"the main difference of this method over all the past methods is the quantification of the benefits of voltage profile improvement,"

An Improved Method for Optimizing the Benefits of Capacitor Application in Radial Feeders*

by Arsenio A. Abellana**

Abstract

The sizing, location and combination of capacitor banks to be placed in a radial distribution feeder for loss reduction and voltage control—is thoroughly investigated in this research work. The quantification of the benefits of voltage profile improvement and a thorough analysis for fixed and switched capacitor combinations are the main features of this work.

Solution algorithms together with the corresponding computer programs and derivations of the equations are presented here. A mathematical model of a single-ended radial feeder, with real and reactive loadings assumed known at one hour intervals, was simulated in the IBM 360 UP Engineering Computer.

The results show that the consideration of monetary gain, due to voltage profile improvement of a radial feeder is a very important aspect in the allocation of capacitors. This gain is even comparable to the savings due to peak-loss and energy-loss reductions. Best results are obtained when a combination of fixed and switched capacitors are used and located at different points along the feeder.

Introduction

Reactive loss minimization has always been a challenging problem to utility engineers. Practically, any load consumes reactive-volt amperes. The load reactive-volt ampere requirements necessitate the increase of the volt-ampere supply; this means a greater amount of current and consequently, higher energy loss (I^2RT) and voltage drop in the lines. The increase in the line

*MSEE Thesis, College of Engineering, University of the Philippines.

**Assistant Dean, Graduate School of Engineering, Cebu Institute of Technology.

voltage drop results into low voltages supplied to the load-centers and this may mean inefficient operation of the loads.

If by some means, the feeder is relieved of the high reactive current, then:

1. Energy losses in the feeder will decrease;
2. Peak power requirements will be reduced thus lessening the burden of generating plants;
3. More loads can be serviced by the increase in KVA input since such increase will no longer be used entirely to supply the kvar requirements;
4. The voltage profile throughout the feeder length will improve because of the voltage drop reduction.
5. The utility company will have an increase in revenue due to the increased voltages at the load centers.

Scope of the Study

This research is an attempt to improve the methods proposed by several authors to meet the problem of reactive loss minimization by the application of shunt capacitors.

A test system, of the low voltage primary distribution radial type where the losses due to the reactive current is great, is investigated.

The author attempts to relate all the practical aspects in the allocation of capacitors. Factors such as voltage control and the cost of the capacitors are thoroughly investigated together with the savings from energy loss reduction and peak power loss reduction and increase in revenue due to the increase in voltage brought about by the addition of capacitors.

Description of the Approach Used

A mathematical model of a single-ended radial feeder was made. Voltage control is achieved by a combination of switched and fixed capacitors; the fixed capacitors handling the light load conditions and the fixed and switched capacitors connected together during heavy loading conditions. Loss reduction equation, with the different possible combination of fixed and switched capacitors, were formulated. A significant characteristic of the approach is the analysis of the monetary gain or revenue increase due to the rise in voltage brought about by the addition of capacitors.

The objective function includes the loss reduction savings, the revenue increase and the capacitor cost. All possible combinations of capacitors are tested with the combination giving the maximum objective function as the solution.

Although a cut-and-try method, this approach shows a very favorable result in terms of increased savings and monetary gain using a fixed and switched capacitor combination in radial feeders.

Symbols used in the text of this paper are defined as follows:

A — distance of the investigated fixed capacitor size from the nearest left node.
AA — distance of the optimal fixed capacitor size from nearest left node.
AALF — segment reactive load factor before the addition of fixed capacitors.
AALFS — segment reactive load factor after the addition of fixed and switch capacitors beyond the segment.
AEISX — segment reactive current after the addition of fixed and switch capacitors beyond the segment.
AICA — current rating of maximum available capacitor size.
AICF — fixed capacitor current rating
AICS — switched capacitor current rating
AIL — real component of load current
AILX — reactive component of load current
AIS — real component of segment current
AISX — reactive component of segment current
AIXL — total feeder reactive current at light load
AIXP — total feeder reactive current at peak load
AK1 — constant to convert peak-loss reduction to pesos per year
AK2 — constant to convert energy-loss reduction to pesos per year
AK3 — ratio of switched capacitor cost to fixed capacitor cost
AK4 — price of energy sold in pesos per kilowatt-hour
ANCBF — number of fixed capacitor banks
ANCBS — number of switched capacitor banks
ARA — intercept of the linear approximation of the capacitor cost function
B — distance of the investigated switched capacitor size from the nearest left node
BB — distance of the optimal switched capacitor size from the nearest left node
CC — total capacitor node
CI — increase in energy delivered to a load due to an increase in voltage
DEI — energy-loss reduction savings
DIC — current rating of lowest rated capacitor unit
DLI — peak-loss reduction savings
DP — change in power delivered at a node due to a variation of the node voltage
DR — resistance per span of the distribution line
DX — reactance per span of the distribution line
EISX — segment reactive current after the addition of fixed and switched capacitors beyond the segment
I — number of nodes
IT — time throughout the load cycle
ITP — time when peak load occurs
ITS — total switching time per load cycle of switched capacitors
ITS1 — time switched capacitor is switched on
ITS2 — time switched capacitor is switched off
LM — pole number where optimal fixed capacitor is to be connected
LN — segment number where optimal fixed capacitor is to be connected
LLM — pole number where optimal switched capacitor is to be connected
LLN — segment number where optimal switched capacitors is to be connected

PK — power constant of the node during the time considered
RI — total increase in revenue due to the increase in voltage
RR — resistance of the line segment
SP — maximum value of objective function
SS — objective function value for a trial capacitor allocation
TT — total number of hours per year
V — node voltage before the addition of capacitors
VC — node voltage for a trial capacitor allocation
VRI — voltage at farthest node with one hundred percent compensation of the feeder
VVC — node voltage when optimal capacitor size are connected at proper locations

Review of Literature

Neagle and Samson[1] developed equations for the optimal capacitor allocations on uniformly distributed radial feeders for different numbers of capacitor banks. With a very ideal uniformly loaded feeder as a model, they drew these following conclusions.

1. The maximum loss reduction on a feeder with uniformly distributed load is obtained by locating the capacitor bank where its ckva is equal to twice the load reactive volt-amperes. The same principle holds where more than one capacitor bank is applied to the feeder.
2. Deviations of the capacitor bank location from the point of maximum loss reduction by as much as 10% of the total feeder length do not appreciably affect the loss benefits.
3. Capacitor ckva up to 70% of the total kvar load of the feeder can be applied as one bank with little sacrifice in the maximum feeder loss reduction possible with several capacitor banks.
4. In practice, in order to make the most out of the capacitors loss reduction and voltage benefits, it is best to move the capacitor bank just beyond the optimum loss reduction locations. Their approach did not consider voltage control and only peak-loss reduction was considered.

y A simplified approach to the problem of uniform loading was developed by L. J. Rankin.[2] Here, he developed the so-called two-thirds rule wherein he showed that for best reduction in line I^2R loss, a capacitor bank whose capacity is two-thirds of the peak reactive load is to be placed at two-thirds per unit distance from the source (with the total feeder length taken as one per unit distance). His next paper[3] suggested a method of handling non-uniform loading. This method is a moment method with the distances of the different loads from the source considered as the moment arm. Thus,

$$x_c q_t = \Sigma\, q_i x_i$$

where

X_c = location of capacitor

q_i = peak reactive load at point i
x_i = distance of the load from the source

Rankine's method did not consider the energy losses in a load cycle. Also, voltage control was never a factor in his analysis.

The first paper that investigated the effect of load-cycle in determining the capacitor allocations was by R. F. Cook.[5] Energy-loss reduction and peak-loss reduction were now incorporated to determine the optimal allocation. For n number of capacitor banks, the following equations will give minimal total losses.

With total feeder length considered as one per unit the optimal locations can be found by,

$$C_{11} = 1 - \frac{(2n-1)\,Cr}{2LF}\ \text{pu}$$

$$C_{12} = 1 - \frac{(2n-3)\,Cr}{2LF}\ \text{pu}$$

$$C_{13} = 1 - \frac{(2n-5)\,Cr}{2LF}\ \text{pu}$$

where

Cr = capacitor rating
C_{1n} = capacitor location
LF = feeder load factor
n = number of banks

The optimum capacitor ratings for maximum loss reduction are given by (total reactive load taken as one per unit),

$$Cr = \frac{2nLF}{2n+1}$$

Voltage control was not included in his method and only fixed capacitors were considered.

His next paper,[6] however, investigated thoroughly the general application of fixed and switched capacitors for reactive volt-ampere control and loss reduction. The cost of energy and the demand charge were included in the determination of the optimal capacitor allocation. The following equations give the optimum locations for different capacitor combinations.

With only fixed capacitors used

$$C_{11} = 1 - \frac{(2n-1)\ C_r\ (k1 + k2)}{2\ k1\ (LF) + k2}$$

$$C_{12} = 1 - \frac{(2n-3)\ C_r\ (k1 + k2)}{2\ k1\ (LF) + k2}$$

$$C_{1n} = 1 - \frac{C_r\ (k1 + k2)}{2\ k1\ (LF) + k2}$$

where

k1 = cost of energy per kilowatt-hour

k2 = demand charge per kilowatt

with switched and fixed capacitors used, the optimum locations are found by

$$C_{1f} = 1 - \frac{C_{rf}\ (k1 + k2) + C_{rs}\ k1\ (1-T_f) + k2}{2\ k1\ (LF) + k2}$$

$$C_{1s} = 1 - \frac{C_{rs}\ k1\ (1-T_f) + k2}{2\ k1\ (LF_s)\ (1-T_f) + k2}$$

where:

C_{1f} = fixed capacitor location

C_{1s} = switched capacitor location

C_{rf} = fixed capacitor rating

C_{rs} = switched capacitor rating

LF = load factor before the connection of the switched capacitor

LF_s = load factor after the connection of the switched capacitor

T_f = time fixed capacitor is connected to the feeder

A flowchart was given to aid engineers in establishing a computer program. Although Cook's analysis are very extensive, his methods neglected the ticklish problem of voltage regulation.

J. V. Schmill[7] made a study of the optimum location, size, and timing of capacitor banks of feeder with uniformly distributed loads and randomly distributed active and reactive losses where voltage regulation is no problem. A new simplifying principle is established, taking moments of the loads with respect to feeder resistances or reactances.

Maximum savings are obtained when

$$\frac{\partial CS}{I_{c1}} = 0 \frac{\partial CS}{I_{c2}} = 0 \ldots\ldots. \frac{\partial CS}{I_{cn}} = 0$$

where

I_{cn} = capacitor rating
CS = savings in dollars

Schmill assumes that there is no problem in voltage regulation. Such assumption is erroneous when distribution feeders are considered, since large voltage drops are encountered.

H. Duran[9] described a new method which determine the optimum number, location and size of shunt capacitors in a radial distribution feeder with discrete lumped loads so as to maximize over all savings, including the cost of capacitors. The optimization process is regarded as a multistage decision process with the desired Markovian property. Dynamic programming is issued. The objective function to be maximized in each step is the return function.

$$F = \Sigma \left[S_{1m}(I_{cm}) - C(I_{cm})\right]$$

where

$S_{1m}(I_{cm})$ = reduction of losses in branch m
$= 3r_m (2AI_{cm}I_{1m} - BI^2_{cm})$

A = k_1 LFT + k_2
B = k_1 T + k_2
k_1 = energy cost
k_2 = demand change
T = duration of load cycle
$C(I_{cm})$ = capacitor cost as a function of capacitor current

Duran's method is an improvement over the previous methods mentioned. However, a serious drawback is the non-inclusion of voltage control in his analysis.

N. E. Chang[10] summarized the losses in a primary distribution feeder so that utility engineers can have a clear perspective as to what losses can be minimized by using shunt capacitors. Chang[11] further developed a method of applying shunt capacitors for voltage control and peak loss reduction and this is extended to the optimization of total monetary savings due to both peak loss and energy loss reduction. His methods set lower and upper limits for node voltages and considers fixed capacitors or switched capacitors but not a combination of the two. Although voltage control is considered in Chang's method, there is no measure or quantification of the benefit of an improved voltage profile. Also, a combination of fixed and switched capacitors was not investigated.

Y. G. Bae[12] improved on the work of Neagle and Samson. He expressed in a simple mathematical equation. The maximum yearly loss reduction and optimum reactive-compensational level. Here, a wide range of reactive load conditions in investigated. The problem of voltage regulation was not tackled and the loading is considered uniform as well as the size of the capacitor banks. These assumptions fall short with the actual operating condition of a primary distribution feeder.

The method presented here is an improvement over all the methods previously mentioned. The main difference of this method over all the past methods is the quantification of the benefits of voltage profile improvement; this is incorporated with the objective function which is maximized to yield the optimal capacitor allocations. The general type of randomly distributed loading is investigated. The system simulated is a single-ended radial feeder with loads of different reactive levels. The method proposes a combination of fixed and switched capacitors with the constraint that at no time will the feeder power factor be leading. This constraint negates the possibility of over-voltages. The voltage constraint is set at + 0.05 pu from nominal. The switching time of the switched capacitor is determined by examining the reactive load cycle of the feeder. The switching time must be so set that no over compensation will happen at any instant throughout the load cycle, as seen from the substation.

Theoretical Considerations

The method used in this study consider the monetary savings due to both peak loss and energy loss reductions as well as monetary gain in increased revenue due to the increased voltage brought about by the addition of capacitors.

There are two constraints used in the study. The first constraint is that no node voltage should be less than 0.95 per unit (with the source voltage considered as one per unit) at any time during the load cycle. The second constraint is the limiting of the capacitor current such that always at any time during the load cycle no leading power factor shall occur. This will ensure against over voltages. This is done by using a combination of fixed and switched capacitors.

Real and reactive demands per hour in a 24-hour load cycle are assumed to be known. The load cycle considered is an average 24-hour loading for the particular month under study.

The angular displacement between load nodes is neglected since only short distances are involved and this (angular displacement) is sometimes less than a degree.

The increase in segment current, due to the rise in real and reactive components of the load current because of the voltage increase at the nodes, is neglected since it is but a small percentage of the original segment current and does not affect much the voltage equations as well as the loss reduction equations.

Loss Reduction with Installation of A Capacitor Bank

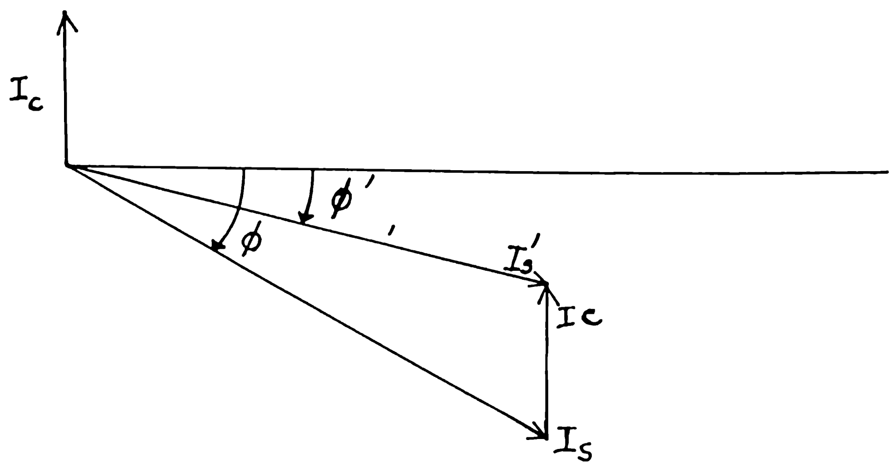


Figure 3.1. Phaser Diagram Showing Decrease in Segment Current with Installation of a Capacitor Bank at the end of the Segment.

The decrease in segment current is depicted in the phasor diagram shown in Figure 3.1.

The I^2R loss in a feeder can be resolved into two parts, namely:

1. I^2R loss due to the real current component.
2. I^2R loss due to the reactive current component.

So,

$$I^2R = (I\cos\Theta)^2R + (I\sin\Theta)^2R \quad (3.1)$$

When a shunt capacitor with current I_c is introduced, resulting in a new line current I_1, the power loss of the line is:

$$I_1{}^2R = (I\cos\Theta)^2R + (I\sin\Theta) - I_c{}^{2R} \quad (3.2)$$

The loss reduction due to the effect of the capacitor is:

$$L = I_1{}^2R - I^2R$$

or

$$L = 2\,(I \sin\Theta)\,I_cR - I_c{}^2R \qquad (3.3)$$

Equation (3.3) shows that only the quadrangle component of the line current, is ...., sin Θ, is to be considered in the loss reduction analysis due to the introduction of a shunt capacitor. The proceeding formulations of loss reduction considers the quadrature component only.

