# STRENGTH DESIGN FOR COMBINED TORSION' SHEAR, AND BENDING**

by

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## Introduction

Recent trends in architecture have resulted in the structural engineer being called upon to design structures in concrete in an ever-widening variety of shapes. Cantilevered balcony beams, curved balcony girders, spiral staircases, skewed bridges, are but a few of the many structural members which confront the structural engineer with the task of proportioning these to withstand not only the action of bending and shear but also the primary action of torsional moments arising from their geometry or from unsymmetrical loading. In the more traditional forms of structures the effects of torsion were masked, either by increasing the beam dimensions over and above that required for bending and shear in order to provide for the ill-effects of torsional action, or by treating torsional action of secondary importance only. In the 1963 edition of the ACI code, for example, only one paragraph was devoted to torsion design, quote:

"SEC. 921 – Torsion

(a) In edge or spandrel beams, the stirrups provided shall be closed and at least one longitudinal bar shall be placed in each corner of the beam section, the bar to be at least the diameter of the stirrup or 1/2 in., whichever is larger."

In monolithic construction of concrete where secondary beams frame into primary beams on one side only and in beams unsymmetrically loaded, the effect of torsional action can be considerable. The simultaneous action of bending, shear and torsion always occurs in frames supporting loads normal to their planes, and in these cases, torsional moment also becomes a primary effect.

Especially so with the advent of strength design method in reinforced concrete members in comparatively recent years and with the increasing attention being paid to limit design methods, the structural engineer will undoubtedly be continually designing more slender members in which the effects of torsional action can no longer be marked. Recognition of the influence of torsion on the load-carrying capacity of reinforced conrete sections has prompted the incorporation of more detailed provisions for torsion design some ten years ago in conjunction with ultimate strength design in the ACI 318-71 edition of the Building Code Requirements for Reinforced Concrete.

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The problem with concrete as a construction material is complicated by the fact that it does not have the necessary ductility required of "plastic" analysis aside from a non-linear stress behavior. Add to this the fact that the behavior in tension is not identical to that in compression, the problem of analysis for torsional action becomes much more complicated. Reinforced concrete structural members are most often rectangular rather than circular, in which case warping of the member cross section accompanies torsional action. In the case of circular sections, plane circular sections before twisting remain plane and circular after twisting, and the maximum shear stress is developed at the outermost fibers. This is not so in the case of rectangular sections; maximum shear stresses are developed at the *middle of the longer side*. With concrete as the material, developers of code provisions have to rely too much on experimental research data and the enormous amount of research interest being paid to the study of the behavior and evaluation of the ultimate strength in torsion of concrete, plain or reinforced or prestressed, cannot be overemphasized.

Although the first recorded investigations of concrete in torsion began at the turn of the present century pioneered by German engineers (153), it was not until about the second post-war period that the first recorded investigations under combined loading were carried out. Since forced concrete in torsion has grown as design philosophy has moved towards greater consideration of structural inter-action and limit design. Despite the rapid growth of research interest in the subject, consideration for torsional action in reinforced concrete is fairly recent as evidenced in the international symposium of torsion in structural concrete in 1968 (179). This resulted in the inclusion of torsion clauses into the proposed revision of ACI 318-63 in 1970 and in the British Unified Code of Practice. Some countries in Europe, however, incorporated torsion clauses in their codes of practice as early as thirty years ago (151).

While pure torsional action seldom occurs in a member of a structure, an understanding of the behavior of concrete under pure torsional loading becomes an essential tool in the analysis under combined loading. The earliest of these theories include those of a German (Rausch, 1929). Americans (Turner and Davies, 1954), (Andersen, 1935), Marshall and Tembe (1949) and a British (Cowan, 1950). The works of Rausch and Cowan became the basis of the German and the Australian codes respectively. The more recent works of Hsu (1968) and Swann (1970) became the basis for the torsion clauses in the ACI Code and in the British Unified Code of Practice, respectively.

For a more comprehensive review of the studies carried out in torsion of structural concrete, the reader is referred to a bibliography appended to this paper.

## Analysis and Design for Torsion According to the ACI Code

These provisions are all based on the premise that the torsional strength of a reinforced concrete beam is a contribution of the strength due to the concrete and of the strength due to the reinforcements, which is analogous to the phenomenon of shear. Thus, Eq. 11-21 of a reinforced concrete section is given by

$$T_n = T_c + T_s \qquad \text{(Eq. 11-21)}$$

where $T_n$ = nominal torsional moment strength
$T_c$ = nominal torsional moment strength provided by concrete
$T_s$ = nominal torsional moment strength provided by torsion reinforcement

## Torsional Moment Strength Provided by Concrete

In arriving at the torsional moment strength due to the concrete, ACI utilized the classical theories of elasticity and plasticity. According to these theories, a rectangular section of dimensions x by y subject to pure torsional moment can sustain a torque equal to

$$T = \alpha x^2 y s_{max}$$

where $\alpha$ = coefficient dependent upon the ratio y/x
$y$ = longer overall dimension of rectangular part of cross section
$x$ = shorter overall dimension of rectangular part of cross section
$s_{max}$ = maximum shear stress of the material

Under pure torsional action, the shearing stress is numerically equal to the tensile stress and since concrete is weaker in tension than in shear, the tensile strength of the concrete initiates failure of a section subject to torsion. Thus, if $f_t$ = tensile strength of the concrete,

$$T = \alpha x^2 y f_t$$

The coefficient $\alpha$ varies from 0.208 to 0.333 in the *elastic* theory and from 0.333 to 0.5 in the *plastic* theory. Since concrete is neither plastic nor elastic, the ACI Code adopted a value of 1/3 for $\alpha$ for simplicity. Since in almost all cases, a concrete beam is monolithic with a floor slab and often acts as a T-beam, and even in some cases where isolated T-beams are used, ACI modified the expression $x^2y$ to $\Sigma x^2 y$ on the assumption that the torsional moment strength of a flanged member is equal to the sum of the torsional strength of the web and the flanges. Tests on isolated T-beams have shown that this assumption is conservative provided that the effective overhanging flange width does not exceed 3 times the thickness of the flanges.

