DESIGN OF HEAT EXCHANGERS

By

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Introduction

A heat exchanger is a device which effects the transfer of heat from a hot fluid to a cold fluid. It is one of the most important and highly utilized piece of equipment found in power generating plants, chemical plants, refrigeration and air conditioning systems and in general, processes which necessitate cooling or heating systems.

Generally, heat exchangers may either be of the open type where the hot and the cold fluid are mixed directly or the closed type where the hot fluid is separated by a wall from the cold fluid. In the open or mixing type, both fluids will reach the same final temperature and the amount of heat transferred can be estimated by equating the energy lost by the hotter fluid to the energy gained by the cooler one. Open feed-water heaters, desuperheaters and jet condensers are examples of heat-transfer equipment employing direct mixing of fluids.

The more common type, however, are heat exchangers in which one fluid is separated from the other by a wall. These types are also called recuperators. There are many configurations of such recuperators ranging from the simple flat wall separating the two fluids and the concentric tube type to the more complex as the shell-and-multipass tube type and the crossflow type. For the concentric tube and the shell-and-single pass tube types the fluids may flow either parallel to each other in the same sense or parallel to each other in the opposite sense. The former is more commonly called parallel-flow heat exchanger, while the latter is often times called counterflow heat exchanger. For the crossflow heat exchanger, the two fluids flow at right angles with each other. Further, crossflow heat exchangers may be identified whether the flow of one fluid is unmixed (constrained) or mixed (unconstrained). The flow of the other fluid may similarly be characterized. Some of these heat exchangers are illustrated below.

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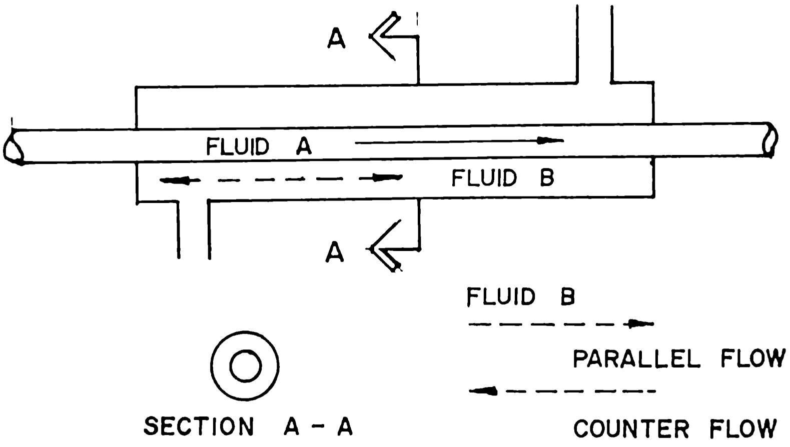


FIG. 1 CONCENTRIC TUBE TYPE

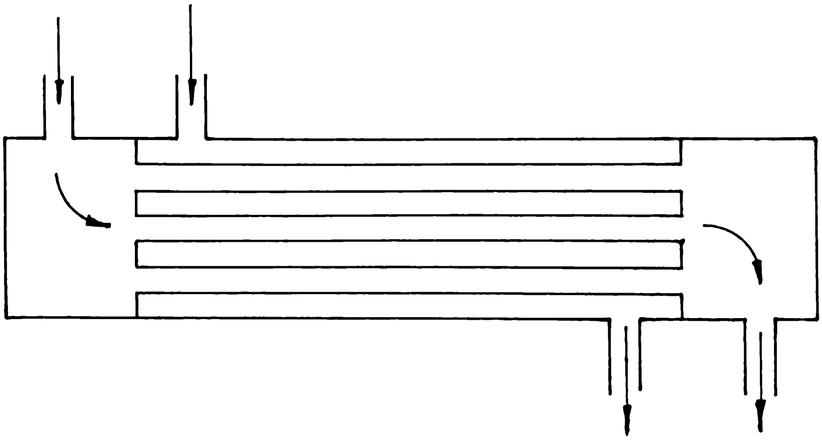


FIG. 2 SHELL-AND-TUBE TYPE SINGLE PASS

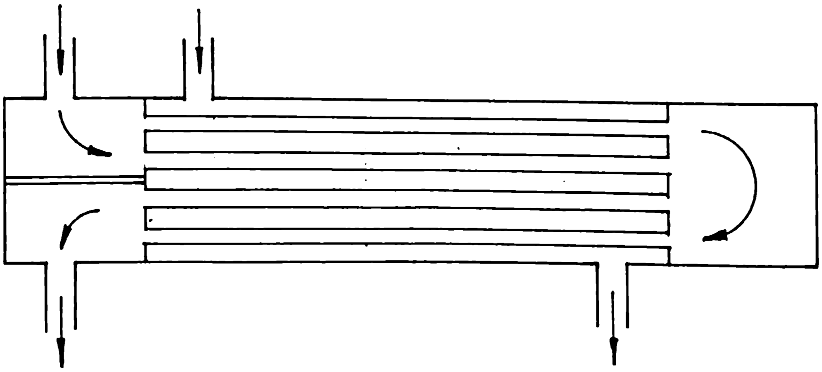


FIG. 3 SHELL-AND-TUBE TYPE DOUBLE PASS

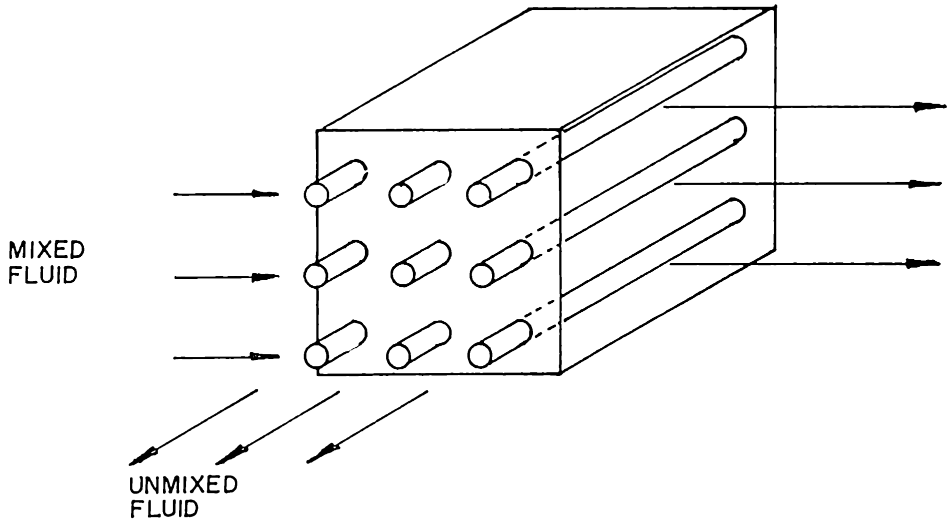


FIG. 4 CROSS FLOW

Design Procedure

The procedure in arriving at an optimum heat exchanger design is a complex one, not only because of the arithmetic involved but more particularly because of the many qualitative judgement that must be introduced. The design methodology may be illustrated in the following diagram.

The complete design of a heat exchanger can be broken down into three major phases, namely:

1. the thermal analysis;
2. the preliminary mechanical design; and
3. design for fabrication or manufacture.

The thermal design is primarily concerned with the determination of the heat transfer surface area required to transfer heat at a specified rate for given flow rates and temperatures of the fluids.

The mechanical design involves considerations of the operating temperatures and pressures, corrosive characteristics of one or both fluids, the relative thermal expansions and accompanying thermal stresses and the relation of the heat exchanger to other equipment concerned.