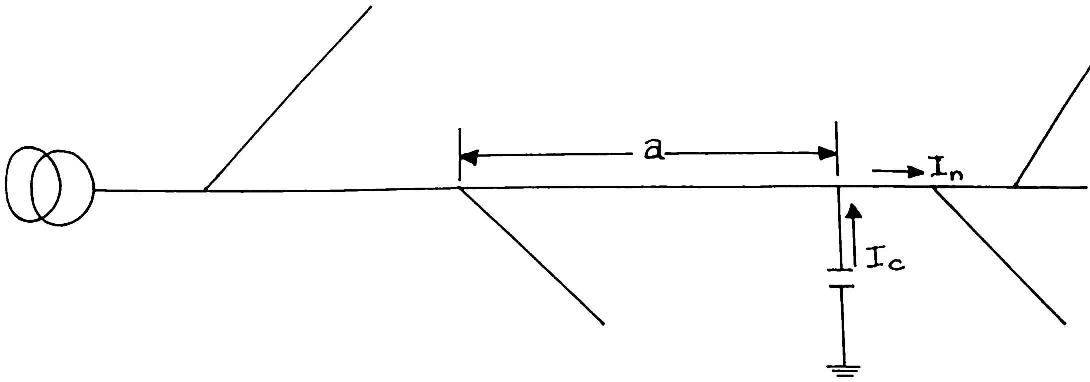


Fig. 3.2. Radial Feeder with Capacitor Bank at Segment n

Peak-loss and Energy-loss reductions due to fixed and Switched Capacitors located at a single point. *

The peak power loss reduction in the line segment is,

$$\Delta P_l = \left\{3aR_n\,[I_l - (I_{cs} + I_{cf})]^2 + 3(1\text{-}a)\,I_q^2R_n\right\}/1000 \text{ KW} \quad (3.4)$$

The peak loss reduction in any line segment j on the source side of line segment n[(c)] due to adding fixed and switched capacitors at a single point in line segment n, is,

$$\Delta P_j = \frac{3R_j}{1000}[2I_j\,(I_{cs} + I_{cf}) - (I_{cs} + I_{cf})^2] \qquad (3.5)$$

---

* All loss reduction derivations are found in Appendix A

The corresponding energy loss reduction in line segment n due to the addition of fixed and switched capacitors is:

$$\Delta Eln = \frac{3aRn}{1000}\left\{T_f\left[(I_q Lf)^2 - I'_q\ LFs)^2\right] - T_s(2I'_q LFsIcs + I^2_{cs})\right\} \qquad (3.6)$$

where

I_{cf} = fixed capacitor current

I_{cs} = switched capacitor current

LF = original reactive load factor

$= \frac{\text{average reactive load}}{\text{maximum reactive load}}$

LFs = load factor when switched capacitor is connected

$= \frac{\text{average reactive load (with fixed capacitors)}}{\text{maximum reactive load}}$

T_f = time fixed capacitor is connected

T_s = time switched capacitor is connected

I'_q = maximum reactive current at segment n when switched capacitor is connected

The corresponding energy loss reduction in any line segment j on the source side of line segment n due to adding fixed and switched capacitors at a single point in line segment n is:

$$\Delta Elj = \frac{3R_j T_s}{1000}(I_j I_{cf} LF - I^2_{cf}) + \frac{3R_j T_s}{1000}(I'_j I_{cs} LF_s - I^2_{cs}) \qquad (3.7)$$

Peak Loss and Energy-Loss Reduction due to Fixed and Switched Capacitors placed at different points in the Line Segment n.

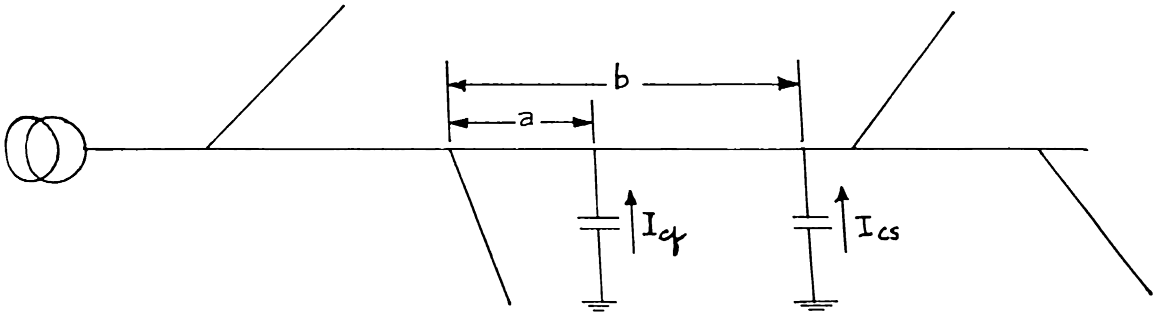


Figure 3.3. Radial feeder with fixed and switched capacitors at different points on segment n

The peak loss reduction is

$$\Delta P_1 = \frac{3aRn}{1000}(2I_n I_{cf} - 2I_{cf} I_n - I^2{}_{cs}) + \frac{3bRn}{1000}$$

$$+ \frac{3bRn}{1000}(2I_n I_{cs} - I^2{}_{cs}) \qquad (3.8)$$

The corresponding energy-loss reduction is

$$\Delta E_1 = \{aR_n T_f[(I_q LF)^2 - (I'_q LF_s)^2]$$
$$+ R_n [T_s\ 2aI'_q\ LF_s I_{cs} - a I^2{}_{cs}$$
$$+ 2bI_q\ LF\ I_{cs} - bI^2{}_{cs} - 2aI_q LF\ I_{cs}$$
$$+ a\ I^2{}_{cs}]\} \frac{3}{1000} \qquad (3.9)$$

Peak loss and Energy-Loss Reduction due to fixed and Switched Capacitors placed at different Segments.

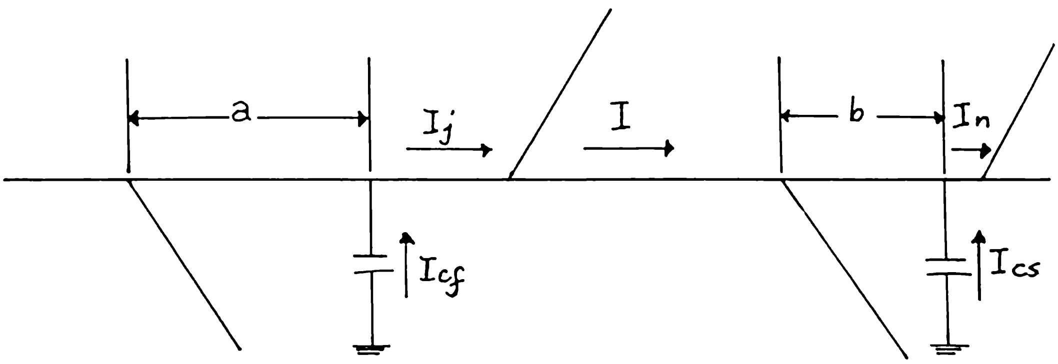


Figure 3.4. Radial feeder with fixed and switched capacitors at different segments

The peak power loss reduction with capacitors is

$$\Delta P_1 = \frac{3}{1000} [R_j (2 I_j I_{cf} - I^2_{cf} - 2aI_{cf} I_{cs}$$

$$+ 2I_j I_{cs} - 1^2_{cs}) + \sum_{n-1}^{n-1} (21_k I_{cs} - I^2_{cs}$$

$$+ 2I_j I_{cs} - I^2_{cs}) + \sum_{K=j+1}^{n-1} (21_k I_{cs} - I^2_{cs}) R_k$$

$$+ R_n b(2I_n I_{cs} - I^2_{cs})] \qquad (3.10)$$

The corresponding energy-loss reduction is

$$E1 = \frac{3aRn}{1000} T_f [I_q LF)^2 - (I'_q LF_s)^2] + \frac{3RnTs}{1000}$$

$$[2a I'_q LF I_{cs} - aI^2_{cs} + 2bI_q LF I_{cs} - bI^2_{cs}$$

$$- 2aI_q LF I_{cs} - aI^2_{cs}] \qquad (3.11)$$

Increase in Revenue due to the Voltage Increase brought about by Capacitor Addition

Most types of loading such as lighting and heating loads can be represented by a constant impedence. The loading at a specific point in time, in this study, is represented by a constant impedance since it is the most simple representation. This is shown in Figure 3.4.

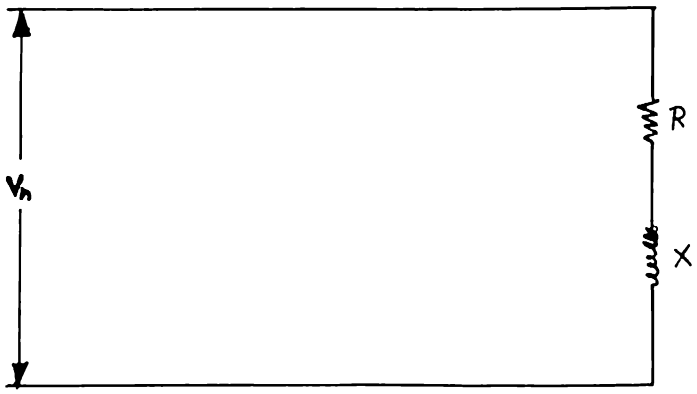


Figure 3.5. Constant Impedance Load at Node n at a Time t

If we let,

V_n voltage across node n before capacitor addition

$V^1{}_n$ voltage across node n after capacitor addition

R resistance load of node n at the particular time considered

X reactance load of node n at the particular time considered.

The power absorbed by the load before addition of capacitors is

$$P = \frac{V_n^2}{R^2 2 X^2} R \tag{3.12}$$

The power absorbed by the load after addition of capacitors is

$$P' = \frac{V'^2_n}{R^2 + X^2} R \tag{3.13}$$

The change in power for the particular time considered is

$$P = P' - P = \frac{R}{R^2 + X^2} (V_n'^2 - V_n^2) \tag{3.14}$$

The factor $R/R^2 + X^2$ is constant for a particular time. To get the power increase at a discrete point in time, we proceed as follows

$$P = K_{(n,t)} (V'^2_{(n,t)} - V^2_{(n,t)})$$

where

$$K_{(n,t)} = \frac{R}{R^2 + X^2} = \frac{\text{real } I(n,t)}{V_{(n,t)}} \qquad (3.15)$$

The change in power, AP, is evaluated at discrete points during the load cycle. A graphical representation is shown in Figure 3.6.

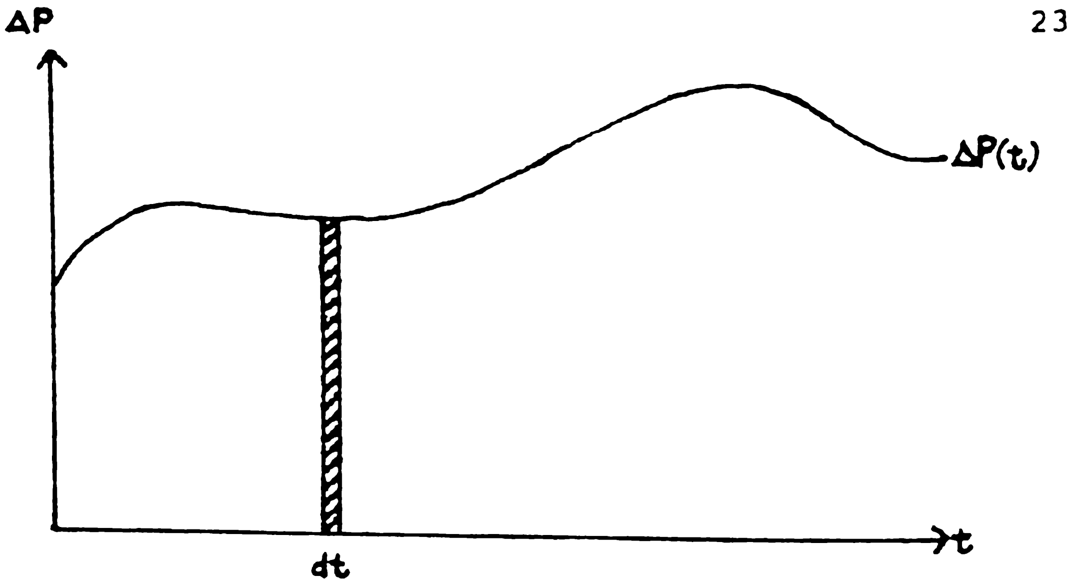


Figure 3.6. Change in Power at Node in throughout the Load Cycle

The increase in revenue is

$$RI = K' \int_{O}^{T} Pdt \qquad (3.16)$$

where K' is the average price of energy

The integration of the P curve is done using numerical integration. Because of its simplicity in programming, the trapezoid rule with end correction is used in the program.

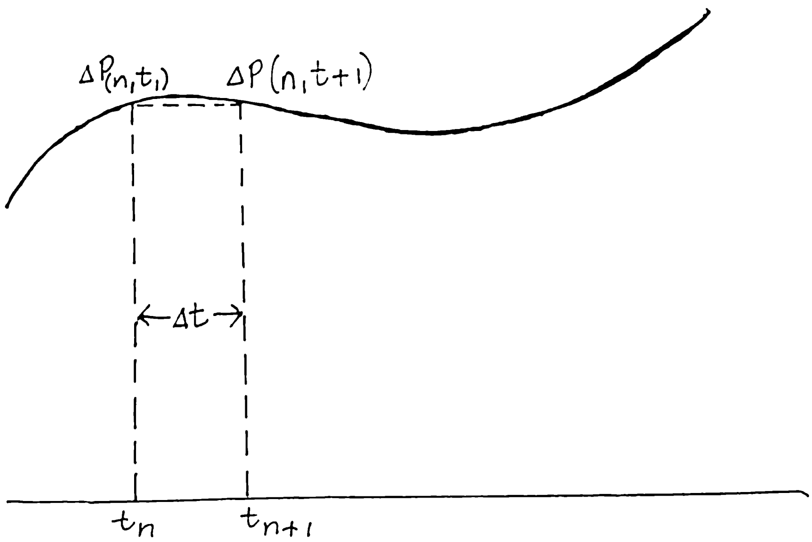


Figure 3.7. Trapezoidal Rule in Solving for the Area Under the P Curve.

The area under the P curve is then**

$$A = \Delta t/_2 \left[f(a) + f(b) + 2 \sum_{n=1}^{j-1} f_n \right] - \frac{(\Delta t)^2}{12} \left[f'(b) - f'(a) \right]$$

where

a = lower limit
b = upper limit

Determination of Capacitor Cost Function

The prices of capacitor banks of different capacities are obtained from the Westinghouse Catalog. These values are shown in Figure 3.1 and plotted in Figure 3.7.

The prices of capacitor banks of different capacities are obtained from the Westinghouse Catalog. These values are shown in Figure 3.1 and plotted in Figure 3.7.

**Derivation found in Appendix C

From the resulting curve behavior of a Bauss least square linear fit is seen to be the best approximation of the curve's equation. This sets the capacitor cost function to be equal to a fixed cost plus a linear variable dependent on the kvar rating.

Using Gauss least square linear approximation, we have

$$\begin{bmatrix} & \Sigma x^2_i \\ \Sigma x_i & x^2_i \end{bmatrix} \begin{bmatrix} a_o \\ a_i \end{bmatrix} = \begin{bmatrix} \Sigma F x_i \\ \Sigma x_i F x_i \end{bmatrix}$$

where

x_i = data points (kvar rating)
Fx_i = corresponding capacitor cost for a rating of x
a_o = constant, equivalent to fixed cost
a_l = slope
n = numbr of data points

From the Westinghouse Catalog the capacitor bank sizes and corresponding prices of 4160 volt delta connected banks are given:

KVAC	*Price ($)*
25	369
50	400
75	438
100	492
125	592
150	692
175	869
200	846
225	1092
250	1200
300	1338
350	1661
400	1785

Table 3.1. Cost of 4.160 KV Capacitor Banks

Solving for the parameters in a least square fit and substituting (3.40), we have

Price = 1611.1 + 262 Ic Pesos

where Ic is capacitor current (3.17)

The capacitor is estimated to last for 20 to 25 years and using straight line depreciation and considering zero salvage value,

$$\text{Cost/year} = ₱(1611.1 + 252\ \text{Ic})/20$$
$$= ₱(80.55 + 12.6\ \text{Ic}) \qquad (3.18)$$

Assuming an installation cost of ₱200 per capacitor unit, the cost will then be

$$\text{Cost/year} = ₱(80.55 + 12.6\ \text{Ic}) \qquad (3.19)$$

The paralleling of capacitor units to increase the capacity results in a change of the constant term of eq. (3.18), thus

$$\text{Cost/year} = ₱(n\,90.55 + 12.6\ \text{Ic})$$

where n is the number of units

Objective Function

The solution of the problem of sizing and locating capacitors for best savings and voltage profile can be expressed as maximizing the objective function

$$F(X, \text{Ic}, V_n) \qquad (3.20)$$

Subject to the constraints

$$0.95\ \text{p.u.} \leqslant V_n \leqslant 1.05\ \text{p.u.} \qquad (3.21)$$

and that the power factor as seen at the substation should in no case be leading.

The objective function is expressed as

$$F(x, I_c, V_n) = K_1 \sum_j \Delta P_L + K_2 \sum_j \Delta E_L + \sum_j RI - CC \qquad (3.22)$$

where

X = distance of the capacitor from the substation
V_n = voltage at any node n
I_c = capacitor current
P_L = peak loss reduction
E_L = energy loss reduction
CC = capacitor cost
RI = revenue increase
j = number of segments
K_1 = constant converting peak loss reduction to Pesos/year
K_2 = constant converting energy loss reduction to Pesos/year

The Test System

The test system considered in the study is a 5 MVA, 4160 volt 3-phase system. There are four concentrated loads. The per hour real and reactive

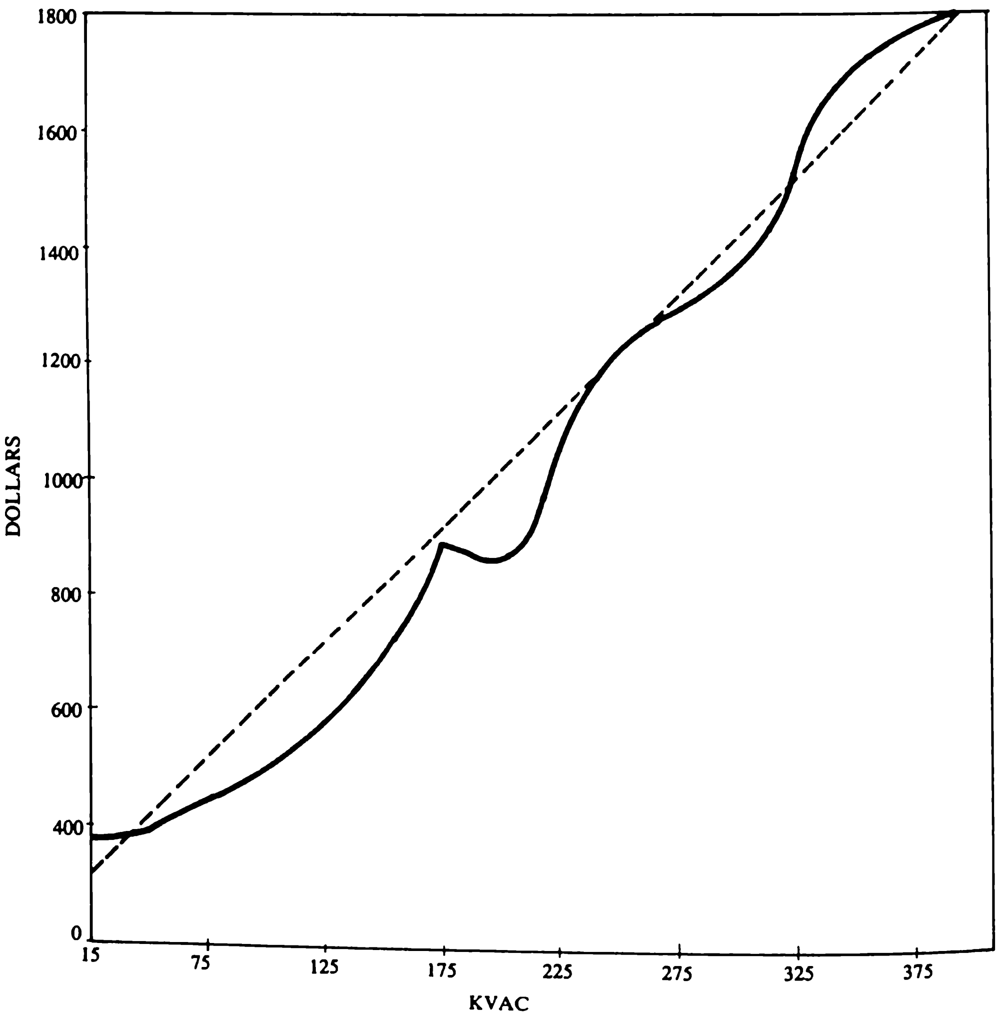


Figure 3.8. Cost of 4.16 KV Capacitors

loading level are assumed to be known. The distance are expressed in resistance and reactance with the pole to pole resistance equal to 0.0025 ohm and the pole to pole reactance equal to 0.005 ohm. The source transformer is assumed to be tap-changing under load transformer; thus the voltage is considered constant. The source is considered as the first node. The first segment is numbered two. The conductor is considered to be of uniform cross-section, although the method is not restricted to this condition.

