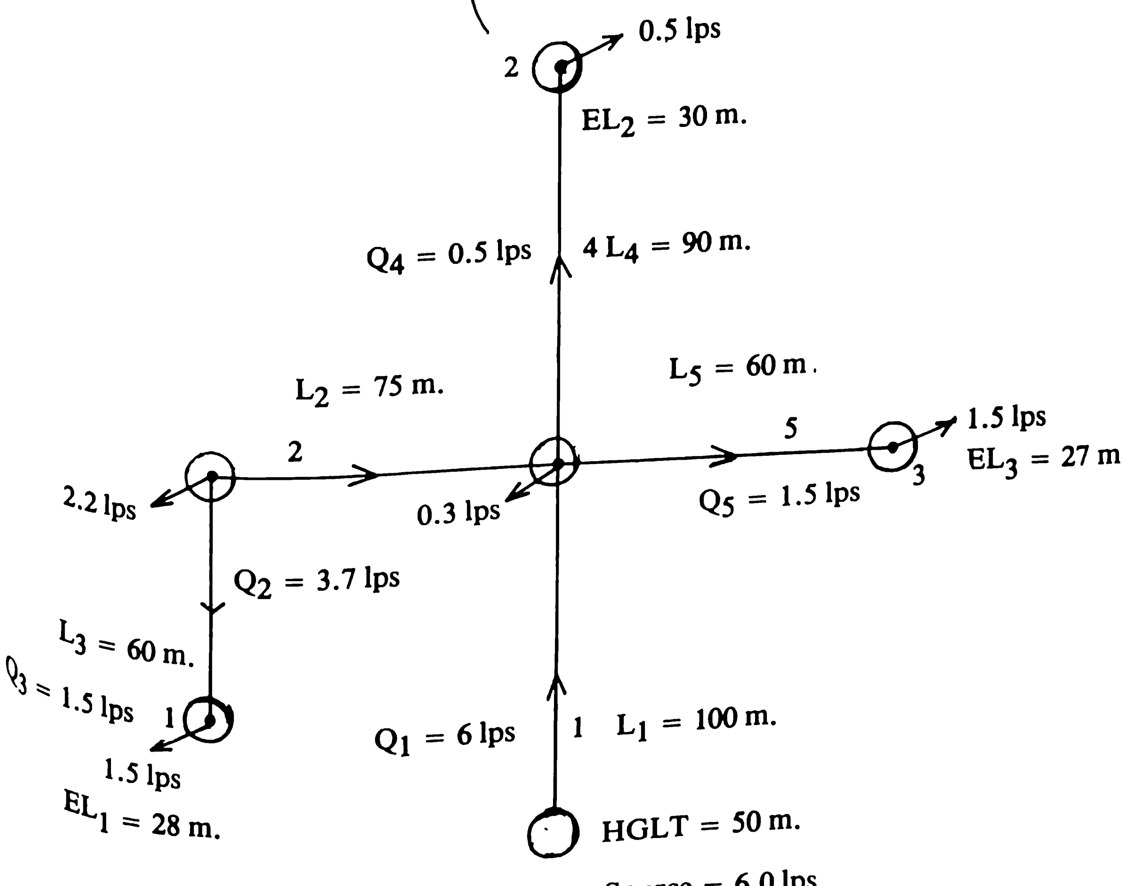


Figure 1 — Pipeline Layout Scheme

We note that there is a cost per unit length associated with every diameter of pipe. For example, using the lastest 1979 bid prices for polyvinyl chloride (PVC), the costs per unit meter, C_j are:

j	Diameter, D_j	C_j, ₱/m
1	50 mm	₱ 84
2	100 mm	₱ 90
3	150 mm	₱160
4	200 mm	₱240

These pipe cost data include: the supplier's costs (labor , raw materials, selling costs), transportation to the project site, pipe laying (excavation, back-filling and surface restoration), hydrostatic testing, disinfection, cost of fittings and taxes. The costs also assume labor-intensive construction methods.