The design for manufacture requires the translation of the physical characteristics and dimensions into a unit which can be built at a low

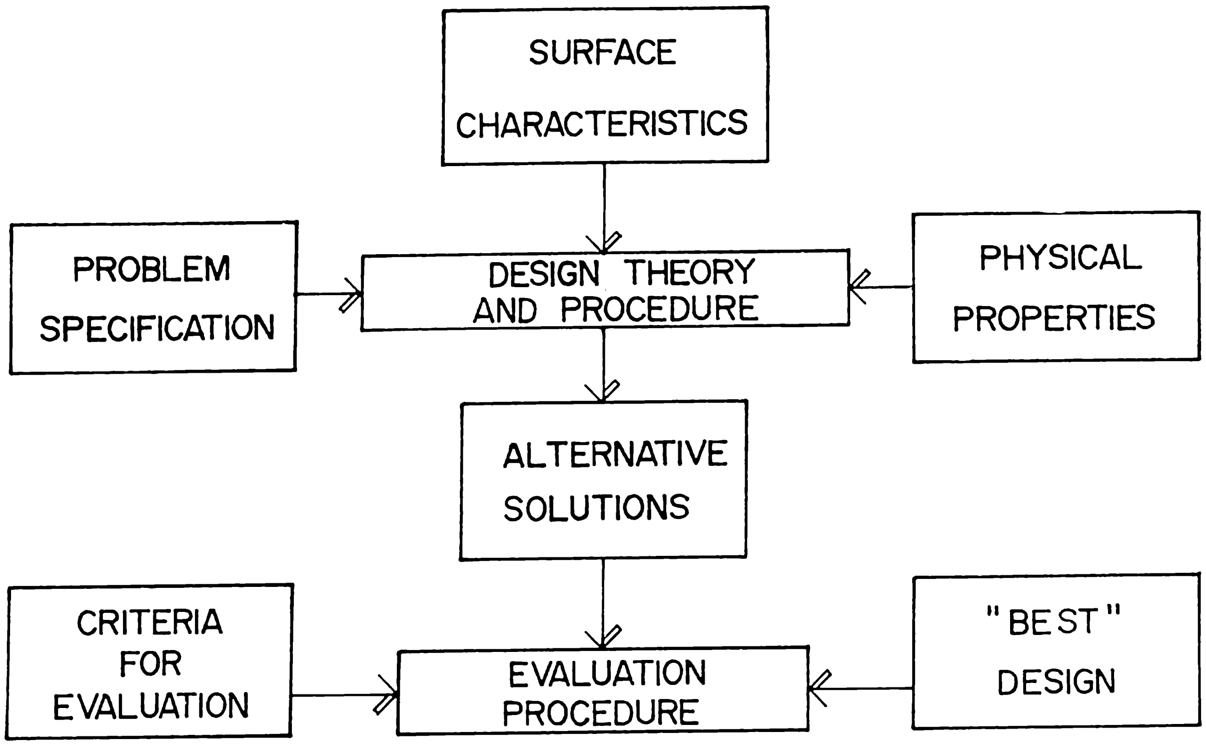


FIG. 5 DESIGN METHODOLOGY

cost. Selection of materials seals, enclosures, and the optimum mechanical arrangement have to be made and the manufacturing procedures must be specified. At all times, safety code requirements (ASME, TEMA, etc.) must be satisfied. The design should likewise take into consideration such other factors as ease of maintenance and interchangeability.

The emphasis in this paper is on thermal analysis. The working equations or relationships needed for thermal analysis may be given by the following:

$$
\begin{aligned}
Q &= UA\Delta Tm \\
&= \dot{m}_c\, C_{p_c}\, (t_{co} - t_{ci}) \\
&= \dot{m}_h\, C_{p_h}\, (t_{hi} - t_{ho})
\end{aligned}
$$

where U = overall coefficient of heat transfer based on heat transfer area A

A = total heat transfer area

ΔTm = true mean temperature difference between the two fluids

$\dot{m}$ = mass flow rate

C_p = specific heat at constant temperature

t = fluid temperature

Subscripts c and h are for cold and hot fluids, respectively.

For plane wall the overall heat transfer coefficient may be represented by:

$$U = \frac{1}{\frac{1}{h_h} + \frac{\Delta x}{K} + \frac{1}{h_c}}$$

For a cylindrical wall,

$$UA = \frac{1}{\frac{1}{A_h h_h} + \frac{\ln(D_o/D_i)}{2\pi\, k} + \frac{1}{A_c h_c}}$$

The mean temperature difference, ΔTm is defined as:

$$\Delta Tm = F \cdot \text{LMTD}$$

where F is some correction factor and LTMD is the log-mean-temperature difference given by the following equation:

$$\text{LMTD} = \frac{(\Delta T)_a - (\Delta T)_b}{\ln \dfrac{(\Delta T)_a}{(\Delta T)_b}}$$

The subscripts a and b refer to the respective ends of the heat exchanger.

Figures 6 to 9 give correction factors for different heat exchanger configurations. Here, P is defined by