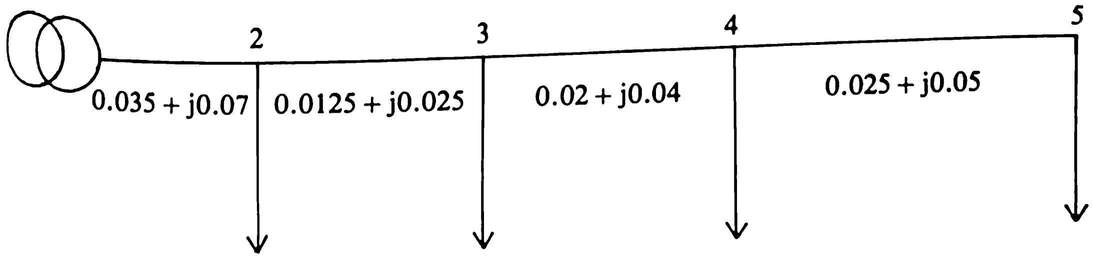


Figure 4.1. Single-line Diagram of the Test System

The following tables and graphs represent the reactive loading levels of the nodes and segments at one-hour intervals in a 24-hour load cycle:

Table 4.1. Real and Reactive loading and voltage at node 2

Time	*Real I*	*Reactive I*	*Voltage*
12	80	60	2371.5
1 AM	80	60	2371.5
2	100	80	2368.7
3	110	85	2364.2
4	120	90	2356.7
5	200	150	2340.6
6	210	160	2334.5
7	220	165	2330.7
8	220	170	2329.6
9	180	140	2336.7
10	180	140	2336.0
11	200	150	2334.8
12	210	160	2333.1
1 PM	190	145	2335.9
2	230	180	2324.0
3	250	185	2311.5
4	260	190	2307.3
5	270	200	2312.4

6	270	200	2318.1
7	180	140	2335.1
8	150	145	2345.5
9	120	90	2362.6
10	100	80	2367.5
11	80	60	2371.7

Table 4.2. Real and Reactive loading and voltage at node 3

Time	*Real I*	*Reactive I*	*Voltage*
12	90	70	2360.7
1 AM	75	50	2372.9
2	75	50	2360.8
3	105	75	2354.9
4	120	80	2345.0
5	172	120	2325.5
6	200	145	2317.6
7	200	145	2312.7
8	200	145	2311.4
9	175	115	2318.9
10	175	115	2319.8
11	180	120	2317.7
12	190	125	2315.7
1 PM	180	120	2318.9
2	210	155	2304.2
3	220	160	2289.8
4	250	180	2284.5
5	250	180	2290.1
6	250	180	2297.1
7	190	125	2317.7
8	160	130	1331.5
9	110	75	2353.0
10	100	70	2359.1
11	80	60	2364.1

Table 4.3. Real and Reactive loading and voltage at node 4

Time	*Real I*	*Reactive I*	*Voltage*
12	100	75	2355.9
1 AM	100	75	2355.9

2	110	80	2351.6
3	140	90	2344.5
4	180	120	2331.8
5	220	130	2309.8
6	230	150	2300.5
7	250	185	2293.7
8	260	190	2299.5
9	260	190	2299.5
10	250	185	2300.8
11	250	185	2298.8
12	250	185	2296.8
1 PM	230	180	2300.1
2	270	210	2282.8
3	290	230	2266.1
4	300	240	2260.1
5	300	240	2265.7
6	280	630	2275.8
7	260	220	2298.5
8	200	170	2317.5
9	150	100	2342.8
10	120	80	2350.3
11	100	75	2355.0

Table 4.4. Real and Reactive loading and voltage at node 5

Time	*Real I*	*Reactive I*	*Voltage*
12	75	50	2348.5
1 AM	80	55	2345.2
2	80	55	2346.9
3	90	55	2339.5
4	110	65	2325.8
5	150	80	2301.0
6	160	85	2292.2
7	160	85	2285.5
8	160	85	2283.8
9	160	85	2291.3
10	160	85	2290.6
11	160	85	2290.5
12	160	85	2288.5

1 PM	170	90	2291.4
2	180	100	2273.3
3	200	120	2255.1
4	200	120	2254.8
5	200	120	2254.8
6	200	90	2267.5
7	140	70	2292.0
8	145	45	2313.5
9	70	45	2338.8
10	80	45	2346.8
11	60	45	2351.9

Table 4.5. Real and Reactive loading of segment 2

Time	*Real I*	*Reactive I*
12	330	235
1 AM	335	240
2	365	265
3	415	305
4	530	355
5	745	480
6	800	540
7	830	580
8	840	590
9	775	530
10	765	525
11	790	540
12	810	555
1 PM	770	535
2	890	645
3	1160	695
4	1210	730
5	1020	740
6	950	700
7	750	555
8	580	490
9	450	310
10	380	275
11	330	240

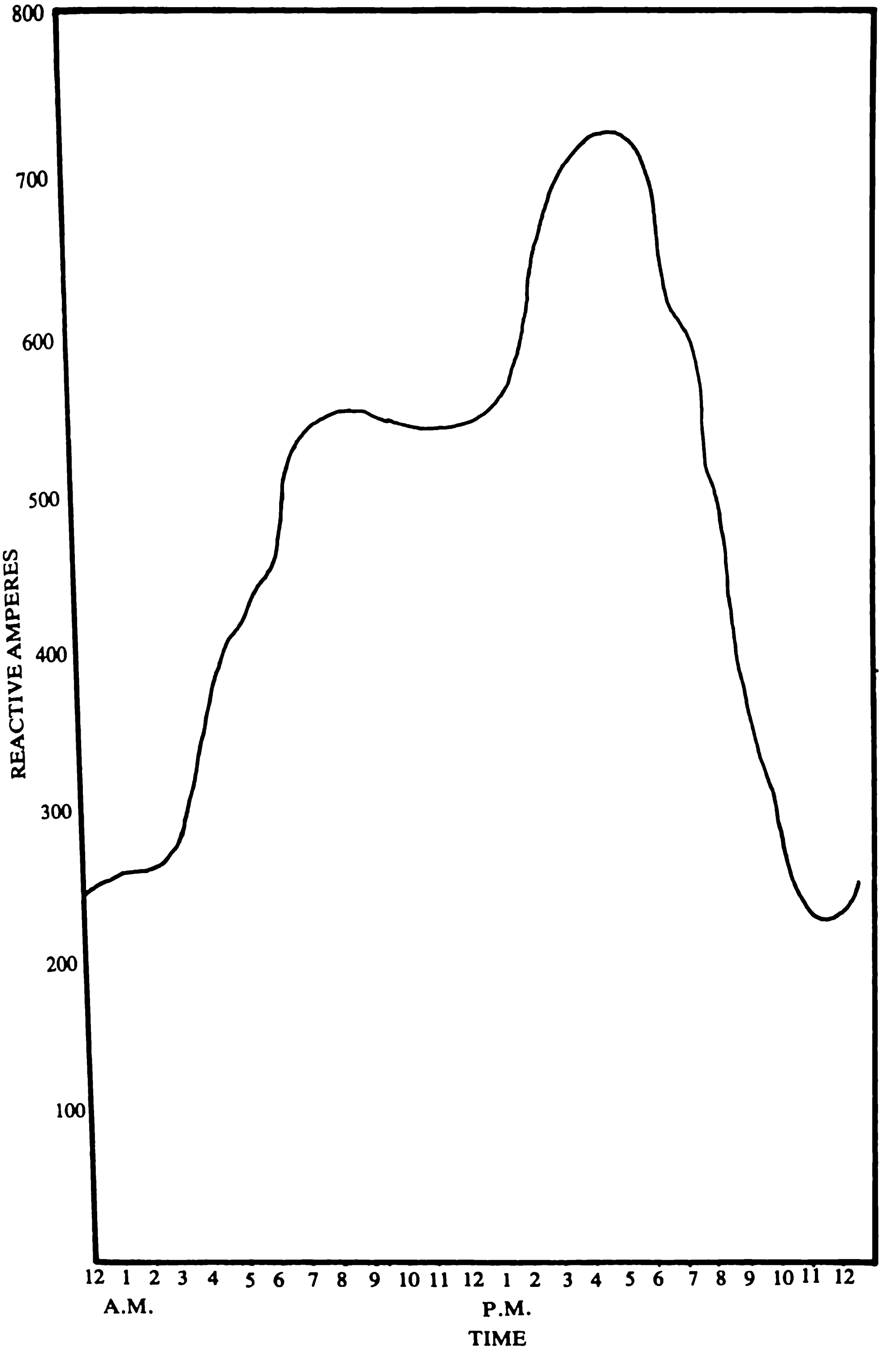


Figure 4.2. Reactive Load Cycle on Test Feeder

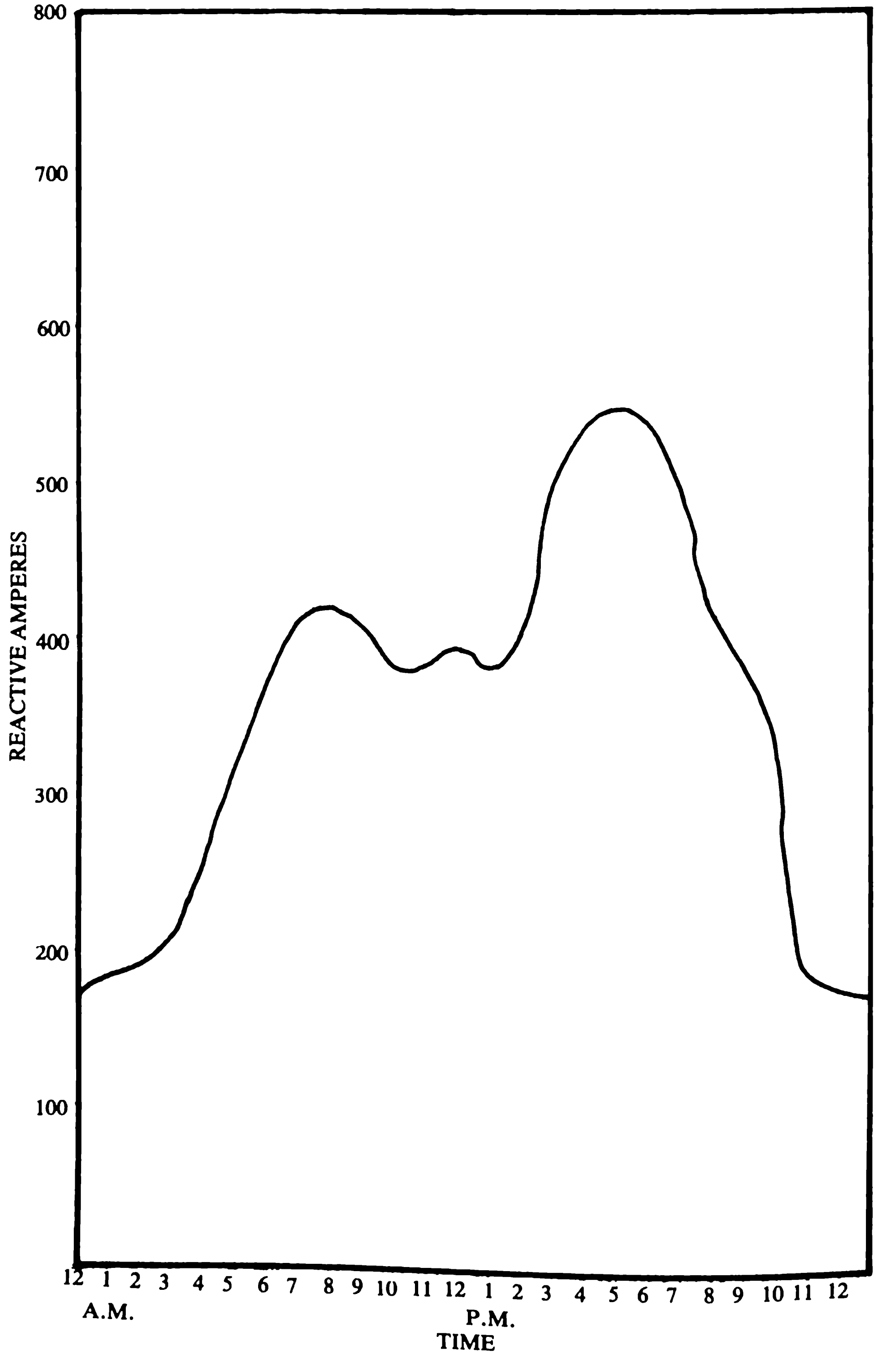


Figure 4.3. Reactive Load Cycle of Segment 3

Table 4.6. Real and Reactive loading of segment 3

Time	*Real I*	*Reactive I*
12	250	175
1 AM	255	180
2	265	185
3	305	220
4	410	265
5	545	330
6	590	380
7	610	415
8	620	420
9	595	390
10	585	385
11	600	395
12	600	395
1 PM	580	390
2	660	465
3	710	510
4	750	540
5	750	540
6	680	500
7	570	415
8	430	345
9	330	220
10	280	195
11	250	180

Table 4.7. Real and Reactive loading of segment 4

Time	*Real I*	*Reactive I*
12	175	125
1 AM	180	130
2	190	125
3	230	145
4	290	125
5	370	210
6	390	235
7	410	270
8	420	275
9	420	275
10	410	270
11	410	270
11	410	270
12	410	270
1 PM	400	270
2	450	310
3	490	350
4	500	360
5	500	360
6	430	320
7	380	290
8	270	215
9	220	145
10	190	125
11	170	120

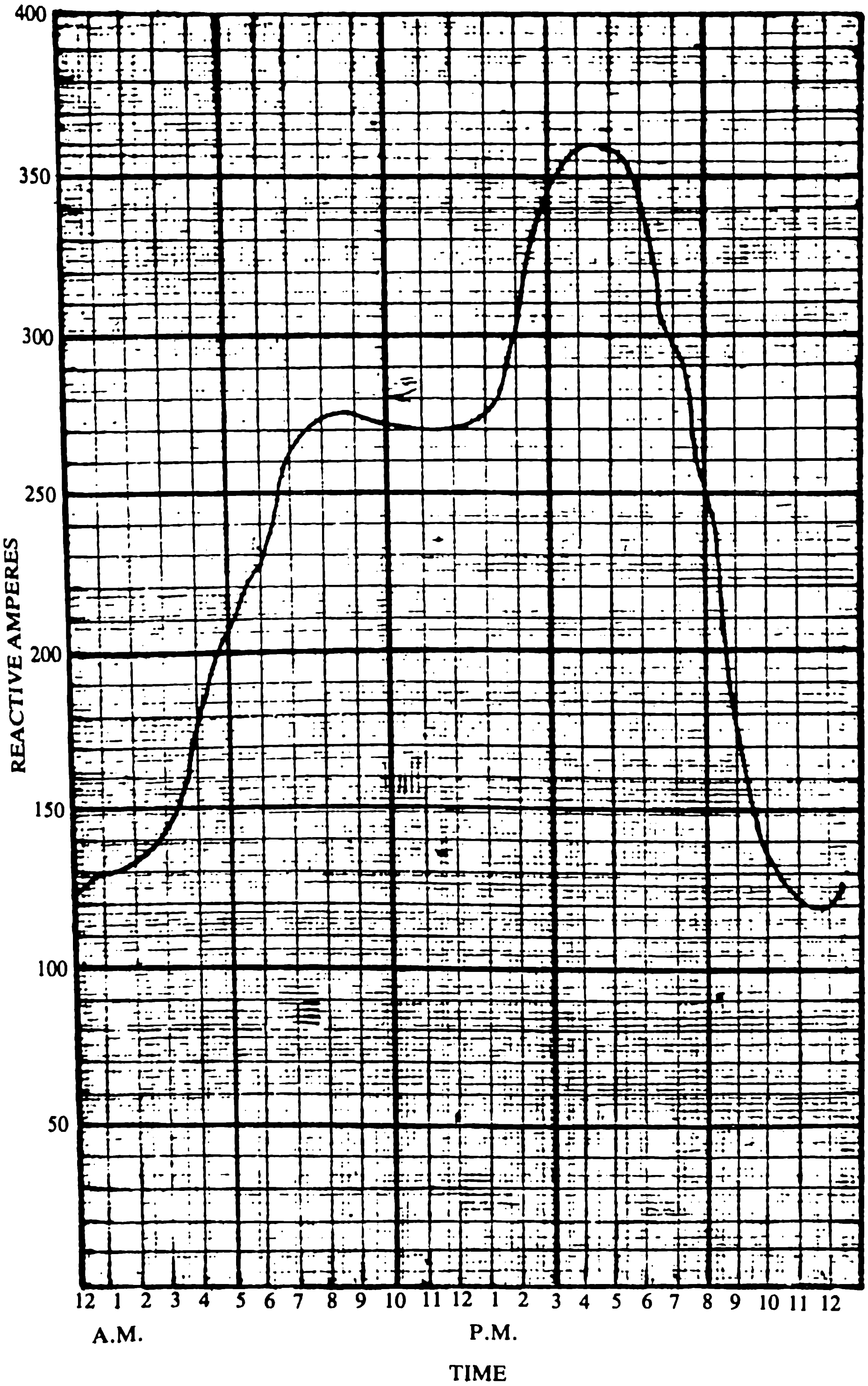


Figure 4.4. Reactive Load Cycle of Segment 4

Table 4.8. Real and Reactive loading of segment 5

Time	*Real I*	*Reactive I*
12	75	50
1 AM	80	55
2	80	55
3	90	55
4	110	65
5	150	80
6	160	85
7	160	85
8	160	85
9	160	85
10	160	85
11	160	85
12	160	85
1 PM	170	90
2	180	100
3	200	120
4	200	120
5	200	120
6	150	90
7	120	70
8	70	45
9	70	45
10	70	45
11	70	45

