RELIABILITY ANALYSIS OF STRUCTURAL SYSTEMS: A PROBABILISTIC APPROACH*

by

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Introduction

Safety is perhaps the most important aspect of structural design. At present, most conventional design procedures treat all loads and strengths as though they are deterministic quantities. The uncertainties associated with the magnitudes of loads and strengths is to be accounted for by the "FACTOR OF SAFETY" – the ratio of structural member strength over member load. This ratio has been accepted by many engineers as a true measure of the safety of a structural system against failure, particularly when all the members are proportioned with identical factors of safety. However, there are several important points to show that the factor of safety in its present context is inadequate to quantitatively indicate structural safety and is not a rational nor economical guide to engineering design.

Among the more important points are: First, both member strength and member load are variables of some uncertainties. Since their statistical properties are not explicitly considered in the design, the factor of safety does not provide the designer with an objective understanding of the chance of failure of each structural member. Second, the chance of failure of a structural system generally increases as the member population making up the system increases. Thus, the survivability of the members can not quantitatively reflect the survivability of the whole structural system. Third, the introduction of vast arrays of new materials makes it extremely difficult to comprehend fully the implication of a given factor of safety. Fourth, the increase in extremes of loading environments such as space and undersea explorations makes it unrealistic, if not impossible, to design the structures for the worst combination of loads and strengths to satisfy certain predetermined factor of safety. Finally, limited natural resources and rising costs of construction make it necessary to balance risk and expenditure. The factor of safety does not indicate the true risks a structural system is being subjected.

In view of the above, it has been recognized in recent years that a more rational criteria for computing structural safety is the probabilistic approach where the statistical properties of all loads and strengths are being considered explicitly. This method involves the assumption of probability distributions for the variables and the construction of computational models to calculate the safety of the structural system.

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The selection of appropriate probability functions for various loads has been the subject of research since mid-1940's. Statistical parameters had been proposed for some types of loads such as wind forces,*[1-2-3] wave forces[4], floor loads in buildings,[5-6] bridge loads[7-8] and earthquake loads[9-10]. Similarly, various structural materials have been investigated in different aspects of strength.[11-15] A literature search of statistical distributions of loads and strengths had been presented by American Society of Civil Engineers.[16]

There are three objectives of this paper. First, the basis of reliability analysis will be presented. Second, the theoretical formulation of some approximate solutions to the reliability of structural systems will be discussed. Of particular interest among the solutions is the Ordering Survivability Method where strength correlations can be incorporated in the analysis. Finally, a short summary of some of the results obtained from a recent study made by the author on the applicability of some approximate solutions will be presented.

Philosophical Foundations of Reliability Analysis

The Fundamental Case

The development of reliability analysis starts with the fundamental case. It consists of a single member with random strength R subjected to a single random load T. The strength R may be defined in any appropriate ways and the nature of load T can be in different forms. Failure is defined as the event that load is greater than strength. Three examples of the fundamental case is shown on figure 2.1.