$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

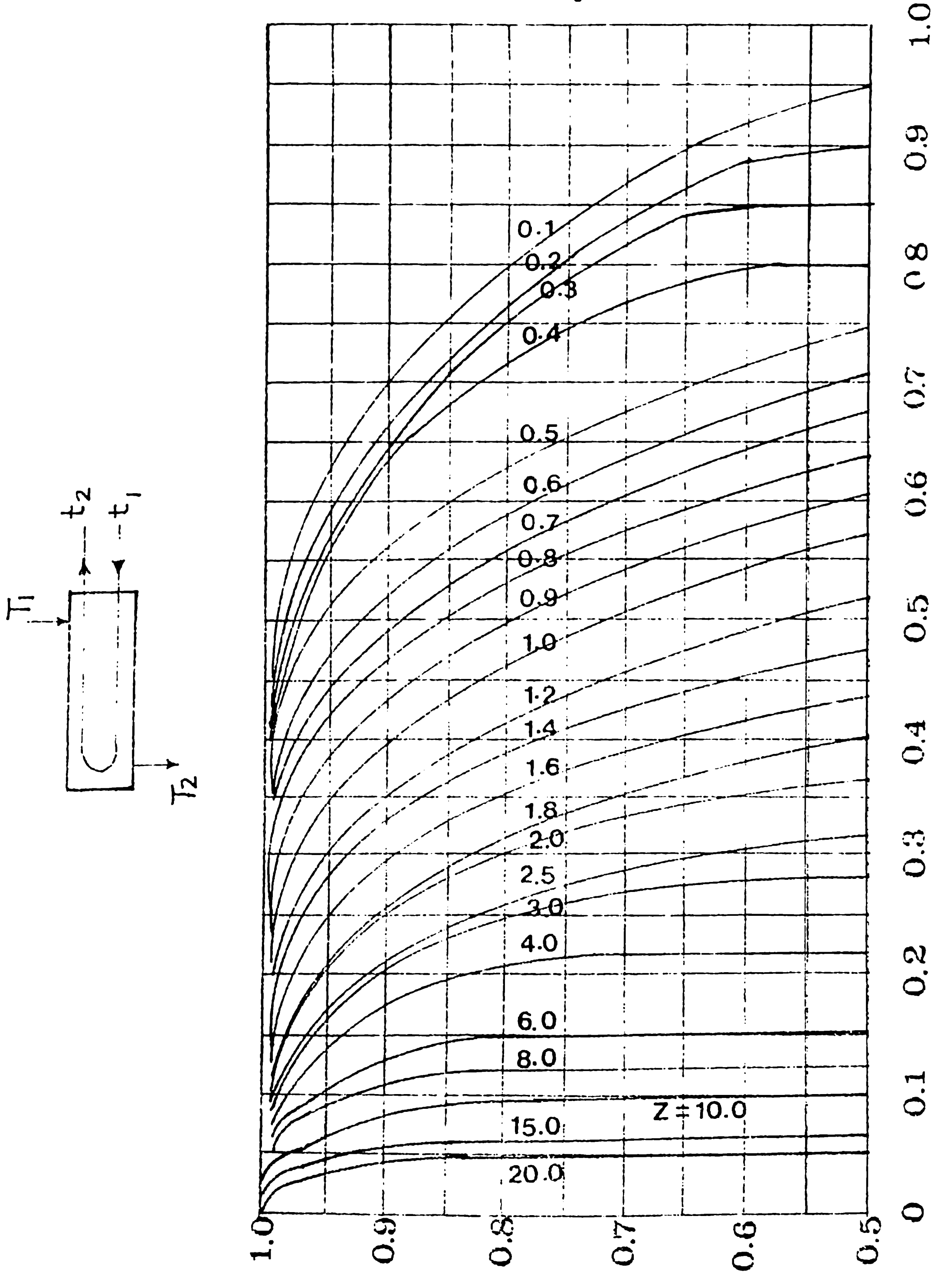
and Z is defined by

$$Z = \frac{T_1 - T_2}{t_2 - t_1}$$

The foregoing expressions will be found convenient when all of the terminal temperatures T_1, T_2, t_1 and t_2 necessary for the evaluation of the true mean temperature difference are known. There are however numerous occasions when the performance of a heat exchanger (i.e. U) is known or can be estimated, but the temperatures of the fluids leaving the exchanger are not known. This type of problem is encountered in the selection of a heat exchanger or when the unit has been tested at one flow rate but service conditions require different flow rates for one or both fluids. The outlet temperatures and the rate of heat flow can only be found by a rather tedious trial-and-error procedure if the above analysis is used.

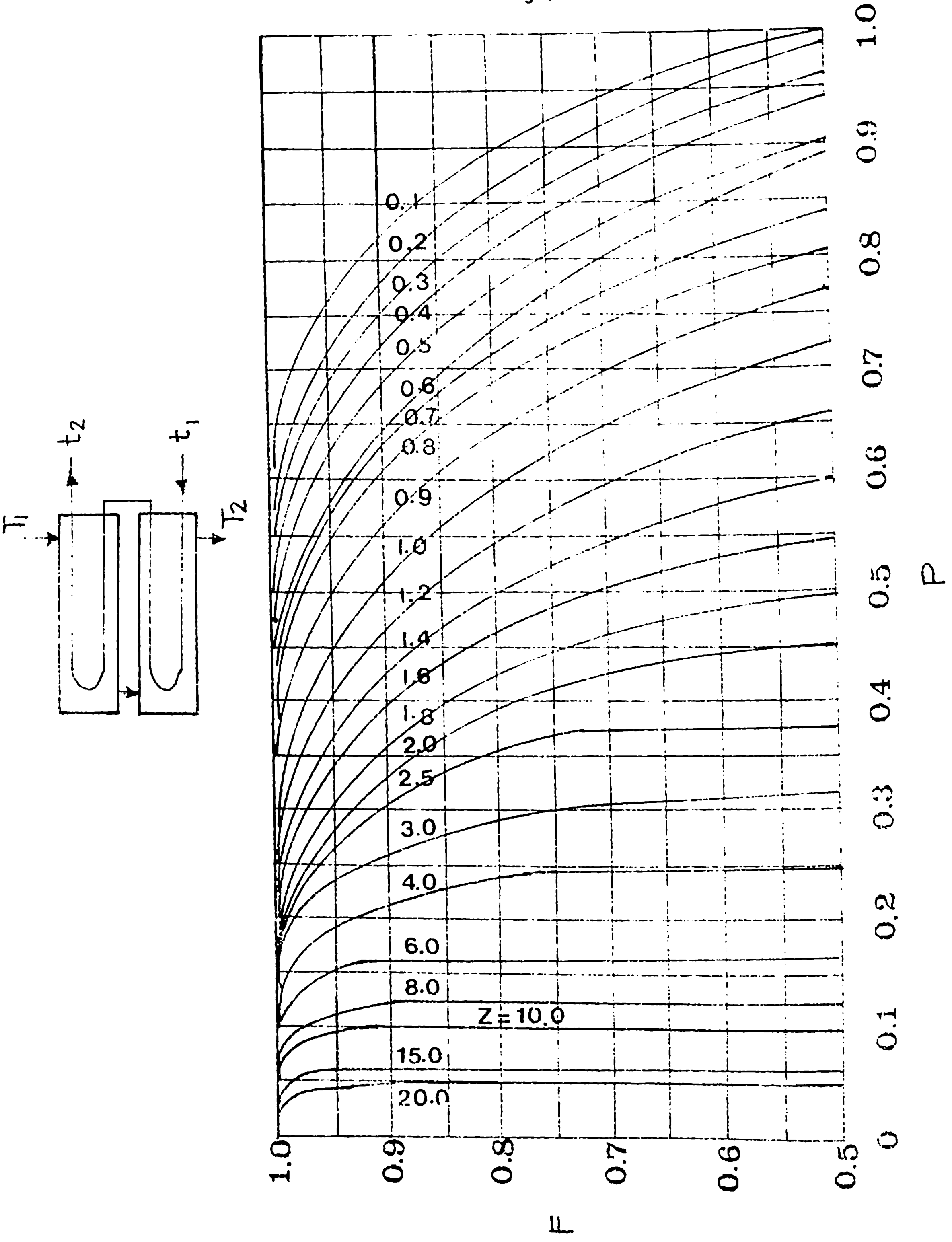
Consider a simple example of U being known for a concentric tube heat exchanger and the outlet fluid temperatures are to be determined (this removes the additional complication of the simultaneous determination of U). Thus if t_{c_i}, t_{h_i}, $\dot{m}_c$, $\dot{m}_h$, C_{p_c}, C_{p_h}, U and A are all specified, the values of Q, t_{co} and t_{ho} may be determined using the foregoing relationships. The simultaneous solution may involve an interative procedure. Thus:

Fig.6

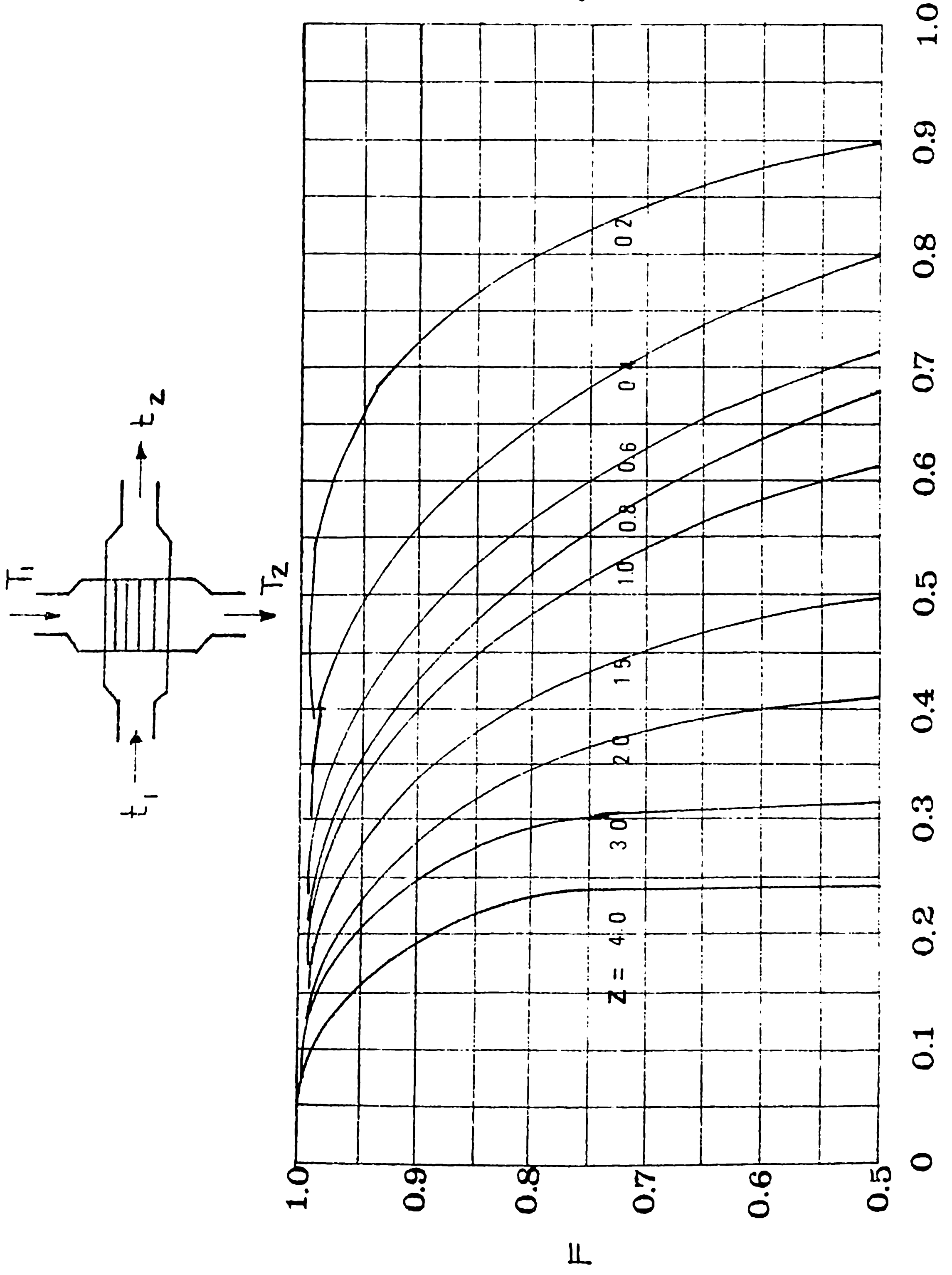


0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
2.5
3.0
4.0
6.0
8.0
Z = 10.0
15.0
20.0

Fig.7



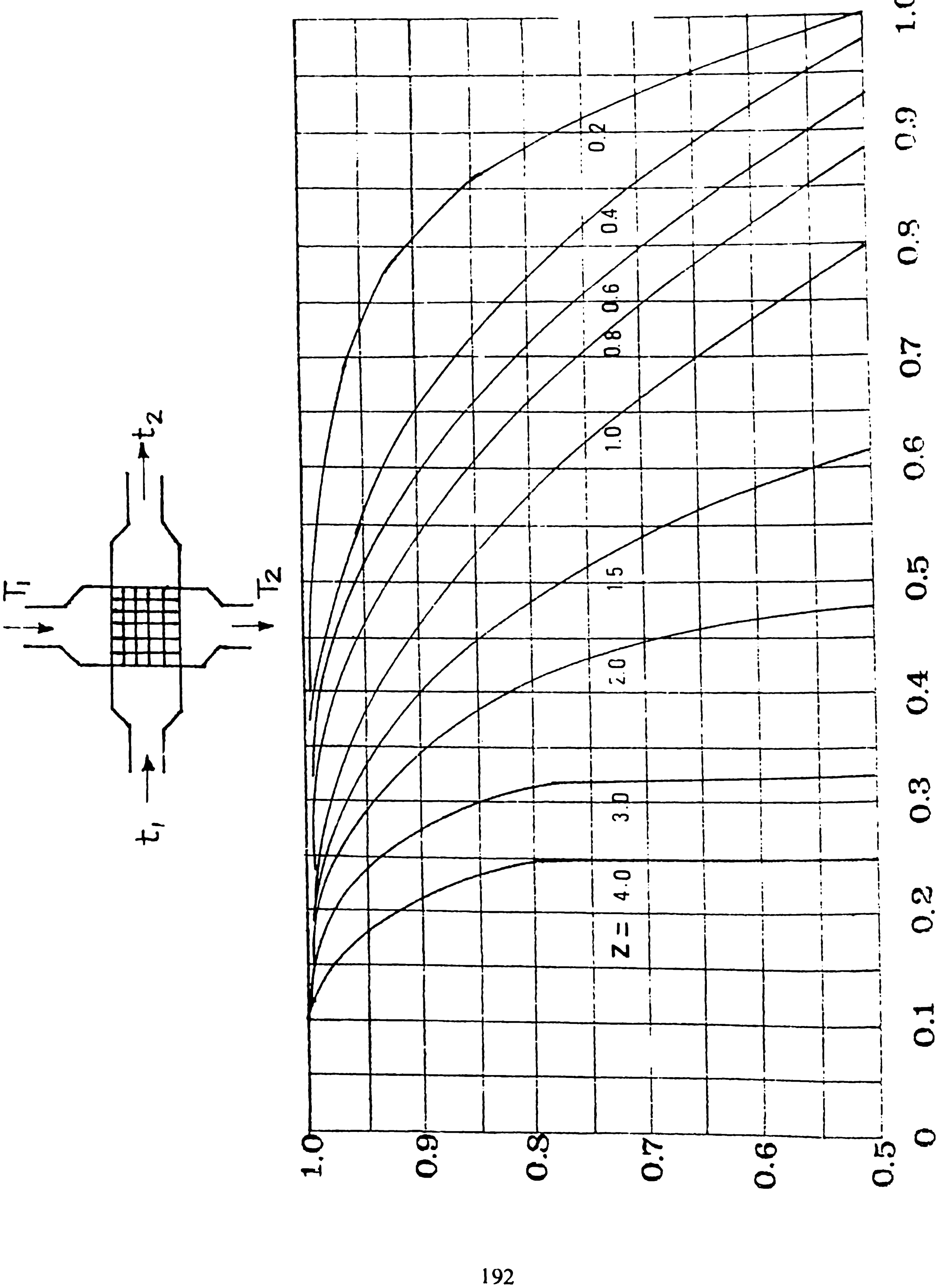
T_1
T_2
t_1
t_2
P
F
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
2.5
3.0
4.0
6.0
8.0
Z=10.0
15.0
20.0



T1
T2
t1
t2
Z = 4.0
3.0
2.0
1.5
1.0
0.8
0.6
0.4
0.2
P
F
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig.8

Fig.9



T_1
T_2
t_1
t_2
z = 4.0
3.0
2.0
1.5
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.0
0.9
0.8
0.7
0.6
0.5