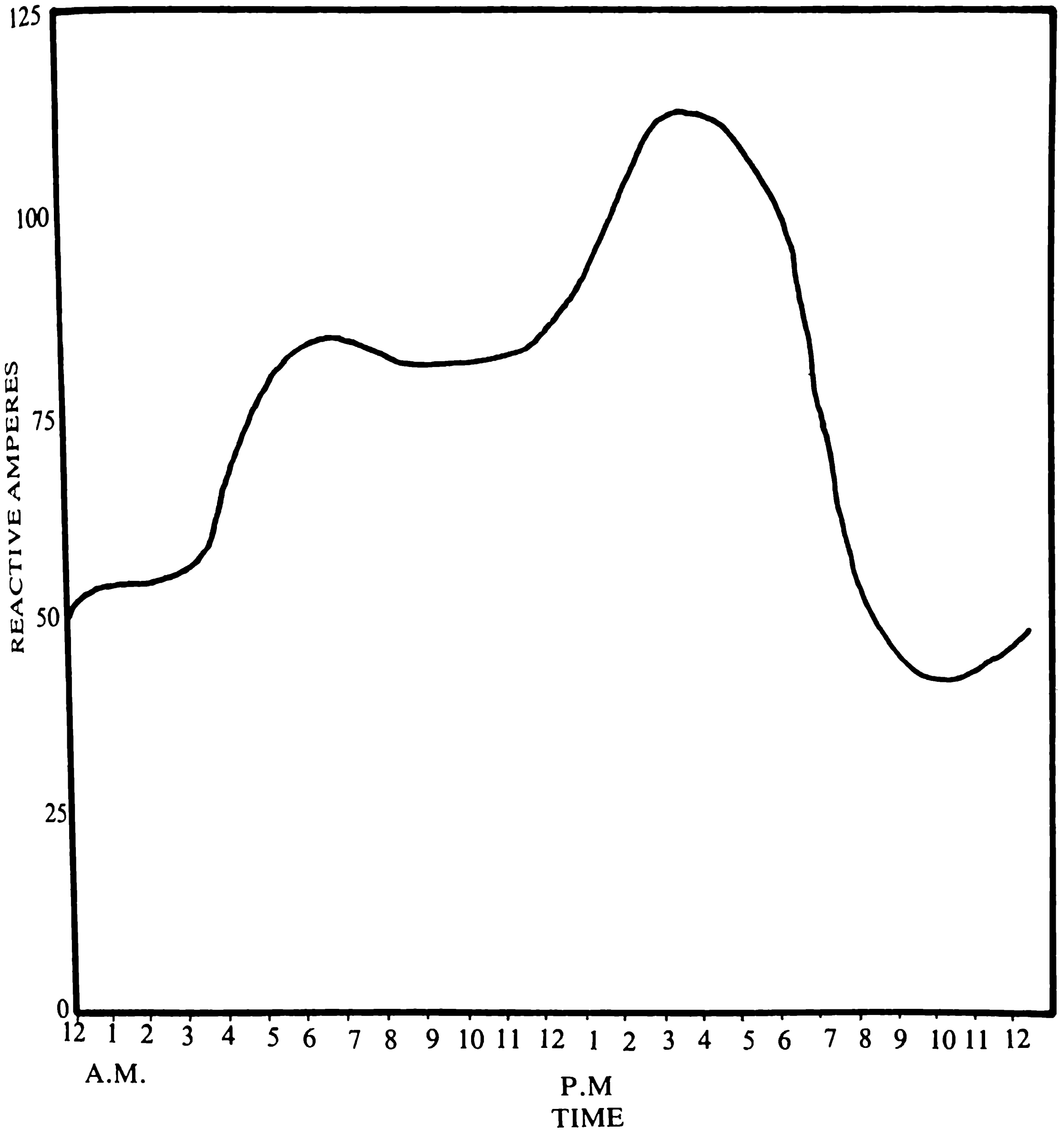


Figure 4.5. Reactive Load Cycle of Segment 5

Solution Methods

Feeders with voltage problem that can be checked by fixed and switched capacitors in one location

The original voltage profile of the feeder throughout the load cycle is first evaluated.

With the feeder power factor considered as unity, the voltage of the last node is calculated. This value determines whether 100% reactive compensation (the maximum compensation level) by capacitors can solve the voltage problem.

Node voltages at peak load are compared to the minimum constraint (0.95 p.u.) to determine violations. If all node voltages are within the limit set then there is no need to add capacitors and the program proceeds to examine the next feeder. If there are constraint violations, capacitors are tested first at the start of the segment before the first node, viewed from the source side, that violated the constraint.

For every trial capacitors, the voltage profile is calculated. Before constraint violations of the node voltages are examined, the peak load voltage of the last node at unity power factor (solved earlier) is compared with the constraint. If the constraint is violated this means that 100% compensation cannot hold the voltage within limits. The objective function value is then calculated. The size of the capacitors is increased until it reaches the maximum feeder reactive current value after which another location (one pole farther) is tested starting with the lowest capacitor size. This is repeated until all possible locations are tested. The combination giving the maximum value of the objective function is the one chosen. Although not all voltages are held within limits, the feeder will still benefit from the monetary savings due to loss reductions and monetary gain from increase revenue due to voltage profile improvement.

If the last node peak load voltage at unit power factor is within the constraints, then the program proceeds to examine for node voltage constraint violations. If there are still constraint violations then the size of the capacitor is increased. As the capacitor size exceeds the feeder light load reactive current, the capacitor is divided into fixed and switched. The fixed capacitor size is set equal to the feeder light load reactive current. The switched capacitor value is then increased until all node voltages are within limits and with this satisfied the objective function value is calculated. The switched capacitor size is increased further with the same process until the total capacitor size is equal to the maximum feeder reactive current after which the capacitor is moved one pole farther and the process is repeated until all possible locations have been tested. The combinations that gives the maximum objective function value is the desired capacitor allocation. The output gives the fixed and switched capacitors sizes; the segment number and pole number where such capacitors are to be connected; the switching time of the switched capacitors, the voltage profile with the capacitors connected, the peak-loss and energy-loss reduction savings and the monetary gain per node due to increase revenue brought about by the voltage profile improvement.

The simplified flow chart is shown in Figure 5.1.

Feeders with voltage problem that can be checked by fixed and switched capacitors at different locations.

As in the method previously discussed the original voltage profile is calculated. The peak load voltage of the last node at unity power factor is also calculated for the same reason as the first method.

This method can be divided into two major processes, namely: The search for best fixed capacitor location and size; and the search for best switched capacitor location and size. The first major process proceeds as follows:

A fixed capacitor size equal to the smallest capacitor rating is tested at the first load node. For every test combination, the voltage profile is calculated. The objective function value is then solved for every trial combination. The capacitor size is then increased until it is equal to the light load feeder reactive current. Another location is tried with the process repeated until all possible locations have been tested. The combination with the highest objective function value is the desired allocation for the fixed capacitor.

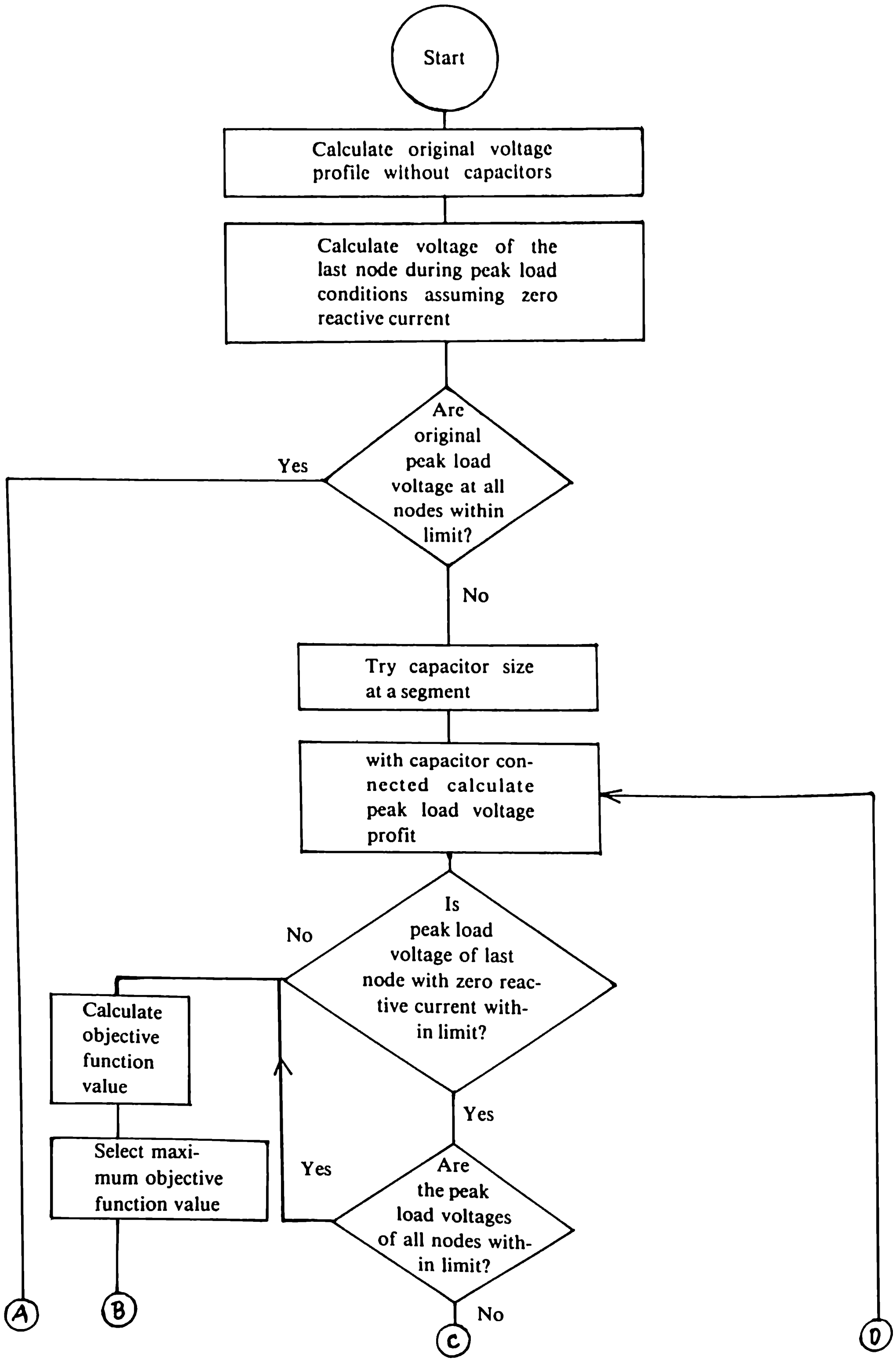
After the fixed capacitor combination is determined, the second major process starts. A switched capacitor corresponding to the smallest available capacitor size is tested on the first load node. For every trial the voltage is calculated and constraint violations are searched. If there are nodes violating the constraints, the switched capacitor size is increased until all the node voltages are within limits. If the capacitor size is equal to the maximum reactive feeder current then another location is tested. The objective function value is calculated only when all node voltages are within limits. All possible locations are tested and the combination that gives the maximum objective function value is the desired allocation for the fixed capacitor.

The output gives the same form of information as the method previously discussed.

Treatment of Feeder to be Tested

For every feeder, the two methods outlined are used. Evidently, the second method is more intensive than the first method and it is expected that the second method gives a better result. The first method is just an extension of N. Chang's method with the addition of increased revenue to the objective function and the combination of fixed and switched capacitors as the only differences.

The second method is the author's proposed method.



Start
Calculate original voltage profile without capacitors
Calculate voltage of the last node during peak load conditions assuming zero reactive current
Are original peak load voltage at all nodes within limit?
Yes
No
Try capacitor size at a segment
with capacitor connected calculate peak load voltage profit
Is peak load voltage of last node with zero reactive current within limit?
No
Yes
Calculate objective function value
Select maximum objective function value
Are the peak load voltages of all nodes within limit?
Yes
No
A
B
C
D

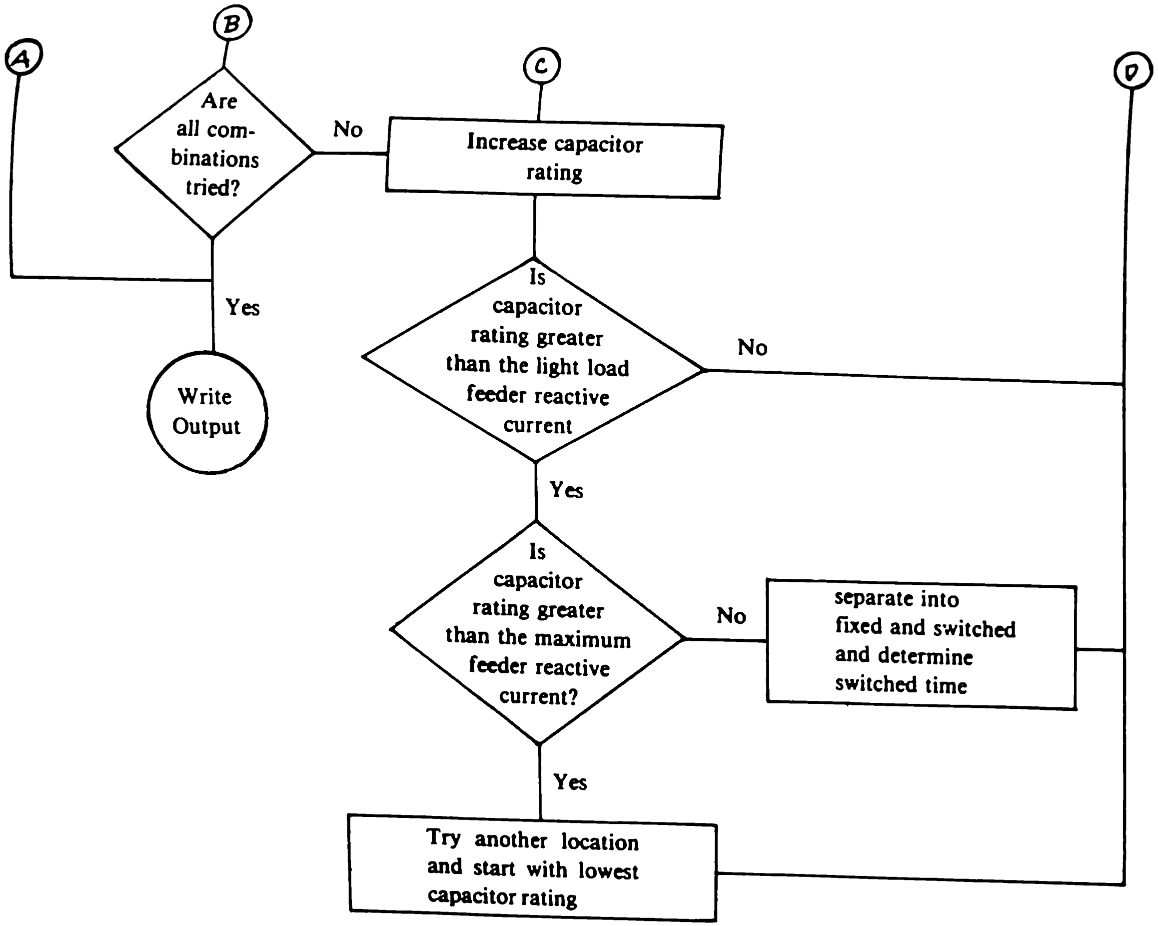
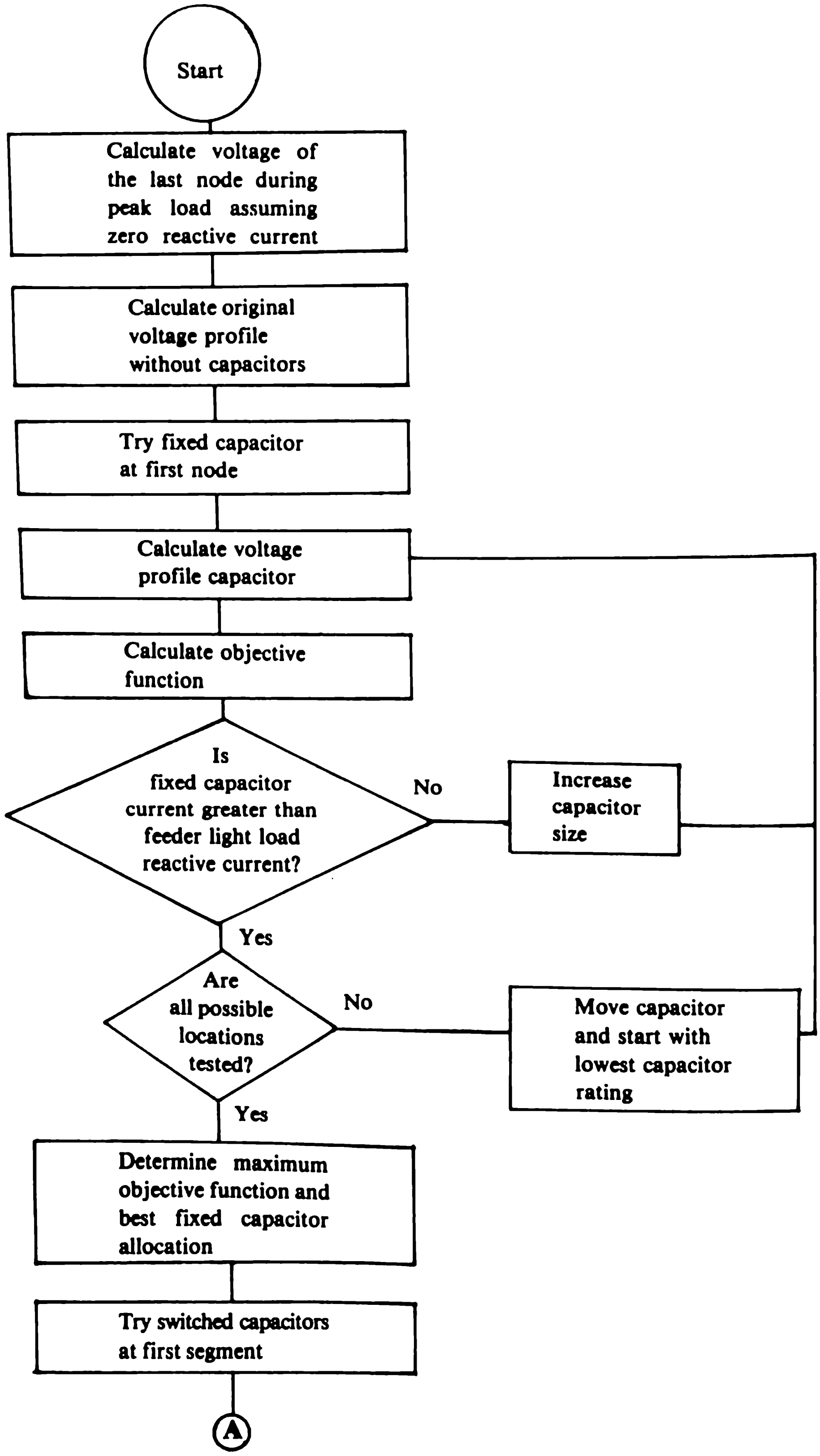