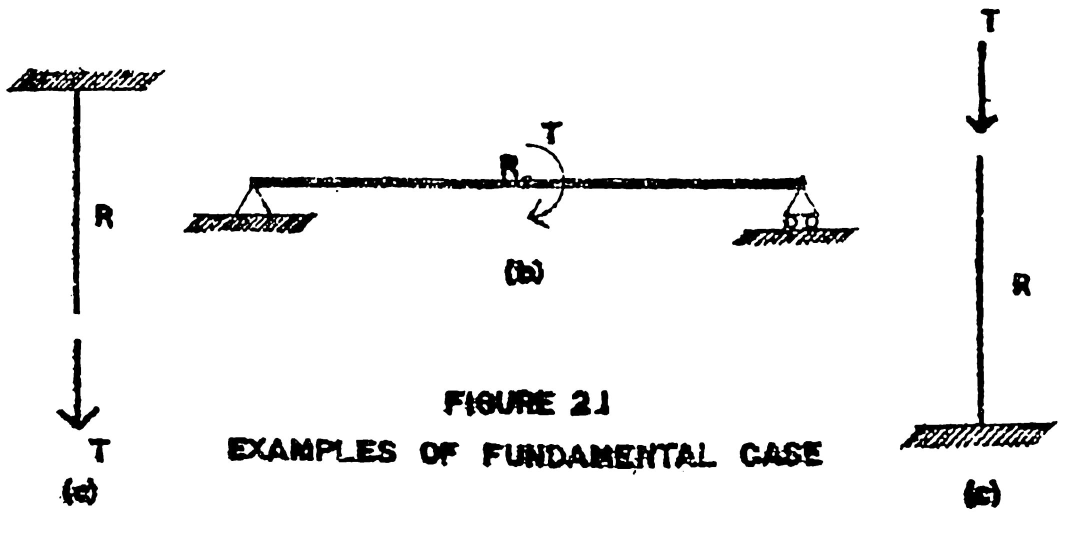


FIGURE 2.1
EXAMPLES OF FUNDAMENTAL CASE

Loads and strengths are random quantities. Therefore, the survivability of the structure is an uncertainty. The primary goal of structural reliability analysis is to define the probability of failure of the system, P_f. In the fundamental case,

$$P_f = Pr\,(R < T) \qquad (2.1)$$

*Parenthetical references placed superior to the line of text refer to the bibliography.

where Pr (X) is the probability that condition X exists.

It has been observed that the relative frequencies of the magnitudes of loads and strengths may be modelled by some well-defined probability density distributions. Figure 2.2 shows an example of these distribution functions. There are infinite

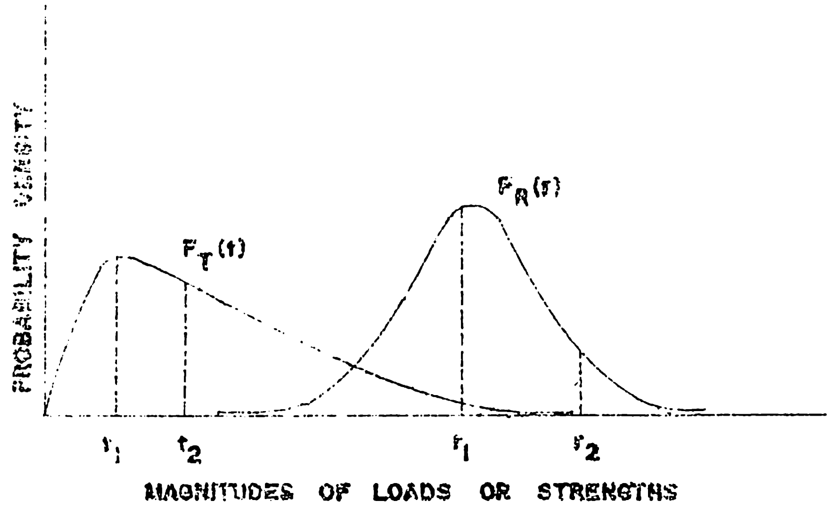


FIGURE 2.2
EXAMPLE OF DENSITY DISTRIBUTIONS OF LOAD AND STRENGTH

number of possible combinations of loads and strengths $t_i - r_j$. The probability of failure of the structure is the probability of occurrence of any one of the $t_i - r_j$ combinations where $r_j < t_i$. This failure probability may be calcuated by

$$P_f = \int_0^\infty \int_0^t f_R(r)\, f_T(t)\, dr\, dt \qquad (2.2)$$

Although the fundamental case is useful in clarifying some aspects of reliability of structures such as the sensitivity of failure probability to input statistical parameters, it is only a single element of a complex structural system consists of many members. Several investigations had been made to extend this classical fundamental case to more realistic structures of multiple elements.[17to28]

In theory, there are basically two types of structural systems: the weakest link systems and the parallel link systems. Most real life structures can be classified under either type or their combination (complex systems).