Using a limiting torsional shear (or tensile) stress of $2.4\sqrt{f'_c}$ the ACI expression for torsional moment strength of a *reinforced concrete section* under pure torsional moment is given by

$$T_c = \frac{\Sigma x^2 y(2.4\sqrt{f'_c})}{3} = 0.8\sqrt{f'_c}\Sigma x^2 y$$

If flexural shear $V_u$ acts simultaneously with a torsional moment $T_u'$ the nominal torsional moment strength is modified to

$$T_c = \frac{0.8\sqrt{f'_c}\Sigma x^2 y}{\sqrt{1 + \left[\frac{0.4V_c}{c_t T_u}\right]^2}} \qquad \text{(Eq. 11-22)}$$

where $V_u$ = factored shear force at section
$T_u$ = factored torsional moment at section
$c_t$ = factor relating shear and torsional stress properties
$= \frac{b_w d}{\Sigma x^2 y}$

At this stage, it must be noted that the value of $T_c$ given above is *not* the strength due to the concrete of a corresponding plain concrete beam of identical properties and dimensions. The quantity $2.4\sqrt{f'_c}$ gives a strength due to the concrete which is 40 percent only of the cracking torque of a plain concrete beam. Consequently, it conservatively predicts torsional strength at cracking and failure of an unreinforced web by 2.5 times. ACI contends that such conservatism is justified for two reasons. First, the torsional strength of a beam without web reinforcement may be reduced by up to one-half due to the simultaneous application of a bending moment and a torsional moment. Therefore, by specifying a limiting torsional shear stress which corresponds to 40 percent of the cracking torque, the effect of bending moment on the torsional strength of beams without web reinforcement may be neglected. Second, any member subjected to a large torsional moment should be designed with torsion reinforcement.

In the case of combined torsion, shear, and flexure, the interaction of torsion and shear is taken into account by means of a circular interaction curve. The square root factor in Eq. 11-22 was derived on this basis.

The effect of bending is not shown explicitly in Eq. 11-22. However, the adoption of a torsional shear stress which corresponds to 40 percent of the cracking torque, also considers the effect of bending. Hence the equation is conservative for any combination of torsion, shear, and bending in beams with stirrups.

Let us consider, as an example, a solid rectangular concrete beam 12 in. x 20 in. cast out of a 3000 psi concrete and devoid of any longitudinal or transverse reinforcement. If this plain concrete beam were tested in pure torsion, it would attain its first crack (and immediately fails! ) when the torsional moment reaches a value equal to

$$T_{cr} = T_u = \frac{1}{3} 12^2 \times 20 \times 6\sqrt{3000}$$

$$= 26.3 \text{ kip-ft}$$

Note that this same beam would have a nominal flexural strength of about 332 kip ft. (assuming 3 in. concrete cover to tension steel with $f_y$ = 40 ksi).

Supposing the section was made hollow instead of solid by leaving a wall thickness of h = 3 inches all around, or reducing the cross-sectional area by 35 percent, ACI suggests that there is *no* reduction in the value of the cracking torque since the wall thickness provided is at least equal to x/4 as provided for in Section 11.6.1.2. If, however, the wall thickness is reduced further to not less than x/10, the cracking torque would be that equal to the cracking torque of a comparable solid section multiplied by the factor 4h/x.

### Torsional Moment Strength Provided by Torsion Reinforcement

Let us now go back to the solid beam and start putting in the reinforcements. If the beam is reinforced with longitudinal steel alone, laboratory tests will show that there is practically no increase in the cracking or failure torque over and above that of the plain section since the longitudinal reinforcements alone are very much ineffective in resisting torsional stresses. On the other hand, if this same beam is going to be reinforced with transverse reinforcement alone, again, there would be registered no increase in cracking or failure torque. These two cases suggest that both longitudinal and transverse steel must be present by some amounts in order to increase the torsional moment capacity.

Let us now reinforce this beam with No. 3 closed stirrups on leg dimensions $x_1$ = 8 in. and $y_1$ = 16 in. Assume the yield strength of the stirrups equal to $f_y = 40{,}000$ psi. Section 11.6.9.3 provides, as defined by Eq. 11-24, the volume of longitudinal reinforcement be equal to the volume of the closed stirrups, unless a greater amount of longitudinal reinforcement is required to satisfy other requirements. Assuming for the the meantime that we provide an equal volume of longitudinal reinforcement for any given amount of transverse reinforcement, let us provide the stirrups within a wide range of spacings.