1. assume t_{co}

2. solve for t_{ho}

3. solve for ΔTm

4. solve for Q

5. check if $Q_h = Q_c = Q = \Delta \quad UA\Delta Tm$

6. If not, repeat with new t_{co}. The symbols i and o are for inlet and outlet, respectively.

Consider somewhat more complicated case where $U \neq$ constant (or may vary along the tubes). This is usually the case when the fluid temperature varies along the length of the tube. Thus if t_{c_i}, t_{h_i}, $\dot{m}_c$, $\dot{m}_h$, C_{p_h}, C_{p_c}, and A are known, the following iterative procedure may be used. Consider first a section in the concentric pipe heat exchanger shown in Figure 10. For the sake of discussion, assume that the cold fluid flows inside the tube 1, while the hot fluid flows outside, 4. Thus:

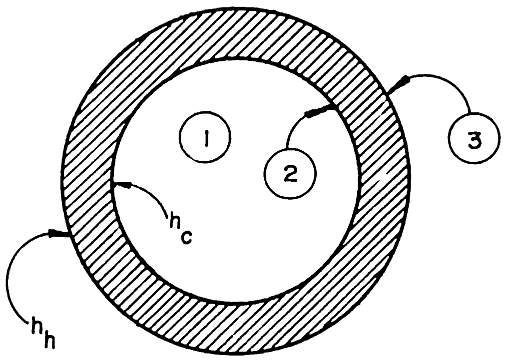


FIG. 10

1. assume t_{co}

2. compute for t_{ho}

3. solve for ΔTm

4. find $t_1 = \dfrac{t_{co} + t_{ci}}{2}$

5. calculate h_c based on t_1

6. assume t_3

   a. find h_h based on t_3

   b. find UA

   c. check the assumed t_3 using:

$$h_h A(t_4 - t_3) = UA(t_4 - t_1)$$

where: U is based on tube outside area (hot side), and

$$t_4 = \frac{t_{ho} + t_{hi}}{2}$$

7. repeat step 6 until satisfactory check of t_3 is obtained

8. solve for new t_{co} using

$$\dot{m}_c \, C_{p_c}(t_{co} - t_{ci}) = UA\Delta Tm$$

9. check if the new t_{co} from step 8 is equal to the assumed t_{co} in step 1.

10. If $(t_{co})_{NEW} \neq (t_{co})_1$, repeat process using $(t_{co})_{NEW}$ in step 1.

The above illustrates the incrased complexity of the procedure, even considering the relatively simple configuration or geometry. If a shell-and-tube type or a crossflow heat exchanger had been chosen in the above analysis that would require the use of the correction factor for the LTMD, the difficulty of the iterative procedure is compounded. In such case, it is desirable to circumvent entirely any reference to the logarithmic or any other mean temperature difference.

To obtain an equation for the rate of heat transfer which does not involve any of the outlet temperatures, the concept of heat exchanger effectiveness, ϵ is introduced. This is defined as follows:

$$\epsilon = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate possible}}$$

The maximum possible heat transfer rate would be obtained in a counterflow heat exchanger of infinite heat transfer area. In this type of unit, if there are no external heat losses, the outlet temperature of the colder fluid equals the inlet temperature of the hotter fluid when

$\dot{m}_c\, C_{p_c} \leq \dot{m}_h\, C_{p_h}$; when $\dot{m}_h\, C_{p_h} < \dot{m}_c\, C_{p_c}$, the outlet temperature of the warmer fluid equals the inlet temperature of the colder one. In other words, the effectiveness compares the actual heat-transfer rate to the maximum rate whose only limit is the second law of thermodynamics. Depending on which of the hourly capacities is smaller, the effectiveness is:

$$\epsilon = \frac{C_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})}$$

or

$$\epsilon = \frac{C_c(t_{co} - t_{ci})}{C_{min}(t_{hi} - t_{ci})}$$

where $C_h = \dot{m}_h C_{p_h}$, $C_c = \dot{m}_c C_{p_c}$

and C_{min} is the smaller of the $\dot{m}_h\, C_{p_h}$ and $\dot{m}_c\, C_{p_c}$ magnitudes. Once the effectiveness of a heat exchanger is known, the rate of heat transfer can be determined directly from the equation

$$Q = \epsilon\, C_{min}\,(t_{hi} - t_{ci}) \text{ since } Q = C_h\,(t_{hi} - t_{ho}) = C_c\,(t_{co} - t_{ci})$$

To illustrate the method of deriving an expression for the effectiveness of a heat exchanger, let us consider the parallel-flow arrangement. For parallel flow,

$$\Delta Tm = \text{LMTD}$$

$$= \frac{(t_{hi} - t_{ci}) - (t_{ho} - t_{co})}{\ln\left[\dfrac{t_{hi} - t_{ci}}{t_{ho} - t_{co}}\right]}$$

therefore

$$Q = UA\Delta Tm$$

Or,

$$Q = UA\left[\frac{(t_{hi} - t_{ci}) - (t_{ho} - t_{co})}{\ln\left[\dfrac{t_{hi} - t_{ci}}{t_{ho} - t_{co}}\right]}\right]$$

$$= C_h(t_{hi} - t_{ho})$$

rearranging,

$$UA\left[(t_{hi} - t_{ho}) + (t_{co} - t_{ci})\right] = C_h(t_{hi} - t_{ho}) \ln \frac{t_{hi} - t_{ci}}{t_{ho} - t_{co}}$$

or

$$\ln\left[\frac{t_{hi} - t_{ci}}{t_{ho} - t_{co}}\right] = UA\left[\frac{1}{C_h} + \frac{(t_{co} - t_{ci})}{C_h(t_{hi} - t_{ho})}\right]$$

but $\quad C_c(t_{co} - t_{ci}) = C_h(t_{hi} - t_{ho})$

therefore

$$\frac{t_{co} - t_{ci}}{C_h(t_{hi} - t_{ho})} = \frac{1}{C_c}$$

so that

$$\ln\left[\frac{t_{hi} - t_{ci}}{t_{ho} - t_{co}}\right] = UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or

$$\left[\frac{t_{ho} - t_{co}}{t_{hi} - t_{ci}}\right] = e^{-UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]}$$

Note that

$$\begin{aligned}
\frac{t_{ho} - t_{co}}{t_{hi} - t_{ci}} &= \frac{(t_{ho} - t_{hi}) - (t_{co} - t_{ci}) + (t_{hi} - t_{ci})}{t_{hi} - t_{ci}} \\
&= \frac{\frac{-C_{min}\epsilon}{C_h}(t_{hi} - t_{ci}) - \frac{C_{min}\epsilon}{C_c}(t_{co} - t_{ci}) + (t_{hi} - t_{ci})}{(t_{hi} - t_{ci})} \\
&= 1 - \epsilon\left[\frac{C_{min}}{C_h} + \frac{C_{min}}{C_c}\right]
\end{aligned}$$

therefore

$$\epsilon = \frac{1 - e^{-UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]}}{\frac{C_{min}}{C_h} + \frac{C_{min}}{C_c}}$$

when $C_h < C_c$ so that $C_h = C_{min}$, then

$$\epsilon_h = \frac{1 - e^{-\frac{UA}{C_h}\left[1 + \frac{C_h}{C_c}\right]}}{1 + \frac{C_h}{C_c}}$$

When $C_c < C_h$ so that $C_c = C_{min}$, then

$$\epsilon_c = \frac{1 - e^{-\frac{UA}{C_c}\left[1 + \frac{C_c}{C_h}\right]}}{1 + \frac{C_c}{C_h}}$$

In the first case $\frac{C_h}{C_c} = \frac{C_{min}}{C_{max}}$ while in the second, $\frac{C_c}{C_h} = \frac{C_{min}}{C_{max}}$

Therefore, finally, for either we can simply write the effectiveness

$$\epsilon = \frac{1 - e^{-\frac{UA}{C_{min}}\left[1 + \frac{C_{min}}{C_{max}}\right]}}{1 + \frac{C_{min}}{C_{max}}}$$

This is the effectiveness for parallel flow concentric tube heat exchanger.