The Weakest Link Systems

Figure 2.3a shows the most basic example of a weakest link system. It consists of several links connected in series. Each link "i" has a strength R_i and is resisting a load T. Failure of this structural system is said to have occured when the load T exceeds any of the member strengths R_i. Denoting by F_i the event that the i^{th} member had failed, the probability of system failure is the probability that any of the members failed. Thus, from the definition of union of probability events, the probability of failure of the system, Pf, may be expressed as:

$$Pf + Pr\,(F_1 \cup F_2 \cup F_3 \cup \ldots \cup F_i \cup \ldots \ldots \cup F_{m-1} \cup Fm) \qquad (2.3)$$

The probability of survival of the structural system is the probability that all the members will survive. Denoting the survivability of each member as S_i, the system probability of survival, P_s, is clearly the intersection of all survival events S_i. It can be expressed as

$$P_s = Pr\,(S_1 \cap S_2 \cap \ldots \cap S_i \cap \ldots \ldots \cap S_{m-1} \cap S_m) \qquad (2.4)$$

and

$$Pf = 1.0 - P_s \qquad (2.5)$$

The survivability of the structure depends on the survivability of the weakest member. Since the strengths of all members are random in nature, every member is a potential weakest link and its probability of failure must be explicitly included in the reliability analysis.

Aside from the chain shown in figure 2.3a, the weakest link system applies to most statistically determinate structures. Two examples of such structures are shown on figure 2.3b and 2.3c. It is obvious in these examples that the loads T_i resisted by the members are not necessarily equal in magnitude as in example 2.3a. Failure of any member constitutes the failure of the whole structure. Thus, both are weakest link systems.

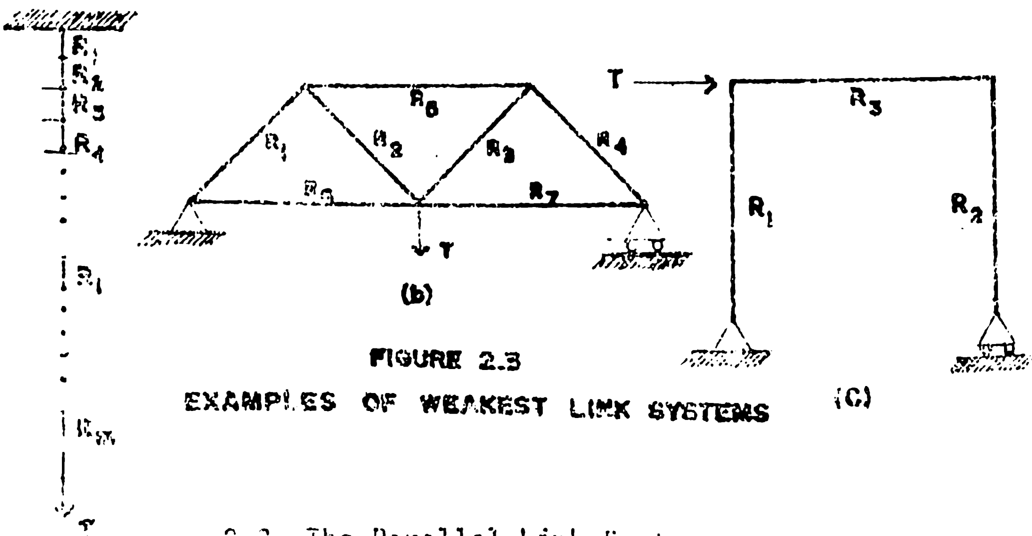


FIGURE 2.3

EXAMPLES OF WEAKEST LINK SYSTEMS

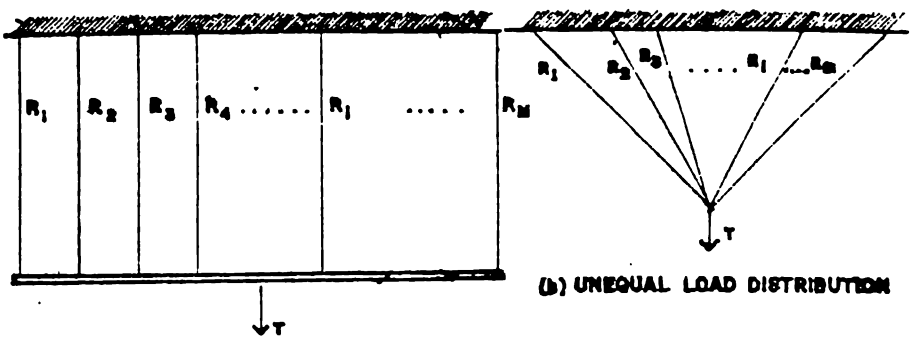
2.3 The Parallel Link System

The Parallel Link System

Basic parallel link systems are shown in figure 2.4. It consists of several members having individual strengths R_i. The applied load T is distributed to and resisted by all members. When a member fails, the portion of the load that the failed member can no longer carry is redistributed to the surviving members. This process of member failure and load redistribution continues until all of the members had failed and the failure of the structural system is said to have occurred. Therefore, the system failure probability is defined as the probability that all members comprising the parallel system failed.