Section 11.6.9 provides that the nominal torsional strength provided by the torsion reinforcement shall be computed by

$$T_s = \frac{A_t \alpha_t x_1 y_1 f_y}{s} \qquad \text{(Eq. 11-23)}$$

where $A_t$ = area of one leg of a closed stirrup resisting torsion

$s$ = spacing of stirrups

$$\alpha_t = \left(0.66 \times 0.33 \frac{y_1}{x_1}\right) \text{ but not more than 1.50.}$$

For our beam under consideration,

$$\alpha_t = 0.66 + 0.33 \times \frac{16}{8} = 1.32 \text{ and}$$

$$T_s = \frac{0.11 \times 1.32 \times 8 \times 16 \times 40}{s}$$

$$= \frac{61.95}{s} \text{ kip-ft, assuming s is in inches.}$$

The foregoing relationship shows that the value $T_s$ approaches the value infinity as s approaches zero, that is, assuming that for all values of s, the stirrups always yield. Assuming this to be true, the relationship between $T_s$ and s, and between $T_n$ and $T_s$

is shown in Fig. 1 where it can be seen that the contribution of the torsional reinforcement to the quantity $T_n$ becomes equal to the cracking torque $T_{cr}$ when the spacing of the stirrups is about 2.4 in. On the other hand, the strength due to the reinforcement becomes equal to the contribution by the concrete as defined by equation for $T_c$ when the spacing is about 914 inches.

Note that Section 11.6.8.1 limits the spacing of closed stirrups to a maximum of $\left(\frac{x_1 + y_1}{4}\right)$ or 12 inches, whichever is smaller. With $\frac{x_1 + y_1}{4} = 6$ in. in our case, then the torsional moment strength due to the reinforcements is at least equal to about 10 kip-ft.

Thus, for this beam the ACI Code provides that the quantity $T_c = 0.4 \times 26.3 = 10.5$ kip-ft, $T_s = 10.3$ kip-ft at maximum spacing of stirrups and that

$$T_n = T_c + T_s = 20.8 \text{ kip-ft}$$

which is even less than the torsional moment that cracks the plain concrete beam to failure!

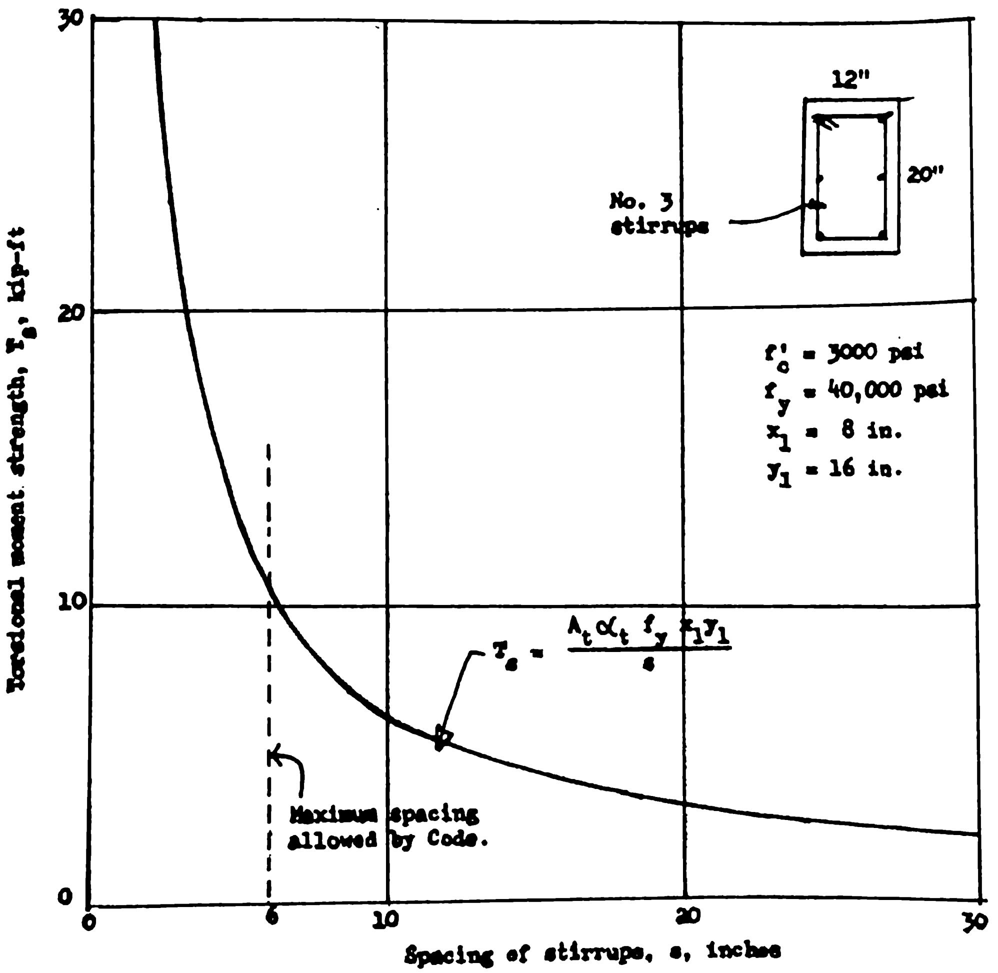

Fig. 1 .

## When to Consider Torsion in Design

Similar to the interaction between an axial load and bending moment (s) in column design, ACI recognizes a close interaction between flexural shear and torsional moment. As indicated earlier, the interaction of flexure with the other two actions is masked by reducing the quantity $T_c$ in evaluating the strength due to the concrete.