The ratio $\frac{UA}{C_{min}}$ is called the *number of transfer units* or NTU, and this method of thermal analysis of heat exchangers is often times called the *NTU method.*

Therefore if we define

$$NTU = \frac{UA}{C_{min}} \quad \text{and} \quad C_R = \frac{C_{min}}{C_{max}},$$

we have for the parallel flow arrangement the effectiveness written as,

$$\epsilon = \frac{1 - e^{-NTU(1 + C_R)}}{1 + C_R}$$

In general therefore the effectiveness for any heat exchanger may be written as

$$\epsilon = f(NTU, C_R)$$

Table 1 provides expressions for ϵ for various types of heat exchangers, including reference to graphs representing effectiveness.

Table 1

Type	$\epsilon =$	Figure
Parallel flow	$\dfrac{1 - e^{-NTU(1 + C_R)}}{1 + C_R}$	11
Counter flow	$\dfrac{1 - e^{-NTU(1 - C_R)}}{1 - C_R e^{-NTU(1 - C_R)}}$	12
One shell — 2, 4, 6 tube passes	$2\left[1 + C_R + \dfrac{1 + e^{-NTU(1 + C_R^2)^{1/2}}}{1 - e^{-NTU(1 + C_R^2)^{1/2}}}(1 + C_R^2)^{1/2}\right]^{-1}$	13
Crossflow with both fluids unmixed	$1 - e^{\{C_R(NTU)^{.22}[\exp[-C_R(NTU)^{.28}] - 1]\}}$	14
Crossflow with one fluid unmixed, C_{min}	$C_R\left\{1 - e^{-CR[1 - \exp(-NTU)]}\right\}$	15 (dashed curves)
Crossflow with one fluid unmixed, C_{max}	$1 - e^{-C_R[1 - \exp(-(NTU)(C_R)]}$	15 (solid curves)

The performance of heat exchangers as developed in the preceding section depends upon the heat transfer surfaces being clean. During operation with most liquids and some gases, a dirt film gradually builds up on the heat-transfer surface. Its effect, which is sometimes termed *fouling*, is to increase the thermal resistance. Generally, refrigerating liquids have fouling resistance of about .001 hr °F ft^2 /BTU.