Figure 5.1. Flowchart for Subroutine No. 1


Start
Calculate voltage of the last node during peak load assuming zero reactive current
Calculate original voltage profile without capacitors
Try fixed capacitor at first node
Calculate voltage profile capacitor
Calculate objective function
Is fixed capacitor current greater than feeder light load reactive current?
No
Increase capacitor size
Yes
Are all possible locations tested?
No
Move capacitor and start with lowest capacitor rating
Yes
Determine maximum objective function and best fixed capacitor allocation
Try switched capacitors at first segment
A

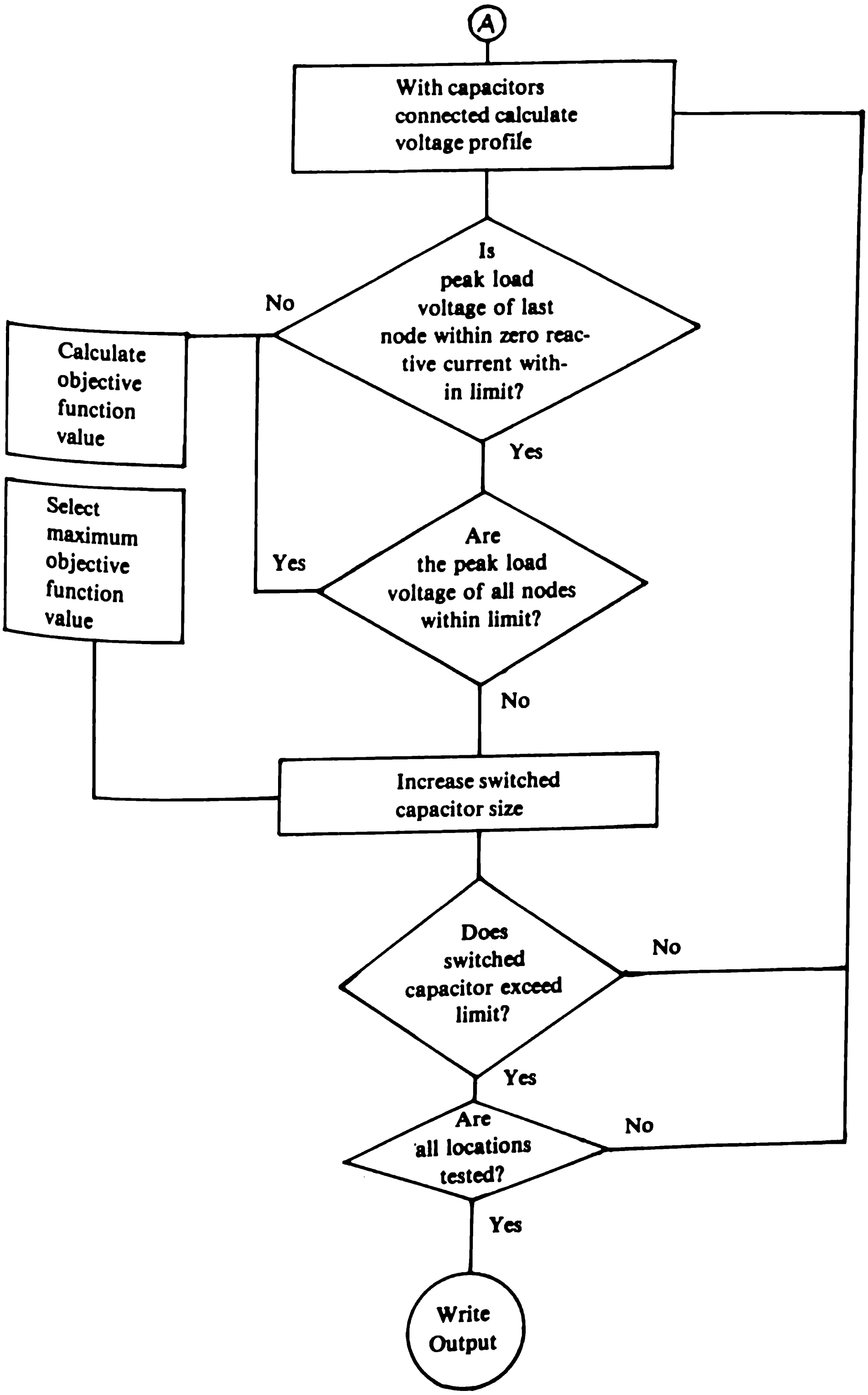


Figure 4.2. Flowchart for Subroutine No. 2

Results of Computer Runs

Shown from table 6-1 to table 6-6 are the computer results of the first algorithm.

Table 6-1 shows the voltage per node throughout the load cycle after the addition of the optimal capacitors.

Table 6-2 shows the fixed and switched capacitor ratings to be connected for maximum savings; the segment number and the pole number where such capacitors are to be connected and the switching time of the switched capacitors.

Peak-loss and Energy-loss reduction savings are given a table 6-3 and table 6-4 gives the increase in revenue per node due to the increase in voltage throughout the load cycle.

In table 6-5, the increase in power, delivered to the different load nodes (at a time interval of one hour) throughout the load cycle, is shown.

For the second algorithm the results are shown from table 6-6 to table 6-10.

The voltage increase, per node per hour throughout the load cycle, after the addition of the optimal capacitors is shown in table 6-6.

Table 6-7 shows the optimal fixed and switched capacitor allocation and the switching time of the switched capacitor.

Table 6-8 shows the peak-loss and energy-loss reduction savings and table 6-9 gives the increase in revenue per node due to the increase in voltage throughout the load cycle.

The increase in power, delivered to the different load nodes throughout the load cycle, is shown in table 6-10.

Corresponding graphs of the said tables are shown in Figure 6.1 to Figure 6.10.

Table 6.1. Node Voltage after addition of capacitors

Time	*Node 2*	*Node 3*	*Node 4*	*Node 5*
1 AM	2388.0	2395.2	2396.9	2397.6
2	2385.2	2383.2	2374.0	2369.2
3	2380.7	2377.3	2366.9	2361.9
4	2373.3	2367.4	2354.2	2348.2
5	2357.2	2348.0	2332.2	2324.5
6	2371.3	2367.5	2350.3	2342.0
7	2367.4	2362.5	2343.5	2335.3
8	2366.4	2361.2	2341.8	2333.6
9	2372.8	2368.7	2350.6	2342.4
10	2373.5	2369.6	2350.6	2342.4
11	2371.6	2367.5	2348.6	2340.3
12	2369.9	2365.6	2346.6	2338.3

1 PM	2372.6	2368.7	2349.9	2341.2
2	2360.8	2354.0	2332.7	2323.2
3	2348.4	2339.8	2316.1	2305.1
4	2344.3	2334.4	2310.1	2299.1
5	2349.8	2340.0	2315.6	2304.6
6	2354.9	2347.0	2325.6	2317.4
7	2371.9	2367.4	2348.3	2341.8
8	2362.1	2353.9	2339.9	2335.9
9	2379.2	2375.4	2365.4	2365.2
9	2379.2	2375.4	2365.2	2361.2
10	2384.0	2381.5	2372.7	2368.7
11	2388.2	2386.5	2378.3	2374.2
12	2389.1	2387.4	2380.4	2376.8

Table 6-2. Optimal Capacitor Allocation

Fixed capacitor rating	235 Amperes
Switched capacitor rating	285.5 Amperes
Segment number	4
Pole number	7
Switching time	14 hours
From	6 AM
To	8 PM

Table 6.3. Peak-loss and Energy-loss Reduction Savings

Peak-loss reduction savings	8,126.18
Energy-loss reduction savings	210,769.32

Table 6-4. Increase in Revenue per Node

Node	*Revenue Increase (Pesos/Year)*
2	₱ 76,812.44
3	97,515.00
4	117,590.32
5	79,225.88

Table 6-5. Increase in Power in KW throughout the Load Cycle

Time	*Node 2*	*Node 3*	*Node 4*	*Node 5*
1 AM*	2.652	3.377	4.502	3.602
2	3.316	3.376	4.952	3.601
3	3.649	4.728	6.304	4.053
4	3.986	5.410	8.115	4.959
5	6.658	7.904	9.936	6.775
6	15.572	20.149	23.173	16.121
7	16.316	20.151	25.190	16.122
8	16.318	20.152	26.199	16.122
9	13.343	17.624	26.189	16.117
10	13.342	17.624	25.180	16.116
11	14.828	17.131	25.184	16.118
12	15.572	19.141	25.188	16.121
1 PM	14.083	18.127	23.165	17.122
2	17.065	21.166	27.215	18.143
3	18.615	22.234	29.312	20.216
4	19.370	25.277	30.334	20.224
5	20.058	25.225	30.274	20.183
6	20.043	25.208	28.236	15.127
7	13.336	19.128	26.177	12.082
8	4.981	7.212	9.015	3.144
9	3.982	4.956	6.758	3.154
10	2.316	4.052	5.402	3.151
11	3.751	3.600	4.500	3.150
12	3.660	3.834	3.511	3.383

Table 6-6. Node Voltages after addition of capacitors

Time	*Node 2*	*Node 3*	*Node 4*	*Node 5*
1 AM	2388	2386.6	2387.4	2396.3
2	2385.2	2383.2	2383.4	2390.4
3	2380.7	2377.3	2376.3	2383.0
4	2373.3	2367.4	2363.6	2369.5
5	2357.2	2348.0	2341.7	2345.7
6	2341.1	2340.1	2332.4	2335.9
7	2367.6	2362.7	2364.8	2368.3

8	2366.6	2361.5	2363.1	2366.5
9	2373.7	2369.9	2371.9	2375.4
10	2373.7	2369.9	2371.9	2374.4
11	2371.8	2367.8	2369.8	2373.3
12	2370.1	2365.8	2367.8	2371.3
1 PM	2372.8	2368.9	2371.1	2374.1
2	2361.0	2354.3	2353.0	2356.1
3	2349.5	2340.1	2337.3	2338.0
4	2344.7	2334.7	2331.3	2332.1
5	2350.0	2340.2	2336.9	2337.6
6	2355.1	2347.3	2346.9	2350.4
7	2372.1	2367.7	2369.5	2374.7
8	2362.1	2353.9	2349.4	2357.1
9	2379.2	2353.9	2374.6	2382.4
10	2384.0	2385.5	2382.1	2389.9
11	2388.2	2386.5	2387.7	2395.4
12	2388.2	2386.6	2387.4	2396.3

Table 6-7. Optimal Capacitor Allocation

Fixed capacitor rating	235 Amperes
Segment number	5
Pole number	1
Switched capacitor rating	288.0 Amperes
Segment number	4
Pole number	7
Switching time	15 hours
From	6 AM
To	8 PM

Table 6-8. Peak-loss and Energy-loss Reduction Savings

Peak-loss reduction savings	₱ 8,238.29
Energy-loss reduction savings	212,426.63

Table 6-9. Increase in Revenue per Node

Node	*Revenue Increase (Pesos/Year)*
2	₱ 78,613.47
3	98,516.00
4	187,594.82
5	98,248.63

Table 6-10. Increase in Power in KW throughout the Load Cycle

Time	*Node 2*	*Node 3*	*Node 4*	*Node 5*
1 AM	2.651	3.372	5.607	7.016
2	3.217	3.371	5.712	7.012
3	3.650	4.730	8.881	7.843
4	3.972	5.401	10.914	8.577
5	6.599	7.991	13.693	12.161
6	15.592	15.978	33.223	28.320
7	16.320	20.121	40.201	28.413
8	16.125	20.125	42.287	28.413
9	13.333	17.632	42.013	28.321
10	13.231	17.625	39.993	28.301
11	14.797	18.032	40.011	28.331
12	15.601	19.150	40.101	28.381
1 PM	14.011	18.134	38.653	30.722
2	17.056	21.176	45.416	34.260
3	18.511	22.211	49.423	38.422
4	19.295	25.371	53.432	28.525
5	20.011	25.217	53.107	28.116
6	20.041	25.108	50.625	28.717
7	13.373	19.215	40.993	22.625
8	4.990	7.216	13.111	6.009
9	3.992	4.943	8.976	6.007
10	2.401	4.007	7.341	6.002
11	2.562	3.607	7.137	6.001
12	2.611	3.826	7.243	6.323

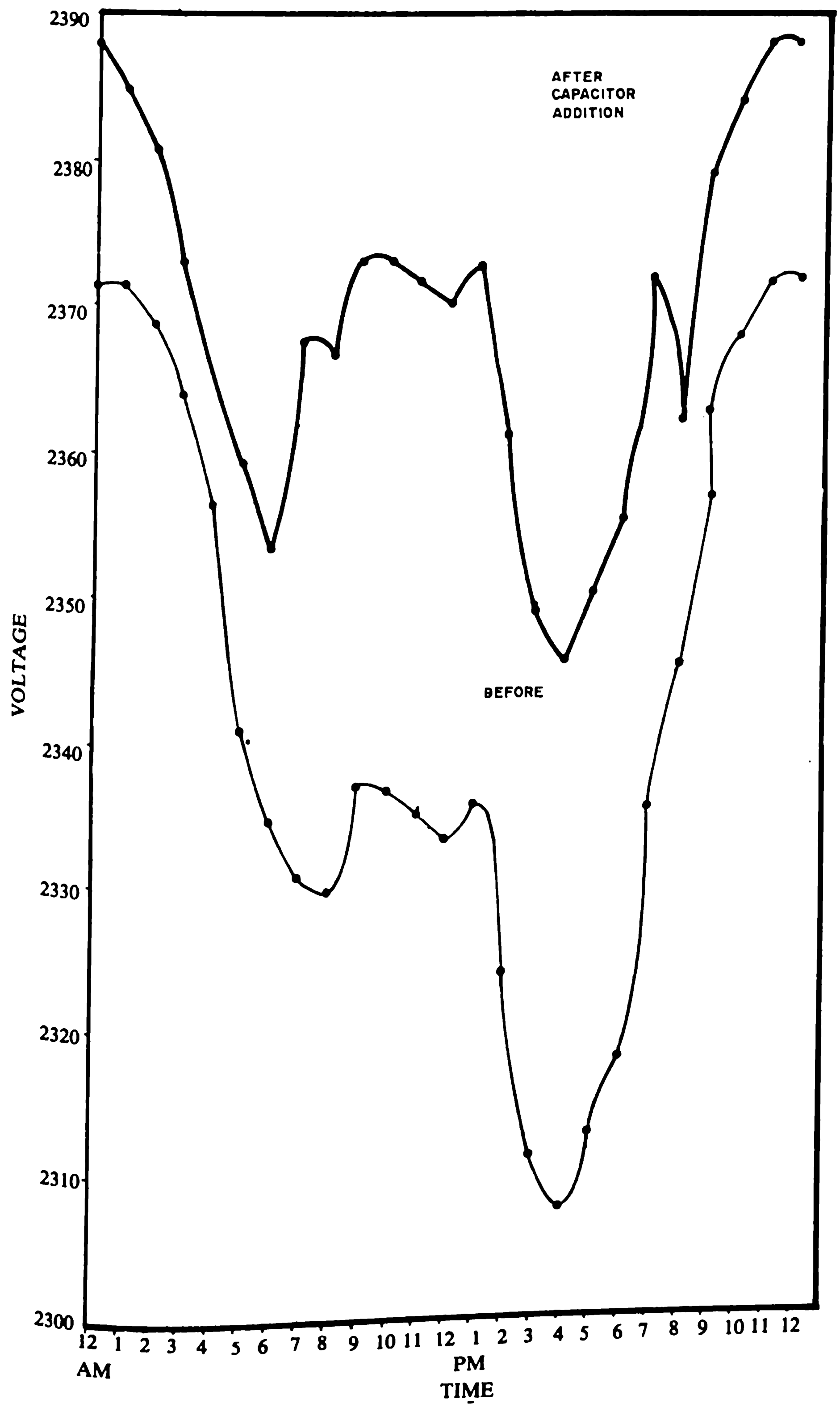


Figure 6-1. Comparison of Voltages at Node 2 throughout the Load Cycle before and after Capacitor Addition