Our objective function will be to

$$\text{Minimize Cost} = \sum_{j=1}^{A} C_j \sum_{i=1}^{P} X_{ij}$$

where p = number of pipe lines
n = number of commercially available pipe diameters
X_{ij} = length of pipe in line i of diameter j (the decision variable)
C_j = cost per unit length of pipe of diameter j

There are two sets of constraints: the head constraints and the length constraints.

The number of head constraints will correspond to the number of nodes where a specific pressure condition is required. We trace the pipelines which will account for head losses or pressure drops occurring in a specific node. The head in a node must be at least equal to the minimum residual pressure head, MRP, say 10 meters. In our case, the first head loss constraints H1 applicable to terminal node 1 is:

$$\begin{aligned} \text{H1:} \sum_{j=1}^{n} h_{1j}\, x_{1j} + \sum_{j=1}^{n} h_{2j}\, x_{2j} + \sum_{j=1}^{n} h_{3j}\, x_{3j} &\leqslant \text{HGLT—EL}_{1\text{—MRP}} \\ &\leqslant 50\text{—}28\text{—}10 \\ &\leqslant 12 \text{ meters} \end{aligned}$$

where h_{ij} = head loss per unit length of a pipe in line i of diameter j.
HGLT = hydraulic grade line elevation at source T
EL_K = elevation of discharge point at the particular node K
MRP = minimum residual pressure head required

Note that pipelines 1, 2, and 3 were used to account for the head loss at terminal node 1.

The hydraulic gradient, h_{ij}, can be calculated using the Hazen-Williams equation:

$$h_{ij} = 1.2 \times 10^{10} \left(\frac{Q_i}{C_H}\right)^{1.85} D_j^{-4.87}$$

where Q_i = flow in line i in liters/second and D_j = pipe diameter in millimeters (mm.). C_H is the Hazen-Williams coefficient, taken as 100 in this case example. Using this equation, a table of hydraulic gradients is computed as shown in Table 4.

The other head constraints, H2 and H3, for terminal nodes 2 and 3, respectively, are:

TABLE 4

Hydraulic Gradients h_{ij}

		Diameter *Diameters*			
Pipeline	*Flow Q_i(lps)*	*50 mm.*	*100 mm.*	*150 mm.*	*200 mm.*
1	6.0	3.5057E-1	1.1988E-2	1.6642E-3	4.0996E-4
2	3.7	1.4334E-1	4.9018E-3	6.8044E-4	1.6762E-4
3	1.5	2.6975E-2	9.2246E-4	1.2805E-4	3.1545E-5
4	0.5	3.5342E-3	1.2086E-4	1.6677E-5	4.1329E-6
5	1.5	2.6975E-2	9.2246E-4	1.2805E-4	3.1545E-5

TABLE 5

Optimal Pipe Sizing

Pipeline	*Length (meters)/Diameter (mm.)*
1	100 meters/100 mm
2	64 meters/50 mm, 11 meters/100 mm
3	60 meters/50 mm
4	90 meters/50 mm
5	60 meters/50 mm

$$\text{H2:} \quad \sum_{j=1}^{n} h_{1j}x_{1j} + \sum_{j=1}^{n} h_{4j}x_{4j} \leqslant \text{HGLT}-\text{EL}_2-\text{MRP}$$
$$\leqslant 50-30-10$$
$$\leqslant 10 \text{ meters}$$

$$\text{H3:} \quad \sum_{j=1}^{n} h_{1j}x_{ij} + \sum_{j=1}^{n} h_{5j}x_{5j} \leqslant \text{HGLT}-\text{EL}_3-\text{MRP}$$
$$\leqslant 50-27-10$$
$$\leqslant 13 \text{ meters}$$