In the design for flexural shear, Section 11.3.1.4 provides that at sections where the factored torsional moment $T_u$ exceeds $(0.5\sqrt{f'_c}\Sigma x^2 y)$, the nominal shear strength $V_c$ provided by the concrete in shear is taken as

$$V_c = \frac{2\sqrt{f'_c}b_w d}{\sqrt{1 + \left(2.5C_t\frac{T_u}{V_u}\right)^2}} \qquad \text{Eq. 11-5}$$

In the same manner, Section 11.6.1 provided that torsion effects may be neglected when the factored torsional moment $T_u$ acting at a given section of a structural member is less than the quantity $\phi(0.5\sqrt{f'_c}\Sigma x^2 y)$ where in both cases $\phi = 0.85$. In the case of the latter, it is implied that a limiting torsional moment is initially assigned to a beam section and if the external torsional moment is greater than this limit, the beam section must be reinforced for torsion effects. This limiting torsional moment is based on a maximum torsional stress of $1.5\sqrt{f'_c}$ which stress corresponds to about 25 percent of the *pure torsional* strength of a member *without torsion reinforcement*. Such a simplification is considered by ACI as possible because torsion of such magnitude will not cause significant reduction in ultimate strength in either bending or shear. With the introduction of the capacity reduction factor $\phi = 0.85$, the limiting value of $T_u$ is reduced further to 21 percent. This means that a factor of about 5 is provided to insure that the torsional moment does not induce torsional cracking. However, Section 11.6.3 provides that in a statically indeterminate structure where reduction of torsional moment in a member can occur due to redistribution of internal forces, a maximum factored torsional moment $T_u$ may be reduced to $\phi(1.33\sqrt{f'_c}\Sigma x^2 y)$.

## Miscellaneous Code Provisions Pertinent to Structural Analysis, Design and Detailing

SEC. 11.6.3.2 – In lieu of more exact analysis, torsional loading from a slab shall be taken as uniformly distributed along the member.

SEC. 11.6.4 – Sections located less than a distance d from face of support may be designed for the same torsional moment $T_u$ as that computed at a distance d.

SEC. 11.6.7.1 – Torsion reinforcement, where required, shall be provided in addition to reinforcement required to resist shear, flexure, and axial forces.

SEC. 11.6.7.2 – Reinforcement required for torsion may be combined with that required for other forces, provided the area furnished is the sum of individually required areas and the most restrictive requirements for spacing and placement are met.

SEC. 11.6.7.3 – Torsion reinforcement shall consist of closed stirrups, closed ties, or spirals, combined with longitudinal bars.

SEC. 11.6.7.4 – Design yield stress of torsion reinforcement shall not exceed 60,000 psi.

SEC. 11.6.7.5 – Stirrups and other bars and wires used as torsion reinforcement shall extend to a distance d from extreme compression fiber and shall be anchored according to Section 12.14 to develop the design yield strength of reinforcement.

SEC. 11.6.7.6 – Torsion reinforcement shall be provided at least a distance (d + b) beyond the point theoretically required.

SEC. 11.6.8.2 – Spacing of longitudinal bars, not less than No. 3, distributed around the perimeter of the closed stirrups, shall not exceed 12 in. At least one longitudinal bar shall be placed in each corner of the closed stirrups.

SEC. 11.6.9.2 – A minimum area of closed stirrups shall be provided in accordance with Sec. 11.5.5.5. which states:

"Where factored torsional moment $T_u$ exceeds $\phi(0.5\sqrt{f'_c}\Sigma x^2 y)$, and where web reinforcement is required, a minimum area of closed stirrups shall be computed by

$$A_v + 2A_t = 50\frac{b_w s}{f_y} \qquad \text{(Eq. 11-16)}$$

where $A_v$ = area of shear reinforcement within a distance s

$A_t$ = area of one leg of a closed stirrup resisting torsion within a distance s.

SEC. 11.6.9.3 – Required area of longitudinal bars $A_l$ distributed around the perimeter of the closed stirrups $A_t$ shall be computed by

$$A_l = 2A_t\frac{x_1 + y_1}{s} \qquad \text{(Eq. 11-24)}$$

or by

$$A_l = \left[\frac{400xs}{f_y}\left(\frac{T_u}{T_u + \frac{V_u}{3C_t}}\right) - 2A_t\right]\left(\frac{x_1 + y_1}{s}\right) \qquad \text{(Eq. 11-25)}$$

whichever is greater. Value of $A_l$ computed by Eq. 11-25 need not exceed that obtained by substituting

$$\frac{50b_w s}{f_y} \text{ for } 2A_t.$$

SEC. 11.6.9.4 – Torsional moment strength $T_s$ shall not exceed $4T_c'$.

## References

### PURE TORSION

1. "The behaviour of concrete under the action of torsion," Concrete and Constructional Engineering, Vol. 18, 1923, pp. 510-514.

2. Young, C.R., Sagar, W.L. and Hughes, C.A., "Torsional strength of rectangular concrete sections." Concrete and Constructional Engineering, Vol. 18, 1923, pp. 674-684.

3. Miyamoto, T., "Torsional strength of reinforced concrete,' Concrete and Constructional Engineering, Vol. 22, 1927, pp. 637-647.

4. Andersen, P., 'Experiments with concrete in torsion,' Proc. Vol. 60, American Society of Civil Engineers, 1934, pp. 641-652.

5. Turner, L. and Davies, V.C., 'Plain and reinforced concrete in torsion.' Selected Engineering Paper No. 165, Institution of Civil Engineers, 1934, 31 pp.

6. Andersen, P., 'Experiments with concrete in torsion,' Transaction Vol. 100, American Society of Civil Engineers, 1935, pp. 949-960.