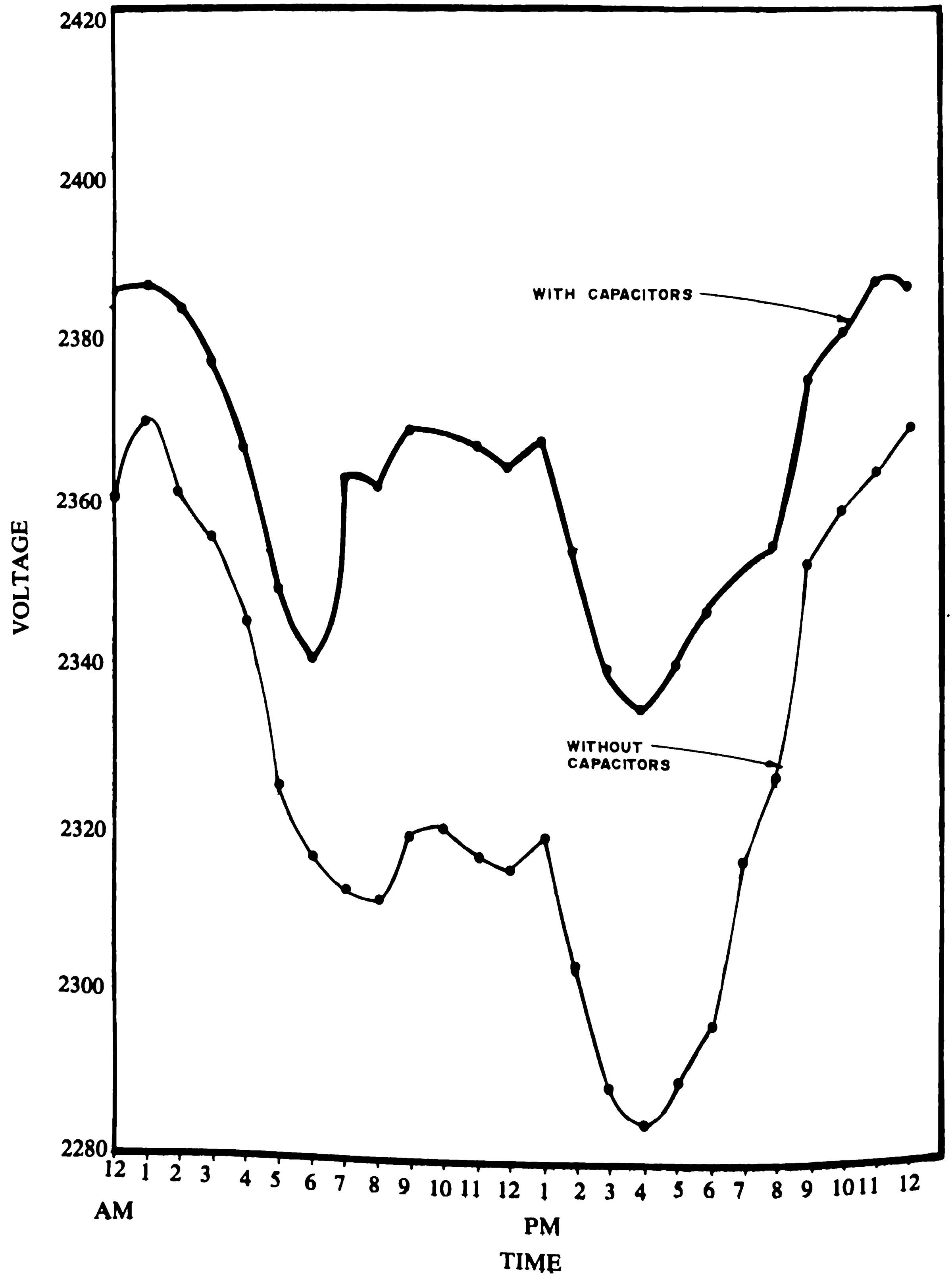


Figure 6-2. Comparison of Voltages at Node 3 throughout the Load Cycle before and after Capacitor Addition

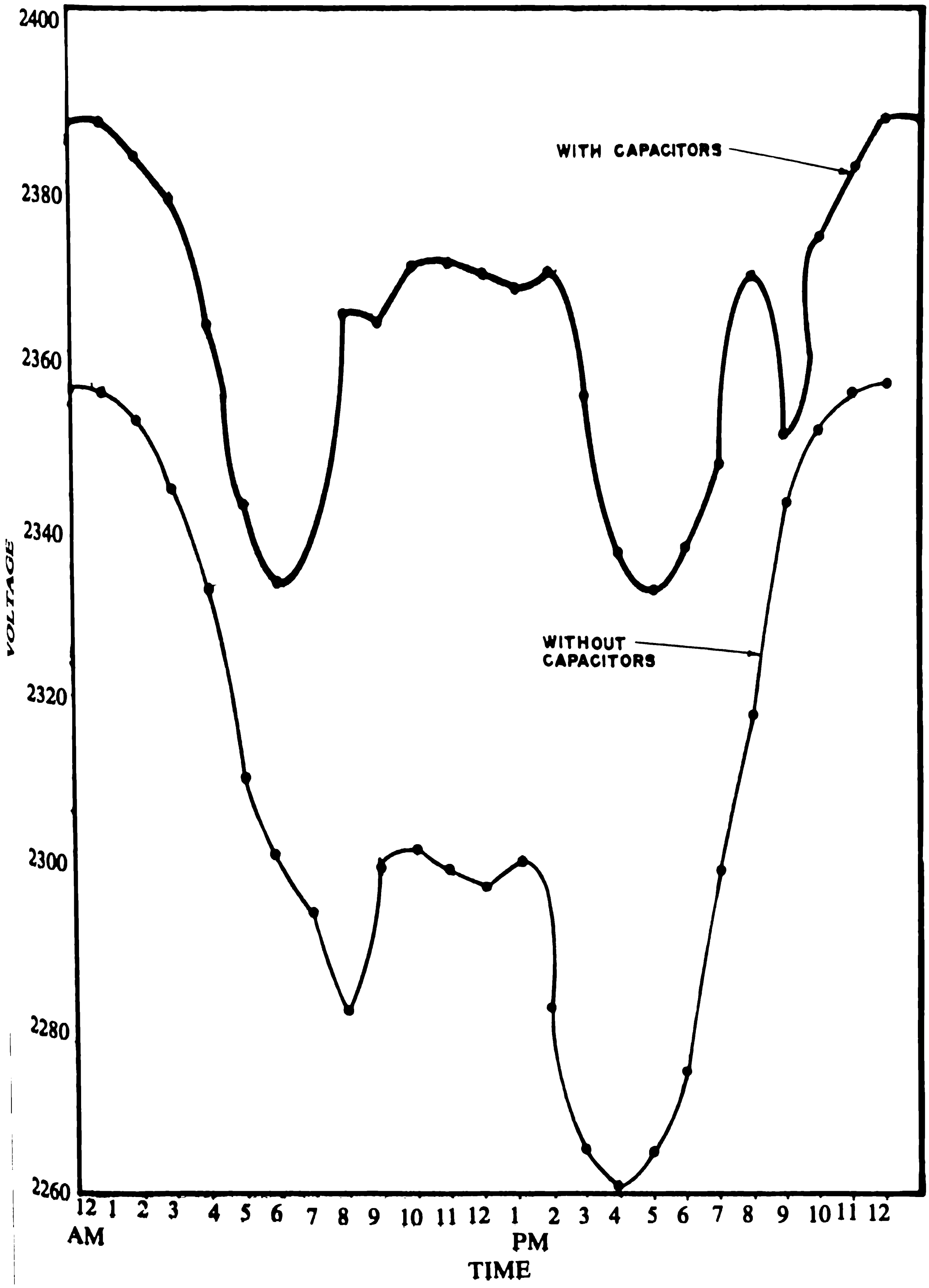


Figure 6-3. Comparison of Voltages at Node 4 throughout the Load Cycle before and after Capacitor Addition

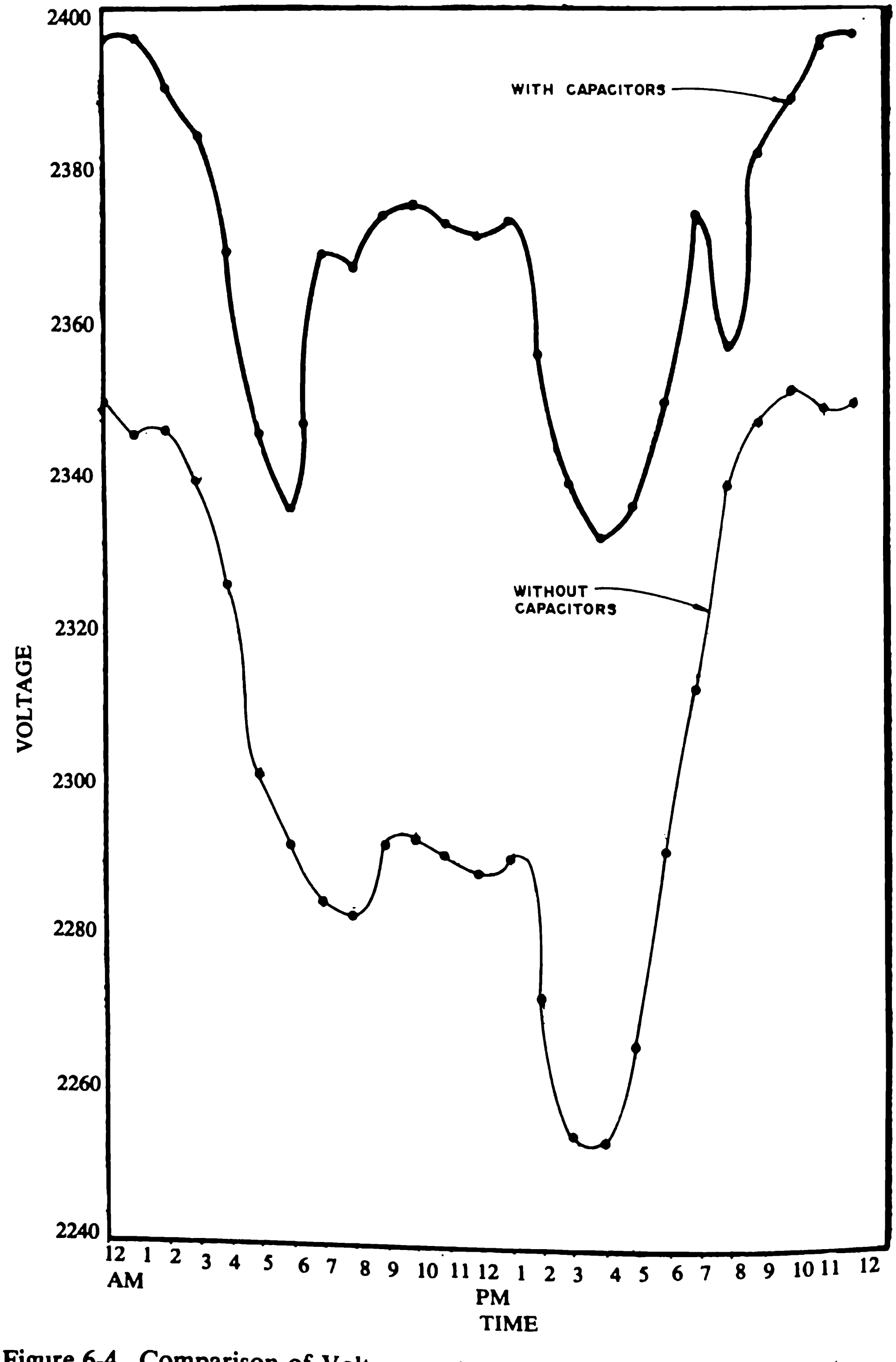


Figure 6-4. Comparison of Voltages at Node 5 throughout the Load Cycle before and after Capacitor Addition

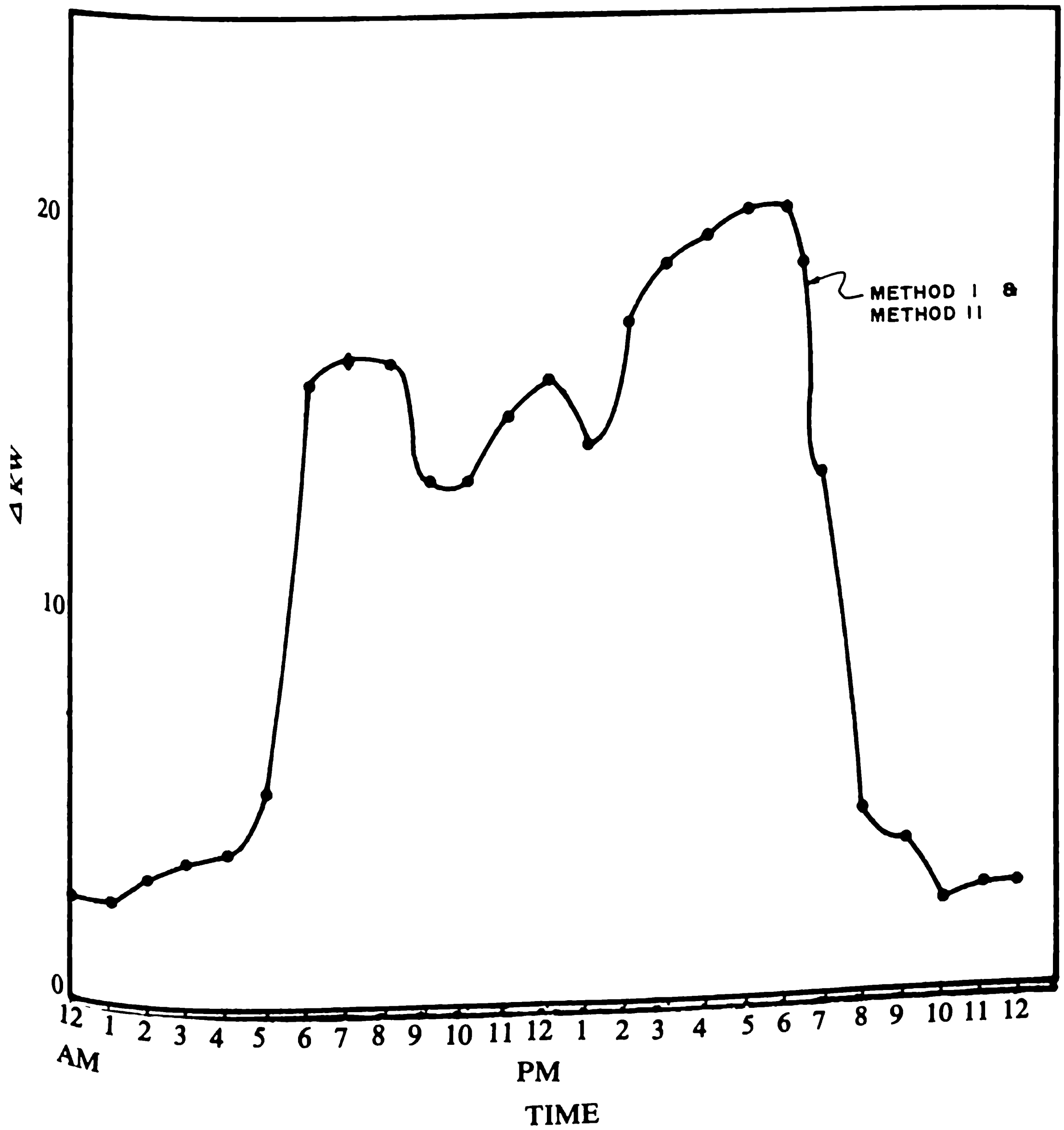


Figure 6-5. Increase in Power at Node 2 throughout the Load Cycle for the Two Methods

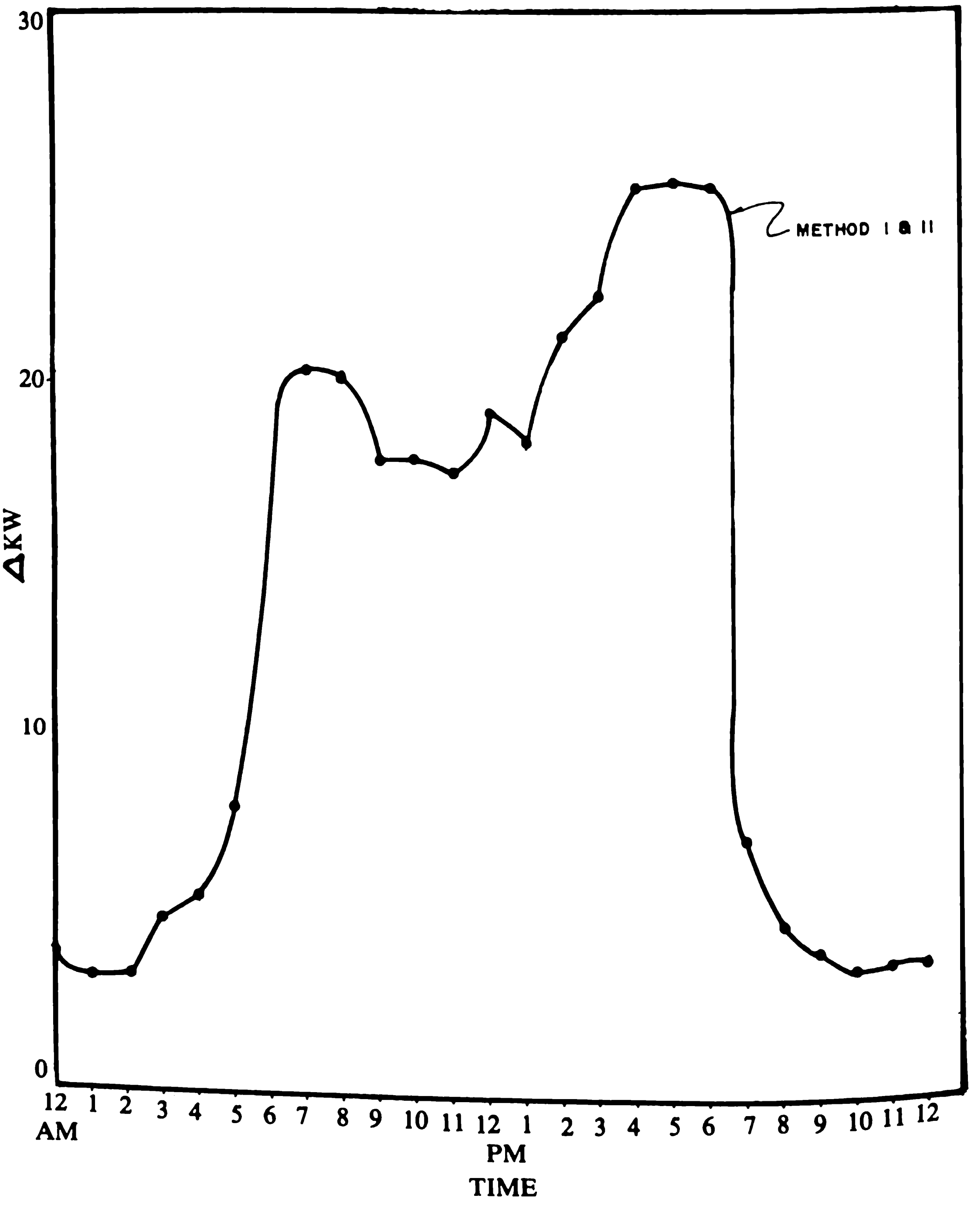


Figure 6-6. Increase in Power at Node 3 throughout the Load Cycle for the Two Methods