$$P_f = Pr\,(F_1 \cap F_2 \cap F_3 \cap \ldots \cap F_i \cap \ldots \cap F_m) \qquad (2.6)$$

Since the final survival of at least one member in the system defines the survival of the structure, the failure probability can also be expressed as



(a) EQUAL LOAD DISTRIBUTION

(b) UNEQUAL LOAD DISTRIBUTION

FIGURE 2.4
EXAMPLES OF PARALLEL LINK SYSTEMS

$$P_f = 1.0 - Pr\,(S_1 \cup S_2 \cup S_3 \ldots \cup S_i \cup \ldots \cup F_m) \qquad (2.7)$$

There are two major difficulties in calculating the probabilities of failure of parallel link systems. First, it is necessary to perform a structural analysis to determine the load redistribution to the surviving members after the failure of a member. Second, all paths leading to the failure of the structure must be considered explicitly.

It is seen, therefore, that the mathematical evaluation of equations 2.6 or 2.7 is extremely complicated in that the member load varies with different failure paths. Furthermore, as the member population and the degree of structural indeterminacy increases, the problem becomes even more difficult to deal with due to the large number of failure paths* and the even larger number of structural analysis that must be performed. This is perhaps the reason why, to date, there is no general solution to the parallel link system reliability analysis being created.

*The total number of failure paths need be considered is equal to the permutation of all m members or equal to m! . Thus, the number of failure paths for a seven member system is 7! = 5,040.

There are two types of failures in parallel link systems that require quite different approach in attempting to calculate the system reliability. They are brittle member failure and yielding failure.

When a member of a load redistributing structural system failed by brittle fracture, its load carrying capacity is reduced to zero. Thus, the portion of the applied load that was being resisted by this member prior to failure must now be totally redistributed to the remaining surviving members.

When a member of a load redistributing structural system failed by yielding, it has some load carrying capacity after failure that is equal to the yield strength of the member. The excess member load after failure is redistributed to the other surviving members.

Reliability of some other real structures that may be represented by the parallel link systems also include sophisticated structures such as suspended bridges and cabled roofs.

The Complex Systems

The complex system arises from the combination of weakest link and parallel link properties that exist in a structural system. Most indeterminate structural systems may be classified under the complex systems. Two examples of this type of systems are shown in figure 2.5. Figure 2.5a clearly demonstrates the complex system. The two modes of failure are the failure of the two separate parallel link sub-systems composed of members 1-2-3 and 4-5 respectively. The failure probabilities of these modes may be calculated by parallel link formulas. Since the occurrence of any of the two modes defines the failure of the structural system, weakest link formulas may be employed to compute for the overall reliability of the system.

Figure 2.5b is a fixed based frame subjected to a horizontal load T. Two of the failure modes are presented in figure 2.5b$_1$, and 2.5b$_2$. Failure mode 1 is defined by the formation of plastic hinges M_1, M_2, M_5, and M_6. The other failure mecha-