7. Andersen, P., 'Rectangular concrete sections under torsion,' Journal of the American Concrete Institute, Vol. 9, 1937, pp. 1-11.

8. Andersen, P., 'Design of reinforced concrete in torsion,' Proc. Vol. 63, American Society of Civil Engineers, 1937, pp. 1475-1483.

9. 'Torsion: Theory and experiment,' Concrete and Constructional Engineering, Vol. 37, 1942, pp. 315-317.

10. Marshall, W.T. and Tembe, N.R. 'Experiments on plain and reinforced concrete in torsion.' The Structural Engineer, Vol. 19, No. 11, 1941, pp. 177-191.

11. Marshall, W.T., 'The torsional resistance of plastic materials with special reference to concrete.' Concrete and Constructional Engineering, Vol. 39, 1944, pp. 83-88.

12. Cowan, H.J., 'Elastic theory for torsional strength of rectangular reinforced concrete beams.' Magazine of Concrete Research, Vol. 2, 1950, pp. 308.

13. Cowan, H.J., 'Tests of torsional strength and deformation of rectangular reinforced concrete beams.' Concrete and Constructional Engineering, Vol. 46, No. 2, 1951, pp. 51-59.

14. Cowan, H.J. 'Torsion in reinforced and prestressed concrete members.' Journal of the Institution of Engineers, Vol. 28, 1956, pp. 235-240.

15. Ernst, G.C., 'Ultimate torsional properties of rectangular reinforced concrete beams.' Journal of the American Concrete Institute, Vol. 54, 1957, pp. 341-356.

16. Sarkar, S., 'Torsion in reinforced concrete sections.' Indian Concrete Journal, Vol. 33, 1959, pp. 391-393.

17. Cowan, H.J., 'Design of beams subject to torsion related to the New Australian Code.' Journal of the American Concrete Institute, Vol. 31, 1960, pp. 597-618.

18. Pandit, C.S., 'Ultimate strength of reinforced concrete in torsion.' Cement and Concrete, No. 2, 1962, pp. 5-12.

19. Sundara Raja Iyengar, K.T. and Vijaya Rangan, B., 'Torsion of Tee, Ell and I-sections.' Journal of Science and Engineering Research, Vol. X, Part 1, 1966, pp. 19-37.

20. Hsu, T.T.C., 'Torsion of structural concrete – a summary on pure torsion.' Special Publication No. 18, American Concrete Institute, 1968, pp. 165-178.

21. Hsu, T.T.C., 'Ultimate torque of reinforced rectangular beams.' Proc. Vol. 94, ST2, American Society of Civil Engineers, 1968. pp. 485-510.

22. Hsu, T.T.C., 'Torsion of structural concrete – plain concrete rectangular sections.' Special Publication No. 18, American Concrete Institute, 1968, pp. 203-238.

23. Hsu, T.T.C., 'Torsion of structural concrete – behavior of reinforced concrete rectangular members.' Special Publication No. 18, American Concrete Institute, 1968, pp. 261-306.

24. Pandit, G.S., 'Long-time torsion test.' Journal of the American Concrete Institute, Proc. Vol. 65, 1968, pp. 659-661.

25. Kuyt, B., 'The ultimate torsional resistance of rectangular reinforced concrete beams.' Concrete, 1968, pp. 522-527.

26. Navaratnarajah, V., 'Effect of anchorage efficiency of lateral reinforcement on the torsional strength of reinforced concrete beams.' Journal of the American Concrete Institute, Proc. Vol. 65, No. 11, 1968, pp. 965-968.

27. ACI Committee 438, "Tentative recommendations for the design of reinforced concrete members to resist torsion.' Journal of the American Concrete Institute, Proc. Vol. 66, No. 1, 1969, pp. 1-8.

28. Hsu, T.T.C. and Kemp, E.L., 'Background and practical application of tative design criteria for torsion.' Journal of the American Concrete Institute, Proc. Vol. 66, No. 1, 1969, pp. 12.

29. Behera, U. and Rajagopalan, K.S. 'Two-piece U-stirrups in reinforced concrete beams.' Journal of the American Concrete Institute, Proc. Vol. 66, 1969, pp. 522-524.

30. Victor, D.J., 'Effective flange width in torsion.' Journal of the American Concrete Institute, Proc. Vol. 68, No. 1, 1971, pp. 42-46.

## COMBINED FLEXURE AND TORSION

31. Nylander, H., 'Vridning och vridningsinspaning vid betong-konstruktioner' (Torsion and torsional restraint in concrete structures), Bulletin No. 3, Statens Kommittee for Byggnadsforskning, Stockholm, 1945.

32. Cowan, H.J., 'The strength of plain, reinforced and prestressed concrete under the action of combined stresses, with particular reference to the combined bending and torsion of rectangular beams.' Magazine of Concrete Research, Vol. 5, 1953, pp. 75-86.

33. Cowan, H.J. and Amstrong, S. 'Experiments on the strength of reinforced concrete and prestressed concrete beams and of concrete encased steel joists combined bending and torsion.' Magazine of Concrete Research, Vol. 7, 1955, pp. 3-20.

34. Grassam, N.S.J., 'Experiments on concrete under combined bending and torsion.' Proceedings of the Institution of Civil Engineers, Vol. 5, 1956, pp. 159-165.

35. Bresler, B. and Pister, K.S., 'Failure of plain concrete under combined bending and torsion.' Transactions of the American Society of Civil Engineers, 122, 1957, pp. 1049-1059.