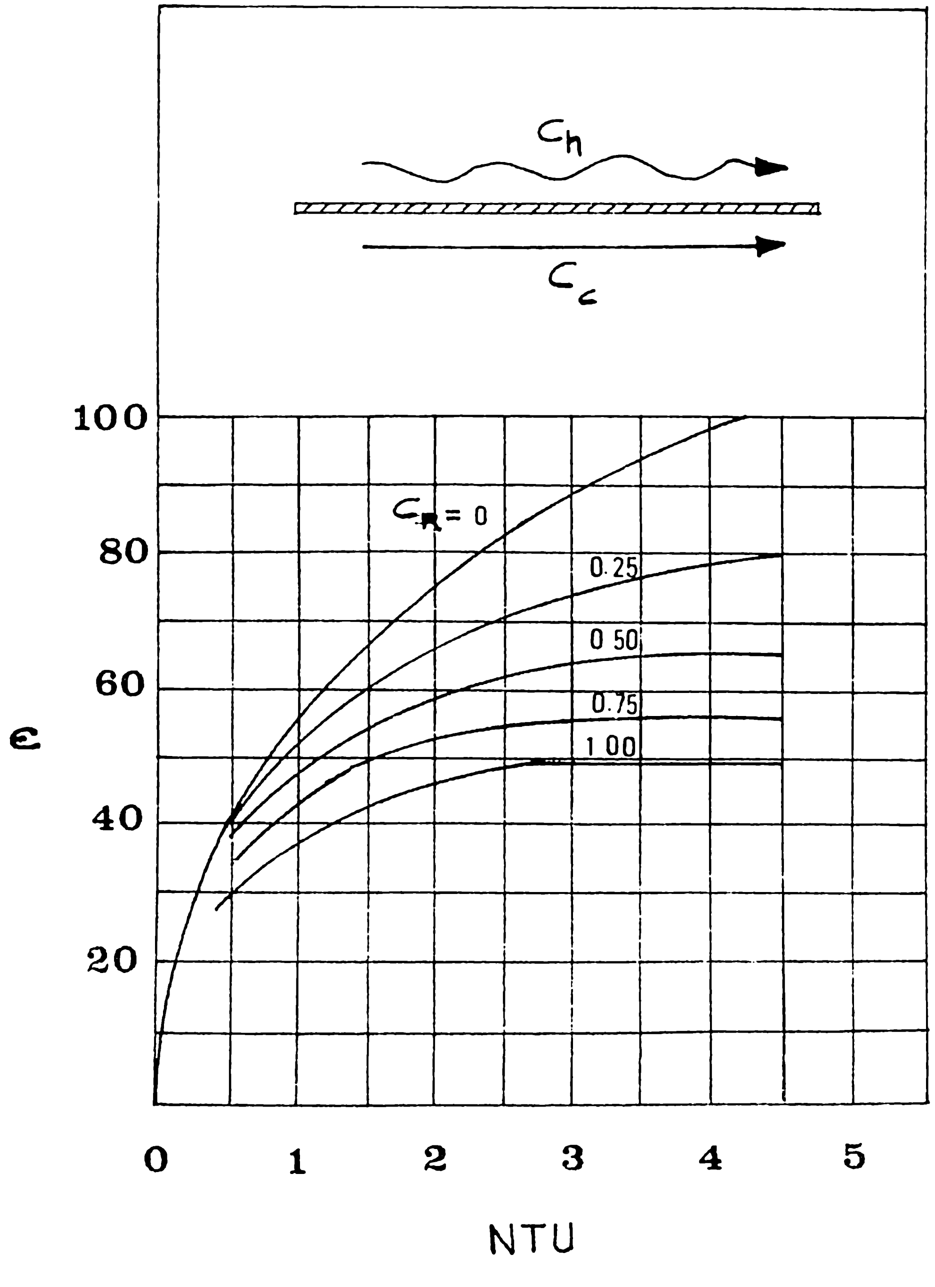


Fig. 11
Parallel Flow

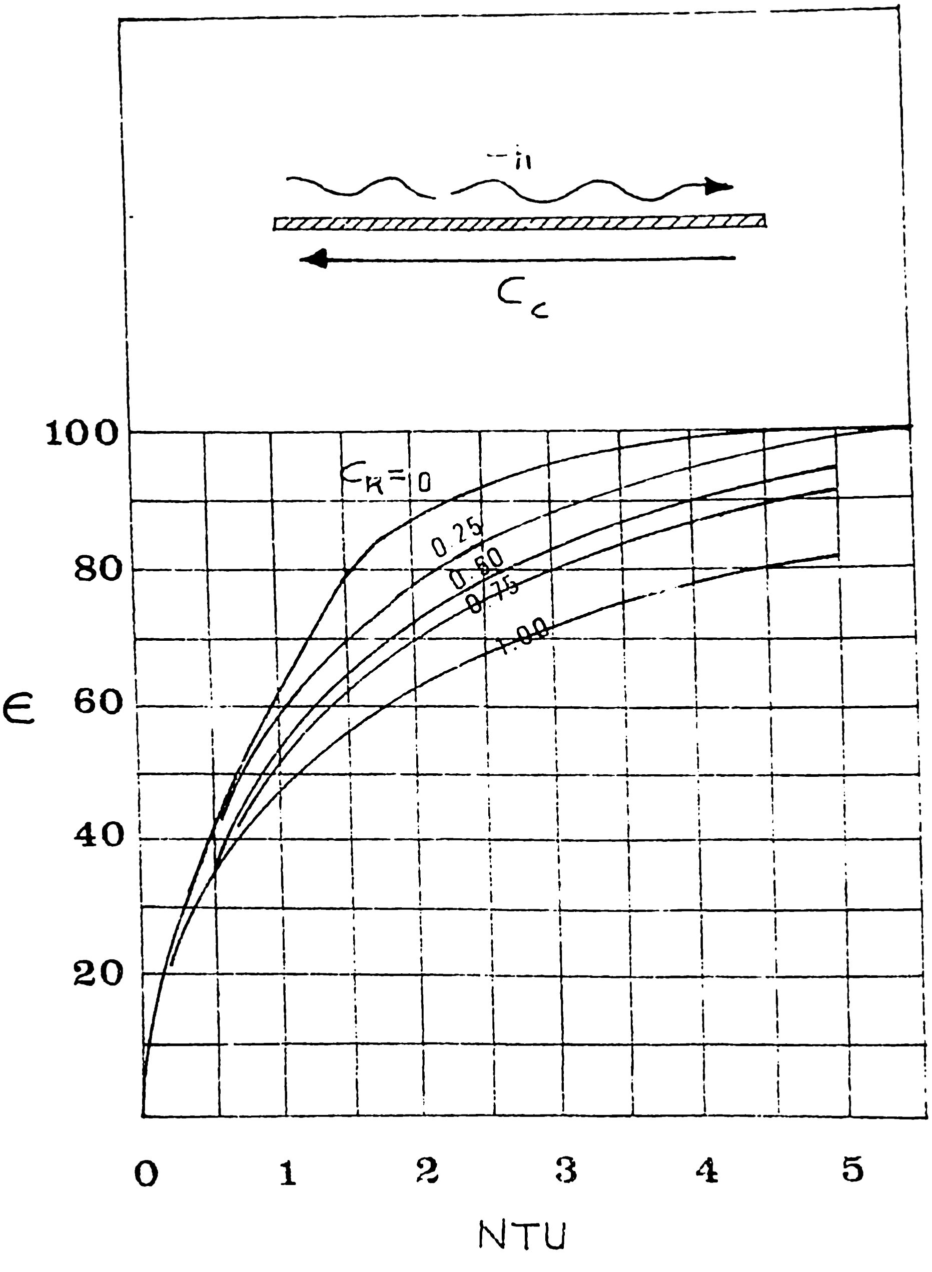


Fig. 12
Counter Flow

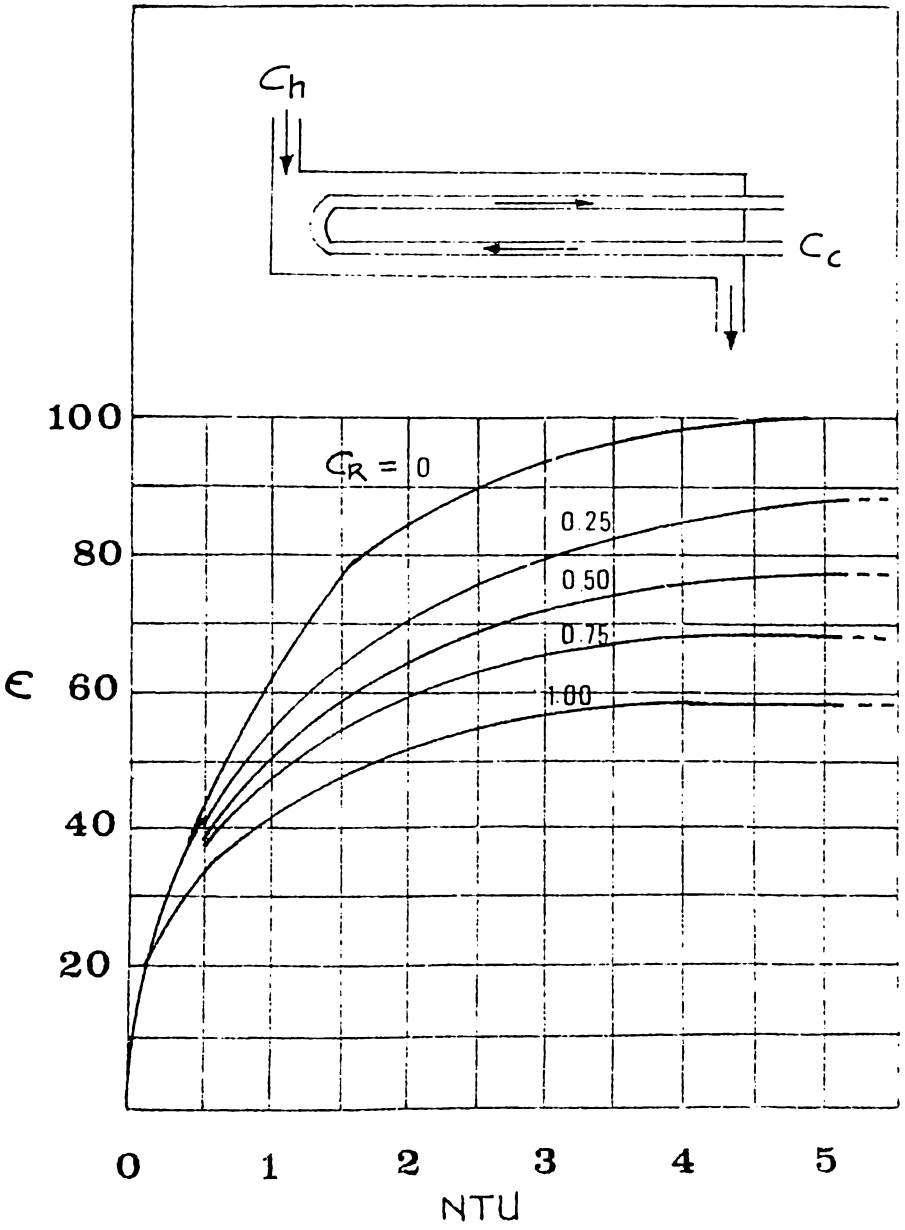


Fig. 13
One shell – two tube passes
(including multiples of 2 passes)