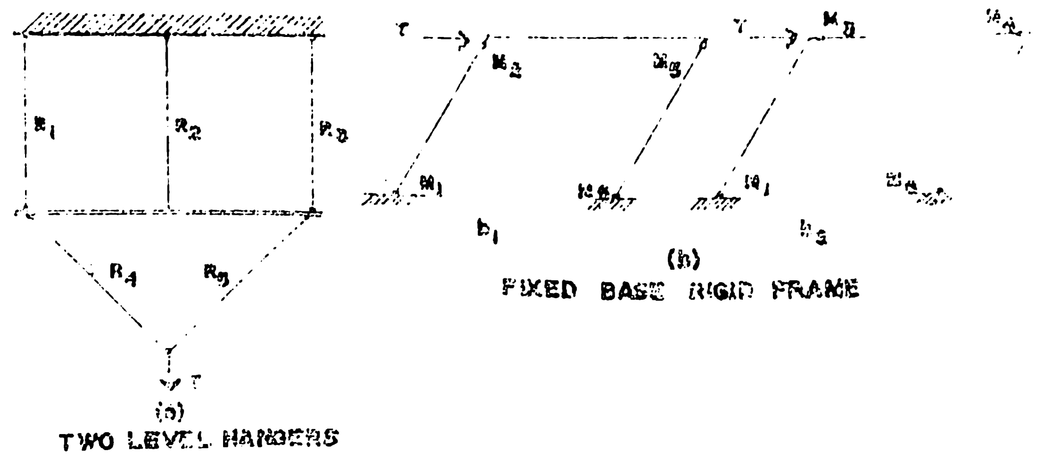


FIGURE 2.5
EXAMPLES OF COMPLEX SYSTEMS

nism is the formation of plastic hinges M_1, M_3, M_4, and M_6. Note that there are other failure modes not shown in the figure. Each failure mode probability can be computed by the parallel link system representation. For example, the failure probability of mode 1 is the probability that plastic hinges formed at M_1, M_2, M_5, and M_6 under the load T. After the probabilities of failure of each individual modes are calculated, the entire system failure probability may be computed by the weakest link solution. In this case, it is to find the probability that one of the failure modes will occur.

COMPUTATIONAL MODELS FOR RELIABILITY ANALYSIS

The Need for Approximate Solutions

Due to manufacturing processes, chemical composition, and/or structural geometric formulation, element strengths within a structural system may be statistically or functionally correlated. As a result, the respective survivabilities of the members can not be independently evaluated to compute the overall survivability of the system. When all the statistical parameters are completely defined, the exact probability of failure of structural system with correlated strengths may be calculated in three ways: (a) closed-form integration of the load and strength functions; (b) numerical integrating the load-strength distribution; and (c) Monte Carlo simulation of large number of trial cases.

Most probability density functions are very complex that closed-form integration can not be performed. The integrals must be evaluated by numerical methods or Monte Carlo simulation. Both methods involve lengthy computer processing. In general, the number of samples, n, required to simulate a structural system with failure probability p at $(1.0 - \alpha)$ confidence level and with a relative error of γ can be expressed

$$n = \frac{(1 - p)K^2{}_{\alpha/2}}{\gamma^2 P} \tag{3.1}$$

where $K_{\alpha/2}$ is that value which a standardized normal random variable exceeds with a probability $\alpha/2$.

The number of numerical steps NS needed for an m dimensional integration with each variable distribution divided into NP spaces is

$$NS = NP^M \tag{3.2}$$

Thus, in deciding whether the numerical integration or simulation will be used, one should compare the number of steps in each case using equations 3.1 and 3.2 based on the same accuracy. As an example, for a four dimensional integral with failure probability in the 10^{-5} range, the required steps for each level of integration is 36 at 1% error. The total number of steps is therefore $36^4 = 1.7$ million steps. For the same problem with 1% error, the required sample size is 27 billion!

Since most structural systems are constructed with large number of elements and are designed for low failure probabilities, numerical integration and simulation became costly. Hence, it is necessary to devise some approximate methods to closely estimate the reliability of structural systems in a more economical fashion.

THEORETICAL BASES FOR SOME APPROXIMATE SOLUTIONS OF STRUCTURAL RELIABILITY

The two basic equations of probability of failure events are:

$$P_f = Pr(F_1 \cup F_2 \cup F_3 \ldots \cup F_i \cup \ldots \cup F_m) \tag{3.3}$$

and

$$P_f = 1.0 - Pr(S_1 \cap S_2 \cap \ldots \cap S_i \cap \ldots \cap S_m) \tag{3.4}$$

By the manipulation of some probability theorems, these two equations may be expressed in many forms. In the derivation of approximate methods, some terms in the equations are discarded.