36. Armstrong, S., 'The strength of concrete members in combined bending and torsion,' Symposium on the strength of concrete structures, Cement and Concrete Association, 1956, pp. 101-122.

37. Lessig, N.N., 'Determination of the load-carrying capacity of reinforced concrete elements with rectangular cross section under simultaneous action of flexure and torsion.' Beton i Zhelezobeton, No. 3, 1959. Translated

from the original Russian as Portland Cement Association Foreign Literature Study No. 348.

38. Lessig, N.N., 'Determination of the load-bearing capacity of reinforced concrete elements with rectangular cross section subjected to flexure and torsion.' Trudy No. 5, Concrete and Reinforced Concrete Institute, Moscow, 1959. Translated from the original Russian as Portland Cement Association Foreign Literature Study No. 371.

39. Chinenkov, U.V., 'Investigation of behaviour of reinforced concrete elements subjected to combined and torsion.' Proc., Concrete and Reinforced Concrete Institute, Moscow, 1959. Translated from the original Russian as Portland Cement Association Foreign Literature Study No. 370.

40. Lessig, N.N., 'Study of cases of failure of concrete in reinforced concrete elements with rectangular cross section subjected to combined flexure and torsion.' Design of Reinforced Concrete Structures, Moscow, 1961. Translated from the original Russian as Portland Cement Association Foreign Literature Study No. 398.

41. Ramakrishnan, V. and Vijayarangan, B., 'The influence of combined bending and torsion on rectangular beams without web reinforcement.' Indian Concrete Journal, Vol. 37, 1963, pp. 412-416.

42. Yudin, V.K., 'Behaviour of reinforced concrete beams with rectangular cross section subjected to combined torsion with flexure.' Beton i Zhelezobeton, No. 1, Moscow, 1964. Translated from the original Russian as Portland Cement Association Foreign Literature Study No. 431.

43. Gesund, H. and Boston, L.A., 'Ultimate strength in combined bending and torsion of concrete beams containing only longitudinal reinforcement.' Journal of the American Concrete Institute, Proc. Vol. 61, No. 11, 1964, pp. 1453-1471.

44. Gesund, H., Schuette, F.S., Buchanan, and Gray. G.A., 'Ultimate strength in combined bending and torsion of concrete beams containing both longitudinal and transverse reinforcement.' Journal of the American Concrete Institute, Proc. Vol. 61, No. 12, 1964, pp. 1509-1521.

45. Ramakrishnan, V. and Vijayarangan, B., 'The influence of web reinforcement on reinforced concrete beams subjected to combined bending and torsion.' Indian Concrete Journal, Vol. 39, No. 3, 1965, pp. 89-93.

46. Evans, R.H. and Sakar, S., 'A method of ultimate strength design of reinforced concrete beams in combined bending and torsion.' The Structural Engineer, Vol. 43, No. 10, 1965, pp. 337-344.

47. Walsh, P.F., Collins, M.P., Archer, F.E. and Hall, A.S. 'Experiments on the strength of concrete beams in combined bending and torsion.' UNICIV Report No. R15, University of New South Wales, 1966.

S'

48. Pandit, G.S. and Warwaruk, J., 'Effect of flexure on the initial torsional stiffness of reinforced concrete beams of rectangular cross section.' Indian Concrete Journal, Vol. 41, No. 9, 1967, pp. 355-358.

49. Goode, C.D. and Helmy, M.A., 'The design of rectangular beams subjected to combined bending and torsion.' Concrete, 1967, No. 7, pp. 241-244.

50. Kittisatra, L., 'Moment-rotation characteristics of reinforced concrete beams under combined bending and torsion.' M. Eng. Thesis No. 182, SEATO Graduate School of Engineering, 1967.

51. Estanero, R.A., 'Moment-rotation characteristics of reinforced concrete sections under combined bending and torsion.' M. Eng. Thesis No. 186, Asian Institute of Technology, 1967.

52. Hsu, T.T.C., 'Torsion of structural concrete – interaction surface for combined torsion, shear and bending in beams without stirrups.' Journal of the American Concrete Institute, Vol. 65, No. 1, 1968, pp. 51-60.

53. Collins, M.P., Walsh, P.F., Archer, F.E. and Hall, A.S., 'Ultimate strength of reinforced concrete beams subjected to combined bending and torsion.' Special Publication No. 18 of the American Concrete Institute, Proc. pp. 379-402.

54. Goode, C.D. and Helmy, M.A., 'Ultimate strength of reinforced concrete beams in combined bending and torsion.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 357-377.

55. Gvozdev, A.A., Lessig, N.N., and Rulle, L.K., 'Research on reinforced concrete beams in combined bending and torsion in the Soviet Union.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 307-336.

56. Pandit, G.S. and Warwaruk, J., 'Reinforced concrete beams in combined bending and torsion.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 133-163.

57. Sundara Raja Iyencar, K.T. and Vijaya Rangan, B., 'Strength and stiffness of reinforced concrete beams under combined bending and torsion.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 48-66.

58. Zia, P. and Cardenas, R., 'Combined bending and torsion of reinforced plaster model beams.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 337-356.

59. Goode, C.D. and Helmy, M.A., 'Bending and torsion of reinforced concrete beams.' Magazine of Concrete Research, Vol. 20 No. 64, 1968, pp. 155-156.

60. Pandit, G.S. and Warwaruk, J., 'A rational approach to the ultimate strength of rectangular reinforced concrete beams.' Cement and Concrete, Vol. 8, No. 4, 1968, pp. 294-303.