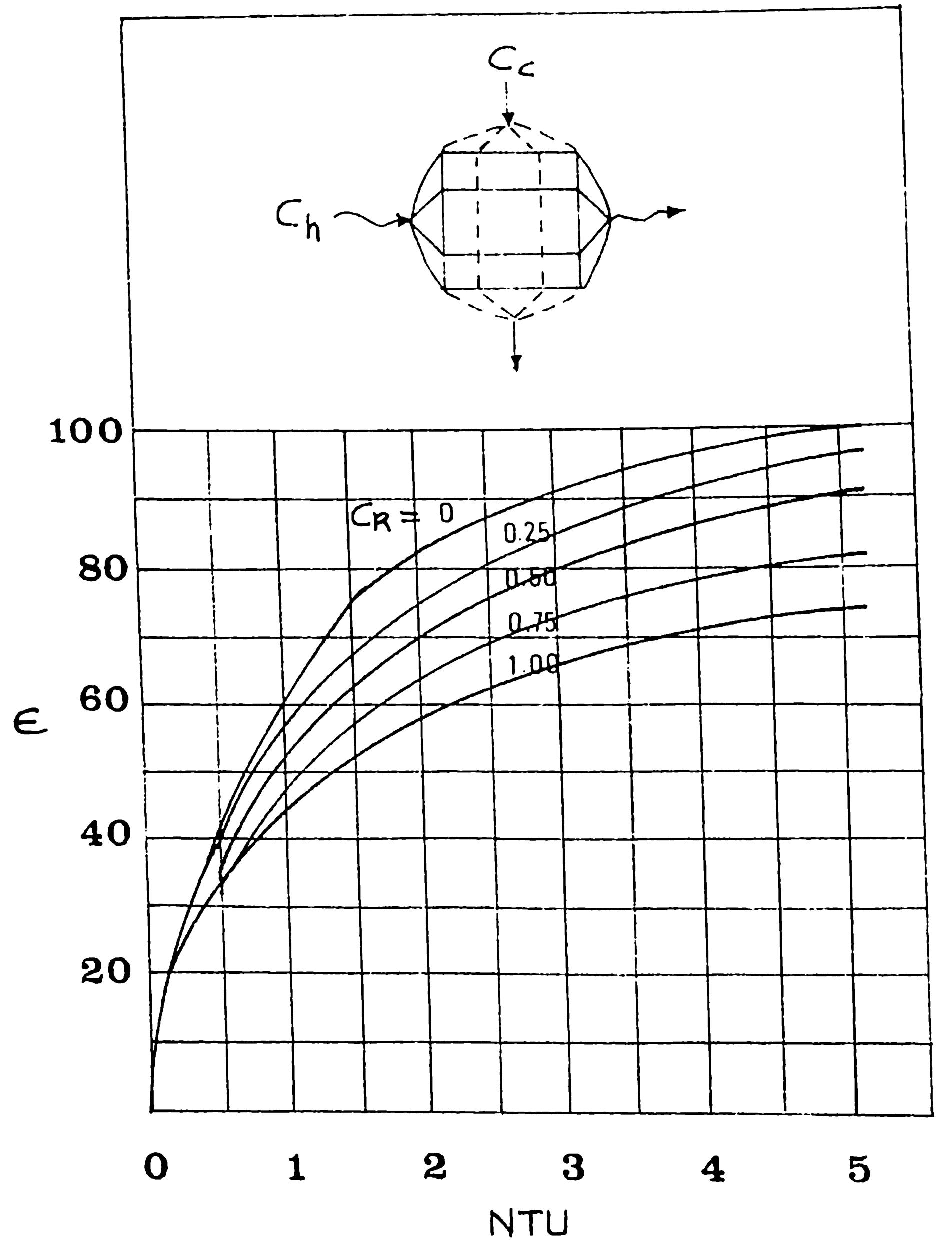


Fig. 14
Cross Flow
with both Fluids unmixed

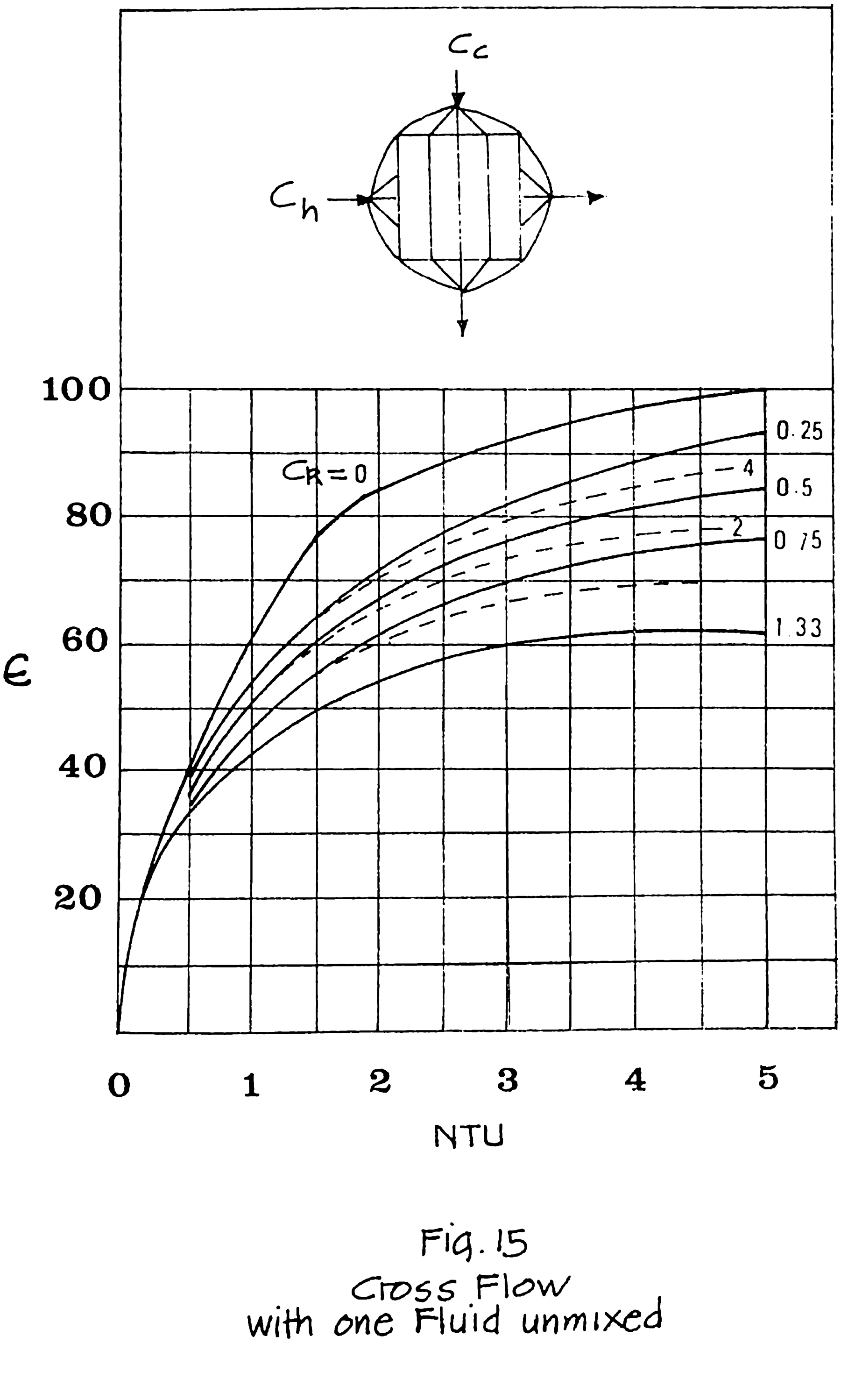


Fig. 15
Cross Flow
with one Fluid unmixed

References

1. Kreith, F. "Principles of Heat Transfer", Intex Press, Inc., 1973.

2. Kays, W.M. and A.L. London. "Compact Heat Exchangers". 2nd Ed. McGraw Hill Book Co., N.Y. 1964.

3. Jakob, M. "Heat Transfer". Vol. 2. John Wiley and Sons, Inc., N.Y. 1957.

4. (Author's personal notes in Heat Transfer.)