Type A Approximation

In the type A approximation, equation 3.4 may be expressed as

$$P_f = 1.0 - Pr(S_1/[S_2 \quad S_3 \ldots S_m]).$$
$$Pr(S_2/[S_3 \quad S_4 \ldots S_m]) \ldots Pr(S_{m-1}/S_m).$$
$$Pr(S_m) \tag{3.5}$$

One way of simplifying the solution of equation 3.5 is by assuming that all the member strengths are uncorrelated. Thus

$$P_f^{(A)} = 1.0 - \prod_{i=1}^{m} Pr(S_i) \tag{3.6}$$

since for structural members, strength correlations are always positive,

$$Pr(S_i) \leqslant Pr(S_1/[S_{i+1} \cap S_{i+2} \cap \ldots \cap S_m]) \tag{3.7}$$

for all values $i = 2$ to m, equation 3.6 always resulted in overestimating P_f and thus is an upper bound estimate of the true probability of failure.

Type B Approximation

In type B, it is assumed that the member strengths are fully correlated. By fully correlated, it is meant that there exist a linear relationship between the true strengths of the members. This linear relationship is usually positive in structural systems. Thus, when the strength of one member is defined, such as by experiment, the strength of the other members can be found by computation. This indicates that when the weakest of all members survive the applied load, there is no chance for the stronger members to fail.

If equation 3.3 is rearrange according to probability laws, it becomes

$$P_f = Pr(F_1) + \sum_{j=2}^{m} \{Pr(F_j/S_1 \cap S_2 \cap \ldots \cap S_{j-2}). \quad Pr(S_1 \cap S_2 \cap \ldots \cap S_{j-1})\} \tag{3.8}$$

If the members were numbered in the order of increasing survivability, all the values with the exception of $Pr(F_1)$ become zero. This resulted in

$$P_f^{(B)} = \underset{i=1}{\overset{m}{Max}}\ Pr(F_i) \tag{3.9}$$

Since all but one term on the right side of equation 3.8 are neglected, the approximation always underestimate the failure probability of the system.

Type C Approximation

Rearranging equation 3.3, the probability of system failure may be expressed as

$$P_f^{(C)} = \sum_{i=1}^{m} Pr(F_i) - \underset{i \neq j}{\overset{all}{\Sigma_1 \Sigma_2}} Pr(F_i \cap F_j) + \underset{1 \neq j \neq k}{\overset{all}{\Sigma_1 \Sigma_2 \Sigma_3}} Pr(F_i \cap F_j \cap F_k) - \ldots + \ldots - \ldots \pm Pr(F_1 \cap F_2 \cap F_3 \cap \ldots \cap F_m) \tag{3.10}$$

In this equation, the magnitude in absolute value of any summation terms in the right hand side is obviously less than its preceding terms because in a venn diagram, the area defined by this term must be enclosed by the area defined by its preceding terms.

Approximations can be made from equation 3.10 by considering only some, but not all, of the summation terms in the right hand side. If one ends the sum with an even sum term, the approximate solution will underestimate the system failure probability. Otherwise, the approximation is guaranteed to be conservative.

To increase the accuracy, one may increase the number of terms included in the analysis. But this might defeat the purpose of approximation because the amount of calculation increases very rapidly as more terms are being considered.

Type D Approximation

The probability of system failure may be expressed as

$$P_f = Pr(F_1) + \sum_{j=2}^{m} Pr(F_j) \ . \ Pr([S_1 \cap S_2 \cap \ldots \cap S_{j-1}]/F_j) \tag{3.11}$$

Since when the strengths of all members are positively correlated,

$$Pr(S_k/F_j) \geq Pr(S_1 \cap S_2 \cap \ldots \cap S_k \cap \ldots \cap S_{j-1}/F_j) \qquad (3.12)$$

an approximation for equation 3.11 may be expressed as

$$P_f \leq Pr(F_1) + \sum_{j=2}^{m} Pr(F_j) \;.\; Pr(S_i/F_j) \quad i>j \qquad (3.13)$$

In order that the approximation be as close to the true solution as possible, it is necessary to select the minimum of all $Pr(S_i/F_j)$. The resulting equation is

$$P_f^{(D)} = Pr(F_1) + \sum_{j=2}^{m} Pr(F_j) \;.\; \{\min_{i=1}^{j-1} Pr(S_i/F_j)\} \qquad (3.14)$$

The Development of the Ordering Survivability Method

The probability of system failure may be expressed as

$$P_f = 1.0 - Pr(S_1) \;.\; \prod_{j=2}^{m} Pr(S_j/[S_1 \cap S_2 \cap \ldots \cap S_{j-1}]) \qquad (3.15)$$