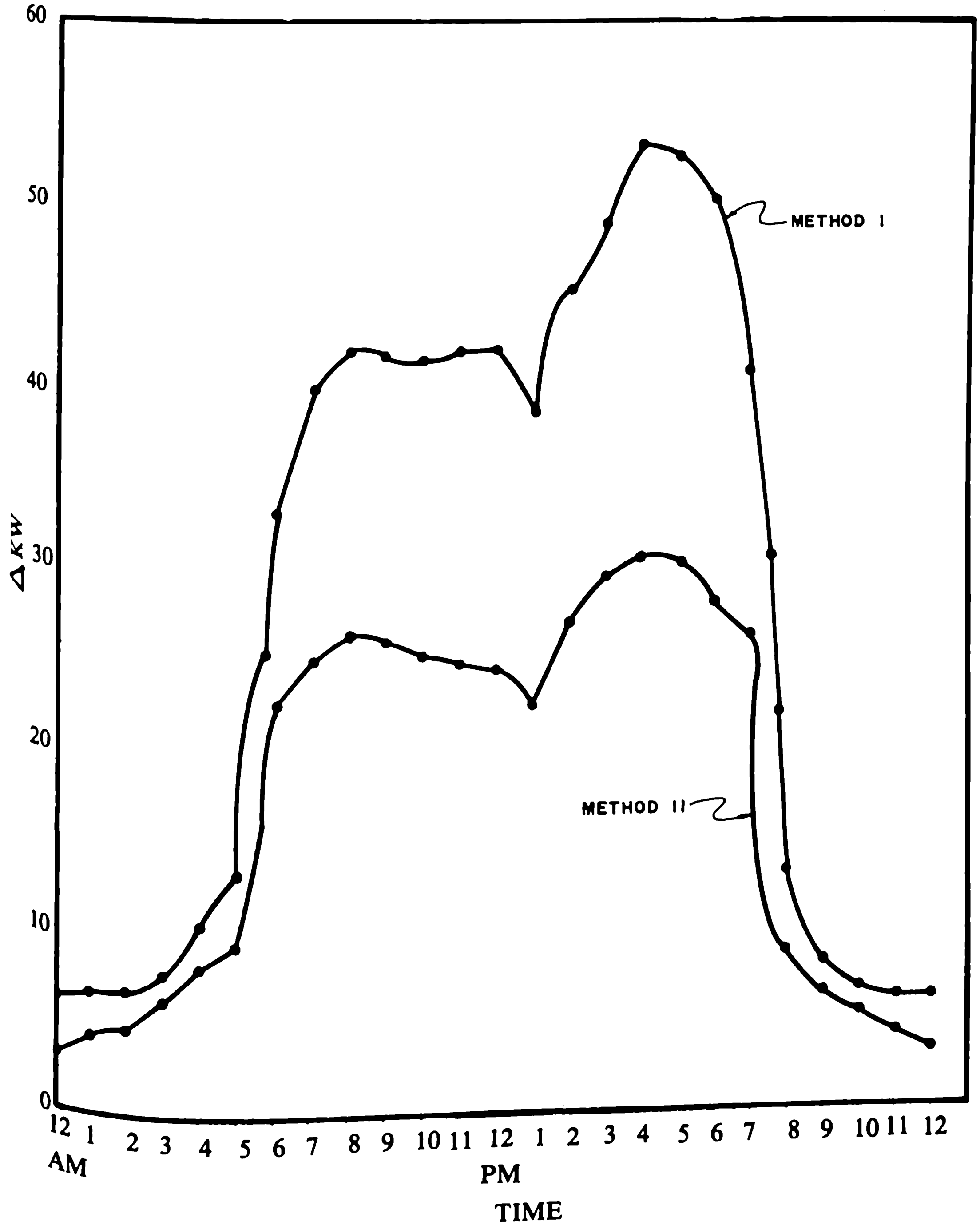


Figure 6-7. Increase in Power at Node 4 throughout the Load Cycle for the Two Methods

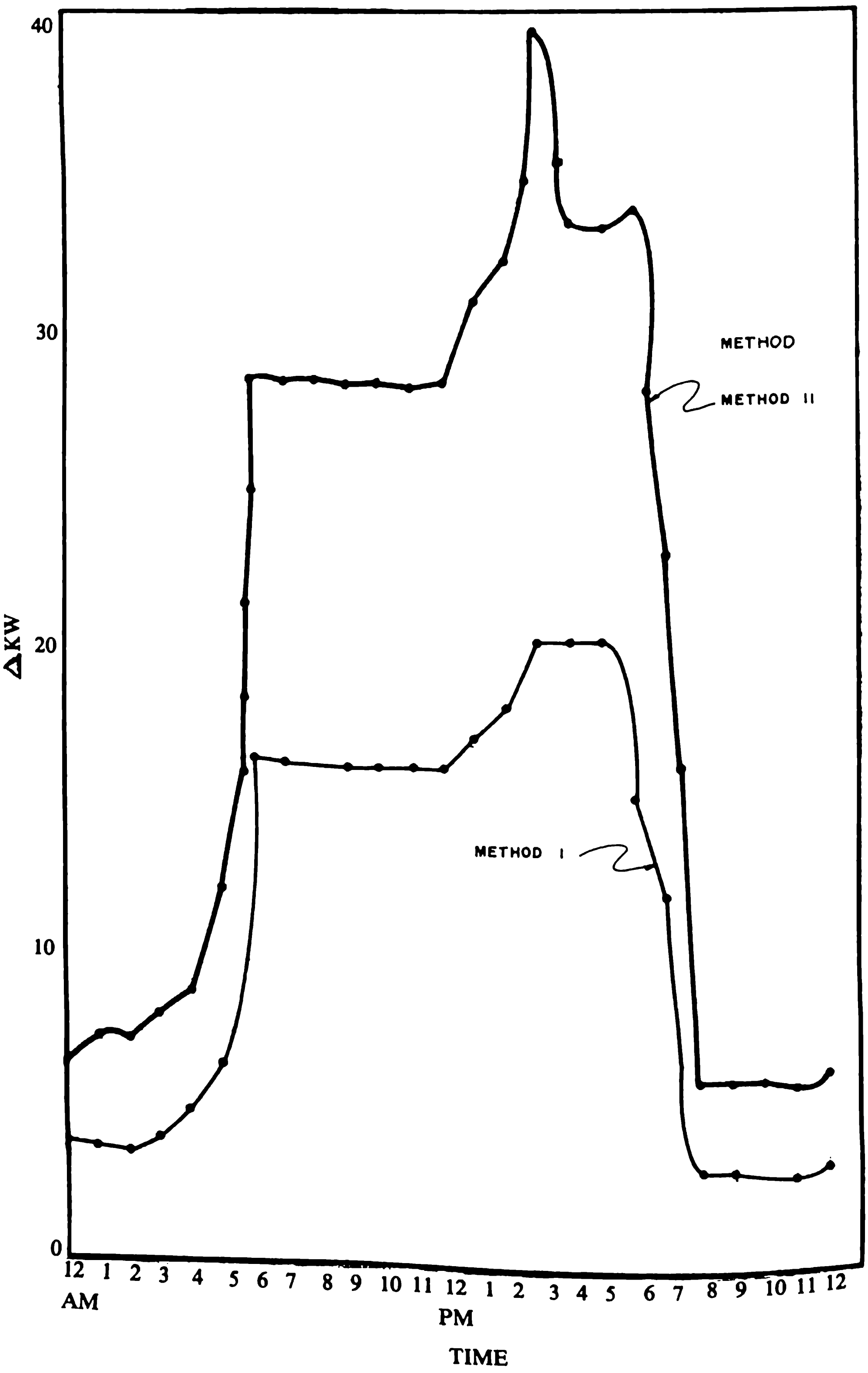


Figure 6-8. Increase in Power at Node 5 throughout the Load Cycle for the Two Methods

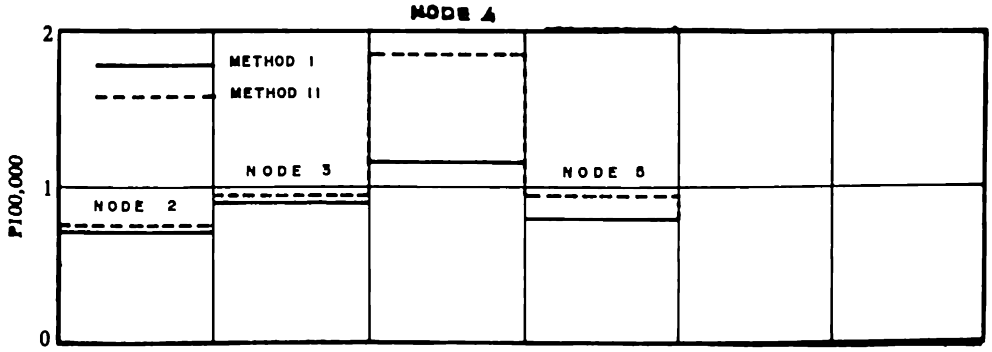


Figure 6-9. Increase in Revenue Per Node of the Two Methods

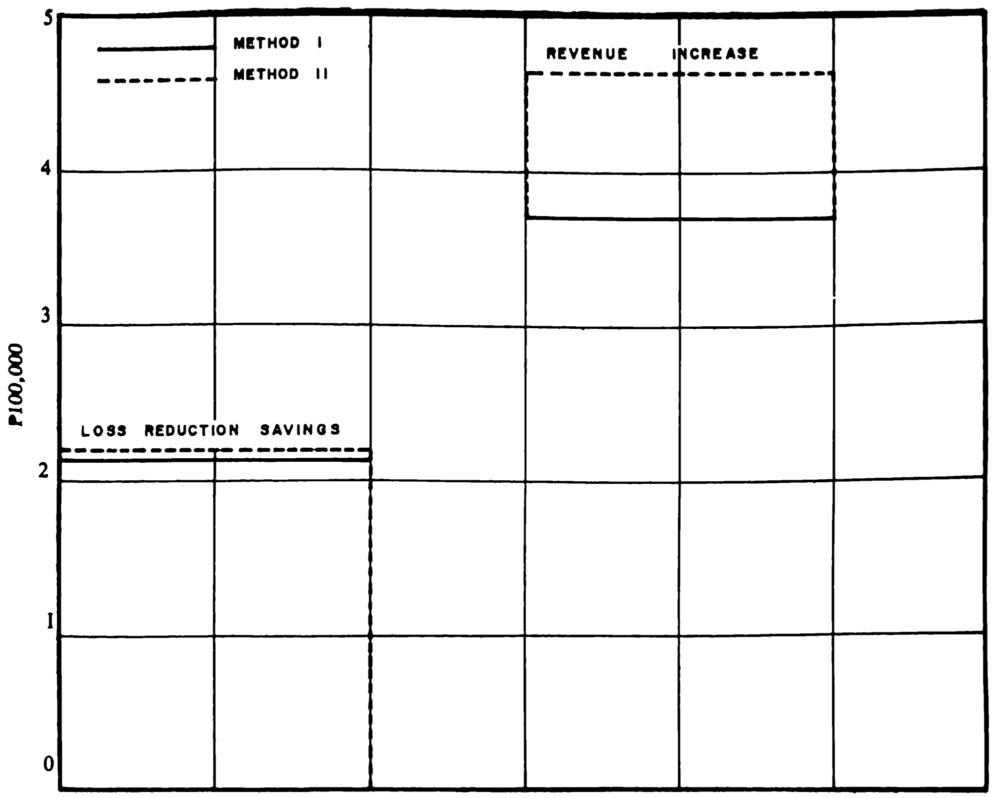


Figure 6-10. Monetary Savings and Monetary Gain of the Two Methods

Discussion of Results

The benefits of voltage control are evident in the results shown. The voltage profile is greatly improved such that the maximum deviation from the base voltage is only $\pm$ 0.02 per unit. Revenue increase brought about by the voltage profile improvement is shown to be very important factor in determining the capacitor allocation.

Results of the two proposed methods indicate that there is not much difference in the energy-loss and peak-loss reduction savings between them. However, the great difference is in the increase in revenue as shown in table 6-4 and table 6-9; with switched and fixed capacitors separated there is a greater increase in revenue. The second method yields a slightly higher switched capacitor rating than the first method. However, this increase results in a greater increase in revenue.

There is no problem with regards to computer storage requirements. The problem lies in the computer time. With all points to be tested it took 17 minutes to compute using the first method and 30 minutes using the second method. With selective testing (say from 0.5 p.u. to 0.75 p.u. distance from the substation) the computation time was reduced to one-fourth the original time. Although the second method requires more computation time the very positive results it yielded negate this drawback.

Conclusions

From the results and discussion of results the following conclusions are drawn:

1. Method II gives better results than Method I in terms of revenue increases.
2. The non-inclusion of the addition revenues, derivable from the voltage profile improvement, in determining the optimal capacitor allocations is the most serious drawback of all the previous methods of capacitor application.
3. The voltage profile with the fixed and switched capacitors separated is greatly improved and correspondingly there is a greater increase in revenue.
4. With the fixed capacitor rating equal to the total feeder reactive current at light load, over voltages are checked; with the highest voltage at 0.999 per unit at light load.
5. The setting of the switching time of the switched capacitors to eliminate the possibility of over compensation results in a very improved voltage profile.

References

1. Neagle, N.M. and Samson, D.R. Loss Reduction from Capacitors Installed on Primary Feeders. AIEE Transactions, Vol. PAS-75, pp. 950-957, October 1956.

2. Rankline, L.J. Two-Thirds Fuel used for Capacitor KV KVAR. Electrical World, p. 72, December 2, 1957.

3. Rankline, L.J. Methods of Locating Shunt Capacitors for Computer Solutions. Electrical World, pp. 72-73, September 26, 1960.

4. Rankline, L.J. Economic Number of Capacitor Banks. Electric Light and Power, pp. 82-83, August 15, 1957.

5. Cook, R.F. Analysis of Capacitor Application as Affected by Load Cycle. AIEE Transactions, Volume PAS-78, pp. 950-956, October 1959.

6. Optimizing the Application of Shunt Capacitors for Reactive Volt-Control and Loss-Reduction. AIEE Transactions, Vol. PAS-80, pp. 430-444, August 1961.

7. Cook, R.F. Calculating Loss Reduction Afforded by Shunt Capacitor Application. IEEE Transaction, Vol. PAS-83, pp. 1227-1230, December 1964.

8. Schmill, J.V. Optimum Size and Location of Shunt Capacitors on Distribution Feeders. IEEE Transactions, Vol. PAS-84, pp. 825-832, September 1965.

9. Duran, H. Optimum Number, Location and Size of Shunt Capacitors in Radial Distribution Feeders, A Dynamic Programming Approach. IEEE Transactions, Vol. PAS-87, pp. 1769-1774, September 1968.

10. Chang, N.E. Determination of Primary-Feeder Losses. IEEE Transactions, Vol. PAS-87, pp. 1991-1994, December 1968.

11. Chang, N.E. Locating Shunt Capacitors in Primary Feeder for Voltage Control and Loss-Reduction. IEEE Transactions, Vol. PAS-88, pp. 1574-1577, October, 1969.

12. Chang, N.E. Generalized Equations on Loss Reduction with Shunt Capacitor. IEEE Transactions, Vol. PAS-91, pp. 2189-2195, September to October 1972.

13. Bae, X.G. Analytical Method of Capacitor Allocation on Distribution Primary Feeders. IEEE Transactions, Vol. PAS-97, pp. 1232-1238, July to August, 1978.