61. Victor, D.J. and Ferguson, P.M. 'Reinforced concrete T-beams without stirrups under combined moment and torsion.' Journal of the American Concrete Institute, Proc. Vol. 65, No. 1, 1968, pp. 29-36.

62. Fairbairn, D.R. and Davies, S.R., 'Combined bending and torsion in reinforced concrete beams.' The Structural Engineers, Vol. 47, No. 4, 1969, pp. 151-156.

63. Sundara Raja Iyengar, K.T. and Vijaya Rangan, B., 'Factors affecting the ultimate strength of reinforced concrete beams under combined bending and torsion.' Magazine of Concrete Research, Vol. 21, No. 67 pp. 91-102.

64. Collins, M.P., 'The normal moment yield criterion applied beams in flexure-torsion.' Paper submitted to the International Conference on Shear, Torsion and Bond, Coimbatore, India, 1969.

65. Jackson, N. and Estanero, R.A., Discussion of a paper 'Bending and torsion of reinforced concrete beams' by C.D. Goode and M.A. Helmy published in the Magazine of Concrete Research, Vol. 21, No. 69, 1969, pp. 228-231.

66. Liao, H.M. and Ferguson, P.M., 'Combined torsion in reinforced concrete L-beams with stirrups.' Journal of the American Concrete Institute, Proc. Vol. o No. 12, 1969, pp. 986-993.

67. Pandit, G.S., 'A note on balanced section in combined loading.' Journal of the American Concrete Institute, Proc. Vol. 67, No. 5, 1970, pp. 420-423.

68. Kirk, D.M. and Lash, S.D., 'T-beams subject to combined bending and torsion.' Journal of the American Concrete Institute, Proc. Vol. 68, No. 2, 1971, pp. 150-159.

69. Jackson, N. and Estañero, R.A. 'The plastic flow law for reinforced concrete beams under combined flexure and torsion.' Magazine of Concrete Research, Vol. 23, No. 77, 1971, pp. 169-180.

## COMBINED FLEXURE, TORSION AND SHEAR

70. Brown, E.I., 'Strength of reinforced concrete T-beams under direct shear and torsion.' Journal of the American Concrete Institute, Proc. Vol. 26, 1955, pp. 889-902.

71. Lyalin, I.M., 'Experimental studies of the behaviour of reinforced concrete beams with rectangular cross-sections subjected to the combined action of transverse force, flexural and torsional moment.' Trudy No. 5, Concrete and Reinforced Concrete Institute, Moscow, 1959, Translated from the original Russian as Portland Cement Association Foreign Literature Study No. 402.

72. Yudin, V.K., 'Determination of the load-bearing capacity of reinforced concrete elements of rectangular cross section under combined bending and torsion.' Beton i Zhelezobeton, No. 6, 1962. Translated from the original Russian as Portland Cement Association Foreign Literature Study No. 377.

73. Collins, M.P., Walsh, P.F., Archer, F.E. and Hall, A.S., 'Reinforced concrete beams subjected to combined bending and torsion and shear.' UNICIV Report No. R14, University of New South Wales, 1965.

74. Farmer, L.E. and Ferguson, P.M., 'T-beams under combined bending, shear and torsion.' Journal of the American Concrete Institute, Proc. Vol. 64, 1967, pp. 757-766.

75. Collins, M.P., Walsh, P.F., Archer, F.E. and Hall, A.S., 'The design of rectangular reinforced concrete beams subjected to combined bending and torsion and shear.' UNICIV Report No. R16, University of New South Wales, 1966.

76. Walsh, P.F., Collins, M.P., Archer, F.E. and Hall, A.S., 'The ultimate strength design of rectangular reinforced concrete beams subjected to combined torsion, bending and shear.' Civil Engineering Transactions, The Institution of Engineers, V. CE8, No. 2, 1966.

77. Ersoy, U. and Ferguson, P.M., 'Behaviour and strength of concrete L-beams under combined torsion and shear.' Journal of the American Concrete Institute, Proc. Vol. 64, 1967, pp. 793-801.

78. Ersoy, U and Ferguson, P.M., 'Concrete beams subjected to combined torsion and shear – experimental trends.' Special Publication of the American Concrete Institute, 1968, pp. 441-460.

79. Kemp, E.L., 'Behaviour of concrete members subject to torsion and to com-

bined bending and torsion shear. Special Publication No. 18 of the American Concrete Institute, 1968.

80. Gesund, H., Mills, D.C. and Martin, V.M., 'Ultimate strength tests of reinforced concrete beams in combined torsion, bending and shear.' International Association of Bridge and Structural Engineers, Vol. 28-II, 1968. pp. 31-48.

81. Victor, D.J. and Ferguson, P.M., 'Beams under distributed load creating moment, shear and torsion.' Journal of the American Concrete Institute, Proc. Vol. 65, No. 4, 1968, pp. 295-308.

82. Osburn, D.L., Mayoglou, B. and Mattock, A.H., 'Strength of reinforced concrete beams with web reinforcement in combined torsion, shear and bending.' Journal of the American Concrete Institute, Proc. Vol. 65, No. 4, 1968, pp. 295-308.

83. Jirsa, J.O., Baumgartner, J.L. and Mogbo, N.C., 'Torsional strength and behaviour of spandrel beams. Journal of the American Concrete Institute, Proc. Vol. 66, No. 11, pp. 926-932.