The second set of constraints are the length constraints. The total of the various sublengths of different diameter pipes must be consistent with the network geometry. Thus, for the five pipelines:

$$\text{L1:} \quad \sum_{j=1}^{n} x_{1j} = 100 \text{ meters}$$

$$\text{L2:} \quad \sum_{j=1}^{n} x_{2j} = 75 \text{ meters}$$

$$\text{L3:} \quad \sum_{j=1}^{n} x_{3j} = 60 \text{ meters}$$

$$\text{L4:} \quad \sum_{j=1}^{n} x_{4j} = 90 \text{ meters}$$

$$\text{L5:} \quad \sum_{j=1}^{n} x_{5j} = 60 \text{ meters}$$

The LPL coding for this problem appears as shown:

```
TASKNAME PIPE SIZING FOR A RURAL WATER SUPPLY SYSTEM
VARIABLES x11(84) x12(90) x13(160) x14(240)
*         x21(84) x22(90) x23(160) x24(240)
*         x31(84) x32(90) x33(160) x34(240)
*         x41(84) x42(90) x43(160) x44(240)
*         x51(84) x52(90) x53(160) x54(240)
FORMAT FREE LIST
CONSTRAINTS H1<12, H2<10, H3<13, L1=100, L2=75, L3=60, L4=90, L5=60
H1 x11(3.5057-1), H1 x12(1.1988-2), H1 x13(1.6642-3), H1 x14(4.0996-4),
H1 x21(1.4334-1), H1 x22(4.9018-2), H1 x23(6.8044-4), H1 x24(1.6762-4),
```

```
H1 x31(2.6975-2), H1 x32(9.2246-4), H1 x33(1.2805-4), H1 x34(3.1545-5),
H2 x11(3.5057-1), H2 x12(1.1988-2), H2 x13(1.6642-3), H2 x14(4.0996-4),
H2 x41(3.5342-3), H2 x42(1.2086-4), H2 x43(1.6677-5), H2 x44(4.1329-6),
H3 x11(3.5057-1), H3 x12(1.1988-2), H3 x13(1.6642-3), H3 x14(4.0996-4),
H3 x51(2.6975-2), H3 x52(9.2246-4), H3 x53(1.2805-4), H3 x53(3.1545-5),
L1 x11(1) L1 x12(1) L1 x13(1) L1 x14(1)
L2 x21(1) L2 x22(1) L2 x23(1) L2 x24(1)
L3 x31(1) L3 x32(1) L3 x33(1) L3 x34(1)
L4 x41(1) L4 x42(1) L4 x43(1) L4 x44(1)
L5 x51(1) L5 x52(1) L5 x53(1) L5 x54(1)
ECHOPRINT PRIMAL
METHOD TWO-PHASE
PRINTFORMAT F
TABLEAUX FIRST LAST*
RANGES ALL
SUMMARY
MINIMIZE PRIMAL DESIGNCOST
TASKEND
```

Table 5 presents the LPL output showing the optimal design. The interesting features in the output are the facts that 100 mm diameter is required for the main pipeline (no. 1), mixed diameters of 50 mm and 100 mm are prescribed for pipeline no. 2 which carries the second largest flow, and 50 mm diameters are required for pipelines no. 3, 4 and 5 which connect to the terminal nodes, 1, 2 and 3.

The discussion is not intended to substitute detailed engineering and design work. It does, however, provide a very rapid and accurate analysis and review of design. The presentation proves the applicability of LP to the very real issues confronting the rural water supply engineer. The formulation presented may be generalized to cover more complicated network configuration and geometry, which are characteristic of bigger systems, as well as to handle wider variation of pipe diameters and materials for finer cost minimization.

Conclusion

This paper has presented two student LP formulations and implementations which utilize the computer package LPL. The decision problems actually solved are medium-scaled, in terms of the numbers of variables and constraints handled. However, it is observed that the LPL output provides decision information which are hardly "trivial" and not at once obvious prior to the LP solutions. Realizing the potential of these and other formulations, one should be encountered to extend and generalize them, given sufficient time and resources, so as to encompass more wide-ranging and complex water resource systems decision problems—in comprehensive planning, feasibility-level studies, and detailed engineering design, including operations studies.

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