APPENDIX A

Derivation of Loss Reduction Equations for Different Capacitor Combinations

A.1 *Peak Loss Reduction in a Line Segment due to Fixed and Switched Capacitors located at a Single Point. Referring to Figure 3.1.*

The Peak-power loss in the line segment before the addition of the capacitor is

$$P_l = 3I_q^2R_n/1000 \text{ KW} \quad \text{(a.1)}$$

where

R_n = resistance of segment n

I_q = maximum reactive current of segment n

the peak power loss in the line segment after the capacitors are added, is:

$$P_l = (3aR_n [I_q - (I_{cs} + I_{cf})]^2 + 3(1-a)I_q^2 R_n)/1000 \text{ KW} \quad \text{(a.2)}$$

The peak loss reduction in the line segment is

$$\Delta P_l = P_l - P_l$$

thus

$$P_l = 3aR_n [2(I_{cs} + I_{cf})I_q - (I_{cs} + I_{cf})^2]/1000 \text{ KW} \quad \text{(a.3)}$$

The peak loss reduction in any line segment j on the source side of line segment n due to adding fixed and switched capacitors at a single point in line segment n is

$$\Delta P_j = P_j - P_j'$$

$$= \frac{3I_5^2R_j}{1000} - \frac{3R_j}{1000}(I_j^2 - [I_j - (I_{cs} + I_{cf})]^2)$$

$$\Delta P_j = \frac{3R_j}{1000}[2I_j(I_{cs} + I_{cf}) - (I_{cs} + I_{cf})^2] \quad \text{(a.4)}$$

where

I_j = peak reactive current at segment j

A.2 *Energy-Loss Reduction in a Line Segment due to Fixed and Switched Capacitors located at a Single Point.*

Referring to Figure 3.1., the energy loss in the line segment throughout the load cycle before the addition of the capacitors is

$$E_1 = 3(I_q LF)^2 R_n T \tag{a.5}$$

The energy-loss with fixed capacitor connected is

$$E_2 = 3a(I_q LF - I_{cf})^2 R_n T_f + 3(1\text{-}a)(I_q LF)^2 R_n T_f$$

where $T_f = T$

with fixed and switched capacitors connected, the energy loss is

$$E_3 = (I_q LF_s) - I_{cs}^2 \quad 3aR_n T_s + 3(1\text{-}a)(I_q LF)^2 R_n T_f + (I_q' LF_s)^2\, 3aR_n (T_f\text{-}T_s) \tag{a.6}$$

The energy loss reduction is then

$$\Delta E_1 = E_1 - E_3$$

or

$$\Delta E_1 = \frac{3aR_n}{1000}[T_f (I_q LF)^2 - (I_q' LF_s)^2 - T_s(2I_q' LF_s I_{cs} + I_{cs}^2)] \tag{a.7}$$

A.3 *Peak Loss Reduction due to Fixed and Switched Capacitors Placed at Different Points in the Line Segment n:*

Referring the Figure 3.2, the peak power loss in the segment before the addition of capacitors is

$$P_1 = 3I_q^2 R_n / 1000 \text{ KW} \tag{a.8}$$

The peak power loss after the addition of capacitor is

$$P_1' = \frac{3aR_n}{1000}[I_q - (I_{cf} + I_{cs})]^2 + \frac{3(b\text{-}a)R_n}{1000}(I_q\text{-}I_{cs})^2 + \frac{3(1\text{-}b)}{1000} R_n I_q^2 \tag{a.9}$$

The peak loss reduction is

$$\Delta P_1 = P_1 - P_1'$$

$$\Delta P_1 = \frac{3aR_n}{1000}(2I_qI_{cf} - 2I_{cf}I_{cs} - I_{cs}^2)$$

$$+ \frac{3bR_b}{1000}(2I_1I_{cs} - I_{cs}^2) \tag{a.10}$$

A.4 *Energy Loss Reduction due to fixed and Switched Capacitors Placed at Different Points in the Line Segment n:*

The energy loss before the addition of capacitors is

$$E_1 = 3(I_qLF)^2 R_nT_f \tag{a.11}$$

The energy loss with fixed capacitor added is

$$E_2 = (I_qLF - I_{cf})^2 3aR_nT_f + (I_qLF)^2 3(1\text{-}a)R_nT_f \tag{a.12}$$

The energy loss with fixed and switched capacitors is

$$E_3 = 3aR_n(T_f\text{-}T_s)(I_q'LF_s)^2 + 3aR_nT_s(I_q'LF_s\text{-}I_{cs})^2$$

$$+ 3(b\text{-}a)R_n(T_f\text{-}T_s)(I_qLF)^2 + 3(b\text{-}a)R_nT_s(I_qLF\text{-}I_{cs})^2$$

$$+ 3(1\text{-}b)T_f(I_qLF)^2R_n \tag{a.13}$$

$$\Delta E_1 = \frac{3aR_nT_f}{1000}[(I_qLF)^2 - (I_qLF_s)^2] + \frac{3R_nT_s}{1000}[2aI_qLF_sI_{cs}$$

$$- aI_{cs}^2 + 2bI_q LFI_{cs}\text{-}bI_{cs}^2\text{-}2aI_qLFI_{cs} + aI_{cs}^2] \tag{a.14}$$

A.5 ***Peak Loss Reduction due to Fixed and Switched Capacitors Placed at Different Segments***

Referring to Figure 3.3, the peak losses before the addition of capacitors are as follows:

The loss in any segment k before the first segment with capacitors

$$Pl_1 = 3I_k^2I_k \tag{a.15}$$

The loss in the segment with fixed capacitor to be connected,

$$Pl_2 = 3I_q^2R_n \tag{a.16}$$

The loss in any segment k between the segments with fixed and switched capacitors,

$$Pl_3 = 3I_k^2R_k \tag{a.17}$$

and the loss in the segment with switched capacitors to be connected

$$Pl_4 = 3I_j^2R_j \tag{a.18}$$

The peak power losses at affected portions of the line with the capacitors connected are as follows: The loss in the segments before the capacitors,

$$P'_{l_1} = 3\sum_{k=1}^{n=k} [I_k-(I_{cf} + I_{cs})]^2 Rk \tag{a.19}$$

the loss in the segment with fixed capacitor,

$$P'_{l_2} = 3a\,[I_q - (I_{cf} + I_{cs})]^2 R_n + 3(1\text{-}a)\,(I_q - I_{cs})^2 R_n \tag{a.20}$$

the loss in the segments between the segments with fixed and switched capacitors connected,

$$P'_{l_3} = 3 \sum_{k=n+1}^{j-1} (I_k - I_{cs})^2 R_k \tag{a.21}$$

and the loss in the segment with switched capacitor

$$P'_{l_4} = 3b(I_j - I_{cs})^2 R_j + 3(1\text{-}b)\, I_j^2R_j \tag{a.22}$$

The peak power loss reduction is then

$$P_l = (P'_{l_1} - P'_{l_1}) + (P_{l_2} = P'_{l_2}) + (P'_{l_3} - PL_3) + (P_{l_4} - P'_{l_4})$$

$$P_l = \frac{3}{1000} \Big[\sum_{k=1}^{n-1} [21kI_{cf}I_{cs} - (I_{cf} + I_{cs})^2]\, R_k$$

$$+ R_n\,(2aI_q I_{cf} - aI_{cf}^2 - 2a\,I_{cf}\,I_{cs}$$

$$+ 2I_q I_{cs} - I_{cs}^2) + \sum_{k=n+1}^{j-1} (2I_k I_{cs} - I_{cs}^2)\, R_k$$

$$+ b\,(2I_q\, I_{cs} - I_{cs}^2)\,\Big] \tag{a.23}$$

A.6 *Energy Loss Reduction due to Fixed and Switched Capacitors Placed at Different Segments*

Before the addition of capacitors the energy losses, at portions to be affected by capacitor addition, are as follows:

The energy losses in the segments before the first segment with capacitor,

$$E'_{l_1} = \sum_{k=1}^{n-1} (I_k LF_k)^2 R_k T \tag{a.24}$$

the loss in the segment with fixed capacitor to be connected,

$$E'_{l_2} = 3\,(I_q LF_n)^2 R_n T \tag{a.25}$$

the loss in the segments between the segments with fixed and switched capacitors to be connected,

$$E_{l_3} = \sum_{k=n+1}^{j-1} (I_k LF_k)^2 R_k T \tag{a.26}$$

and the loss in the segment with switched capacitor to be connected,

$$E_{l_4} = 3(I_j LF_j)^2 R_j T \tag{a.27}$$

The energy losses in the affected segments after the addition of the capacitors are as follows:

The energy loss in the segments before the first segment with capacitors,

$$E'_{l_1} = 3 \{ [I_k LF_k - (I_{cf} + I_{cs})]^2 R_k T_s + (I_k LF_k - I_{cf})^2 R_k (T - T_s) \} \tag{a.28}$$

the loss in the segment with fixed capacitor,

$$E'_{l_2} = 3\,[I_q LF_n - (I_{cf} + I_{cs}))^2 R_n T_s + 3(1-a)\,(I_q LF_n - I_{cs})^2 R_n T_s + (I_q LF_n)^2\, 3(1-a)\,(T-T_s)\, R_n \tag{a.29}$$

The energy loss reduction is then

$$E_l = (E_l - E'_{l_1}) + (E_{l_2} - E'_{l_2}) + (E_{l_3} - E'_{l_3}) + (E_{l_4} - E'_{l_4})$$

or

$$E_l = \frac{3}{1000} \cdot ([(2I_k LF_k I_{cf} - I_{cf}^2) T + (2I_k LF_k I_{cs} - 2I_{cs} I_{cf} - I_{cs}^2) T_s] R_k + [2I_q LF_n (I_{cf} + aI_{cs}) T_s - (I_{cf} + I_{cs})^2 T_s + I_{cs}^2 (a-1) T_s + (I_n LF_n)^2 T_s + a (I_q LF_n)^2 T [R_n + \sum_{k=n+1}^{j-1} (2I_k LF_k I_{cs} - I_{cs}^2) R_k T_s + [(I_j LF_j) T - (I_j LF_j - I_{cs})^2 T_s] bR_j) \qquad (a.32)$$

APPENDIX B

Calculation of Node Voltages for Different Capacitor Combinations

B.1 *Segment with no Capacitors*

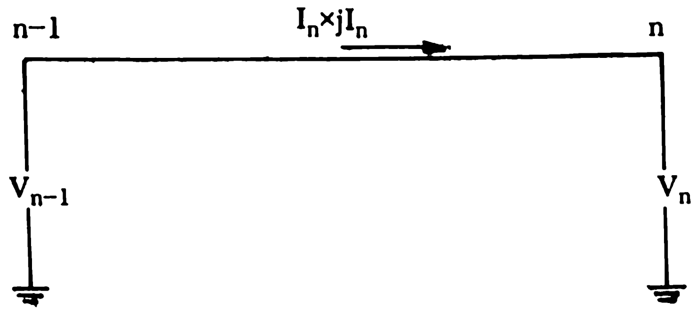


Figure B.1 Feeder Segment with no Capacitors

$$V_n = V_{n-1} - (I_n - jI'_n)\,(R_n + jX_n)$$

$$(V_{n-1} - I_nR_n - I'_nX_n) + j(I'_nR_n - I_nX_n) \qquad \text{(b.1)}$$

B.2 *Capacitors at a Single Location in Segment n*

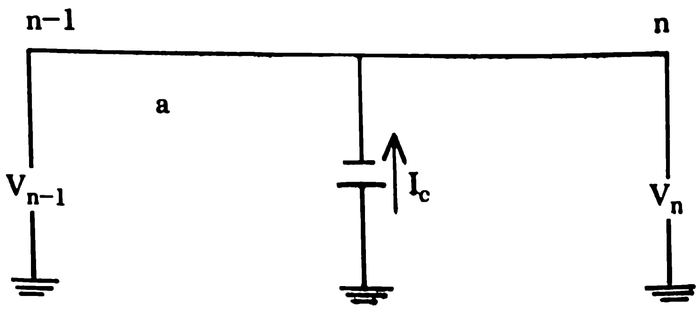


Figure B.2 Fixed and Switched Capacitors at a Single Location in Segment n

The angular displacement between node voltages is assumed to be negligible by Kirchoff's Law:

$$V_n = V_{n-1} - [I_n - j(I'_n - I_c)] \ a(R_n + jX_n)$$
$$- (I_n - jI'_n) \ (I-a) \ (R_n + jX_n)$$

simplifying:

$$V_n = [V_{n-1} - I_n R_n - I'_n - I_c) X_n]$$
$$+ j[R_n(I'_n - I_c) - X_n I_n] \qquad \text{(b.2)}$$

B.3 Fixed and Switched capacitors at different points in Segment n

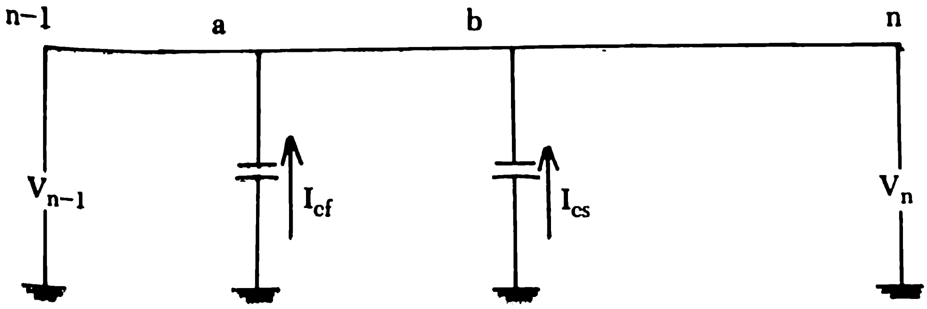


Figure B.3 Fixed and Switched Capacitors at different points in Segment n

by Kirchoff's Law: (a > b)

$$V_n = V_{n-1} - [I_n - j(I_n - I_{cf} - I_{cs})] \, b(R_n + jX_n)$$
$$- [I_{n-j} (I_n - I_{cs})] \, (a-b) \, (R_n + jX_n)$$
$$- (I_n - jI_n) \, (1-a) \, (R_n + jX_n)$$

simplifying

$$V_n = V_{n-1} - bI_nR_n - (I'_n - I_{cf} - I_{cs})\, bX_n$$

$$- (a-b)\, I_nR_n - (I'_n - I_{cf})\, (a-b)X_n$$

$$- (1-a)\, I_nR_n - (1-a)\, I'_nX_n + j(I'_n - I_{cf}$$

$$- I_{cs})bR_n - bI_nR_n - bI_nR_n - bI_nX_n$$

$$+ (a-bR_n\, (I'_n - I_{cf}) - (a-b)I_nX_n + (1-a)I'_nR_n$$

$$- (1-a)I_nX_n \qquad \text{(b.3)}$$

if b > a, then

$$V_n = V_{n-1} - [I_n - j(I'_n - I_{cf} - I_{cs})]\, a(R_n + jX_n)$$

$$- [I_n - j(I'_n - I_{cf})]\quad (b-a)\, (R_n + jX_n)$$

$$- (I_n - jI'_n)\, (1-n)\, (R_n + jX_n) \qquad \text{(b.4)}$$

APPENDIX C

Derivation of Revenue Increase due to the Rise in Node Voltage after the Addition of Capacitors

Taylor's series can be used to determine the area under the curve. Let the indefinite integral be

$$P(t) = \int_{t}^{t'} f(t)\,dt$$

Assuming P(t) is analytic, then $P(t_{n+1})$ can be expressed in a Taylor series expansion about tn,

$$P(t_{n+1}) = P(t_n + \Delta t)$$

$$= P(t_n) + \Delta t P'_{(t_n)} + \frac{(\Delta t)^2}{2!} P''_{(t_n)}$$

$$+ \frac{(\Delta t)^3}{3!} P'''_{(t_n)}$$

where $P(t_n)$ = area under f(t) from a to t_n

$$P(t_{n+t}) = p(t_n) + \text{area under } f(t) \text{ from } t_n \text{ to } t_{n+1}$$

since

$$P(t) = \int_{t}^{t'} f(t)\,dt$$

then

$$P'_{(t_n)} = f(t_n)$$

$$P''_{(t_n)} = f'(t_n)$$

Rewriting the expression for $P(t_{n+1})$

$$P(t_{n+1}) = P(t_n) + \Delta t\, f(t_n) + \frac{(\Delta t)^2}{2!} f'(t_n)$$

$$+ \frac{(\Delta t)^3}{3!} f'''(t_n) + \dots$$

Representing the first derivatives $f'(t_n)$ by a Taylor series expansion to determine $f(t_{n+1})$

$$f(t_{n+1}) = f(t_n) + \Delta t\, f'(t_n) + \frac{(\Delta t)^2}{2!} f''(t_n) + \dots .$$

Solving for $f'(t_n)$

$$f'(t_n) = \frac{f(t_{n+1}) - f(t_n)}{\Delta t} - \frac{\Delta t}{2} f''(t_n) + \dots . \qquad b$$

Substituting to the expression for $P(t_{n+1})$ in eq.

$$P(t_{n+1}) = P(t_n) + \Delta t f(t_n) + \frac{(\Delta t)^2}{2!} \frac{f(t_{n+1}) - f(t_n)}{\Delta t}$$

$$- \frac{\Delta t}{2} f''(t_n) + \dots$$

Collecting like terms:

$$P(t_{n+1}) = P(t_n) + \frac{\Delta t}{2} f(t_{n+1}) + f(t_n) - \frac{(\Delta t)^3}{12} f''(t_n) + \dots .$$

thus, the area of one panel is:

$$A n = P(t_{n+1}) - P(t_n) = \frac{t}{2} f(t_{n+1}) + f(t_n)$$

$$- \frac{(\Delta t)^3}{12} f''(t_n) + \dots .$$

the first term in the right hand side is the trapezoidal rule for a single panel. The next term is the error term with the higher order terms truncated.

To evaluate the integral over the entire interval, the contributions of each panel are added, thus

$$A = \sum_{j=n}^{n} A_j$$

where j = number of panels

or

$$A = \frac{\Delta t}{2}[f(a) + f(b) + 2 \sum_{j=1}^{j-1} f(t_n)]$$

$$- \frac{(\Delta t)^3}{12} \sum_{n=o}^{j-1} f''(t_n)$$

Applying the mean-value theorem to the dominant error term

$$\sum_{n=o}^{j-1} f''_{(t_n)} = j f''_{(t)}$$

$$j f''_{(t)} = \frac{(b-a)}{\Delta t} f''_{(t)}$$

where

a = lower limit

b = upper limit

thus

$$- \frac{(\Delta t)^3}{12} \sum_{n=o}^{j-1} f''_{(t)} = - \frac{(\Delta t)^3 (b-a)}{12 \quad t} f''_{(t)}$$

$$= \frac{(\Delta t)^2}{12} (b\text{-}a)\, f(t)$$

Using simple finite differences, we have

$$f''(t) = \frac{f(b) - f''(a)}{b-a}$$

$$A = \frac{\Delta t}{2}[fo + f1 + 2 \sum_{n=1}^{j-1} f_n]$$

$$- \frac{(\Delta t)^2}{12}[f(b) - f(a)]$$

The preceding formula is the trapezoidal rule with end correction. End corrections may be obtained using finite differences for the derivatives. However, if the number of panels used is many, the non-introduction of the end correction will not give much error.