84. Mirza, M.S. and McCutcheon, J.O., 'Behaviour of reinforced concrete beams under combined bending, shear and torsion.' Journal of the American Concrete Institute, Proc. Vol. 66, No. 5, 1969, pp. 421-427.

85. Ramakrishnan, V. and Ananthanarayana, Y., 'Tests to failure in bending and torsion of reinforced concrete bents.' Journal of the American Concrete Institute, Proc. Vol. 66, No. 5, 1969, pp. 428-431.

86. McMullen, A. and Warwaruk, J. 'Concrete beams in bending, torsion and shear.' Proceedings of the American Society of Civil Engineers, Vol. 96, ST5, 1970, pp. 885-903.

87. Swann, R.A. 'Experimental basis for a design method for rectangular reinforced concrete beams in torsion.' Technical Report No. 42.452, Cement and Concrete Association, 1970.

88. Zia, P., 'What do we know about torsion in concrete members? ' Proceeding of the American Society of Civil Engineers, Vol. 96, ST6, 1970, pp. 1185-1199.

## RELATED TOPICS

89. Mattock, A.H., Kriz, L.B. and Hognestad, E. 'Rectangular concrete stress distribution in ultimate strength design. Journal of the American Concrete Institute, Proc. Vol. 57, 1961, pp. 875-920.

90. Fisher, G.P. and Zia, P., review of foreign and domestic code requirements for torsion design.' Journal of the American Concrete Institute, Proc. Vol. 61, 1964, pp. 1-22.

91. Cowan, H.J. 'Philosophy for design of concrete structures in torsion.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 461-467.

92. Cowan, H.J. Reinforced and Prestressed Concrete in Torsion. Edward Arnold, London, 1965.

93. Fisher, G.P., 'Torsion in structural concrete – a perspective.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 1-6.

94. Klus, J.P. and Wang, C., 'Torsion in grid frames.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 89-102.

95. Mattock, A.H., 'How to design for torsion.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 469-495.

96. Tamberg, K.G., 'Aspect of torsion in concrete structure design.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 7-68.

97. Zia, P., 'Torsion theories for concrete members.' Special Publication No. 18 of the American Concrete Institute, 1968, pp. 102-132.

98. ACI Committee 318, 'Proposed revision of ACI 318: Building code requirements for reinforced concrete.' Journal of the American Concrete Institute, Proc. Vol. 67, No. 2, 1970, pp. 77-186.

99. Heyman, J., 'The limit design of space frames.' Journal of Applied Mechanics, Vol. 18, 1951, pp. 371-378.

100. Heyman, J., 'The limit design of transversely loaded square grid.' Journal of Applied Mechanics, Vol. 19, 1952, pp. 152-158.

101. Baker, J.F., Horne, M.R. and Heyman, J., The Steel Skeleton, Vol. 2 Plastic Behaviour and Design, Cambridge University Press, 1956.

102. Boulton, N.S. and Boonsukha, B., 'Plastic collapse loads for circular-arc bow girders.' Proc. Institution of Civil Engineers, Vol. 13, 1959, pp. 161-178.

103. Hodge, P.G., Plastic Analysis of Structures, McGraw-Hill, 1959.

104. Imegwu, E.O., 'Ultimate strength of plane curved girders.' The Structural Engineer, Vol. 42, No. 4, 1964, pp. 129-134.

105. Gill, S.S. and Boucher, J.K.G., An experimental investigation of plastic collapse of structural members under combined bending and torsion.' The Structural Engineer, Vol. 42, No. 12, 1964, pp. 423-428.

106. Jackson, N., 'The automatic calculation of collapse loads for circular-arc beams.' The Structural Engineer, Vol. 43, No. 12, 1965, pp. 413-421.

107. Jackson, N., 'Collapse loads for circular-arc beams.' Proc. of the American Society of Civil Engineers, Vol. 92, No. ST5, 1966, pp. 1-12.

108. Baker, A.L. , The Inelastic Space Frame, Concrete Publications, Ltd., 1967.

109. Beckett, D., The Ultimate Load Design of Continuous Concrete Beams. Butterworths, 1967.

110. Hill, R. and Siebel, M.P.L., 'On the plastic distortion of solid bars by combined bending and twisting.' Journal of Mech. Phys. Solids, Vol. 1, 1953, p. 207.

111. Johansen, K.W. and Hansen, P.L., 'Studies on the load-carrying capacity of steel structures. Copenhagen, University of Denmark, 1954.

112. Jackson, N., 'Plastic analysis of transversely loaded frames.' Proc. of the American Society of Civil Engineers, Vol. 97, No. ST8, 1971, pp. 2047-2056.

113. Baker, A.L.' and Amarakone, A.M.N., 'Inelastic hyperstatical frame analysis and application of the international correlated tests.' Paper submitted to the Hyperstatic Symposium of the European Concrete Committee, Ankara 1964, and to the ACI Symposium, Miami, 1964.

114. Mattock, A.H., 'The rotational capacity of hinging regions in reinforced concrete beams.' Joint ASCE-ACI Special Publication No. 11, Flexural Mechanics of Reinforced Concrete, 1964.

115. Hoffman, O. and Sachs, G., Theory of Plasticity, Mcgraw-Hill Book Co., 1953.

11. British Standards Institution, CP 114:1957, The Structural Use of Reinforced Concrete in Buildings.

117. British Standards Institution, Draft Unified Code for the Use of Structural Concrete.

118. American Concrete Institute, Torsion of Structural Concrete, Special Publication No. 18, 1968, 505 pp.