It is proposed that the conditional survival of members 3, 4, . . . m be calculated in such a way that it is conditioned only on one of the j-1 survival terms. It can be shown that

$$Pr(S_j/S_k) \leq Pr(S_j/[S_1 \cap S_2 \cap \ldots \cap S_{j-1}]) \qquad (3.16)$$

where k is any member from 1 to j-1. This approximation always overestimates the failure probability of the structural system. In order that the approximation is closest to the true probability of failure, it is necessary to select the largest $Pr(S_j/S_k)$. Thus, a conservative approximation for equation 3.15 is

$$P_f^{(E)} = 1.0 - Pr(S_1) \;.\; \prod_{j=2}^{m} (\max_{k=1}^{j-1} Pr(S_j/S_k)) \qquad (3.17)$$

If the members were numbered in the order of ascending survivability, the maximum of all $Pr(S_j/S_k)$ is computed since all $Pr(S_j)$ are conditioned on the survival of the weakest link.

Equation 3.17 involves the integration of large number of two dimensional (deterministic load) or three dimensional (random load) integrations for the conditional survival probabilities. In order to minimize this calculation, the maximum conditional failure probability may be approximated by

$$\max_{k=1}^{j-1} \; Pr(S_j/S_k) \geq Pr^*(S_j/S_k) \qquad (3.18)$$

where $Pr^*(S_j/S_k)$ is the survival probability of the j^{th} member given that the k^{th} member had survived, k being the member most highly correlated to j. In case of many members equally correlated to j, the member with the highest probability of failure is selected.

Equation 3.17 can now be expressed

$$P_f^{(E)} = 1.0 - Pr(S_1) \cdot \prod_{j=2}^{m} Pr^*(S_j/S_k) \tag{3.19}$$

$$K < j \text{ at } \rho_{jk} = \operatorname*{Max}_{i=1}^{j-1} \rho_{ij}$$

$$Pr(S_1) \leqslant Pr(S_2) \leqslant \ldots \leqslant Pr(S_m)$$

Summary and Conclusions

In section 3.0 some approximate solutions were presented. It is of interest to study the accuracies of the methods, the effect of strength correlations on the reliability of the structures, and the feasibility of using the weakest link solution to approximate the failure probability of parallel link systems In an extensive study made by the author, the following results were obtained.

Accuracies of Approximate Methods

It is observed that the error of type A approximation decreases as the system failure probability decreases. It is also seen that at some low correlation coefficients, the error as compared to the exact solution is very small. Because type A approximation is very simple to apply, its range of applicability in terms of strength correlation coefficients is studied. The conclusions are presented later.

Type B is a very poor approximation of the true probability of failure. The error associated with this approximation is not significantly reduced even at high levels of strength correlations Furthermore, because the error is unconservative, the applicability of type B is limited only to the case of fully correlated systems.

In types A and B, correlations among member strengths are neglected. In type C approximation, correlations may be included and any degree of accuracies achieved. However, greater accuracies are always obtained at disproportionately increased difficulties in mathematical manipulation and increased computational time. In applying type C approximation, only the first term in the right hand side equation can be calculated easily. However, this one-term approximation is always less accurate than type A.

Except for high failure probabilities, $P_f > 0.1$, types D and E both yield results with negligible error. Even at high failure probabilities, the errors are limited to 10%. Since type D computes all intersecting failure events in pair and select the highest values while type E only calculates the intersecting events of the most highly correlated pairs, type E is greatly more efficient than type D.

Effect of Correlation on Failure Probabilities

Figure 4.1 presents four representative normalized curves showing the effect of correlation coefficients on the probabilities of failure The number of members in the structural systems ranges from two to fifty. Failure probabilities calculated ranges from close to unity to 10^{-4} The results were obtained by ordering survivability method. Two parent distributions were used — normal and extreme distributions.

There are two obvious observations that can be made from figure 4.1. First, as the failure probabilities decrease, their values at zero correlation are changed significantly only by high correlation coefficients. Second, as the member population increases, only high correlation coefficients can significantly lower the probabilities of failure from their values at zero correlation.

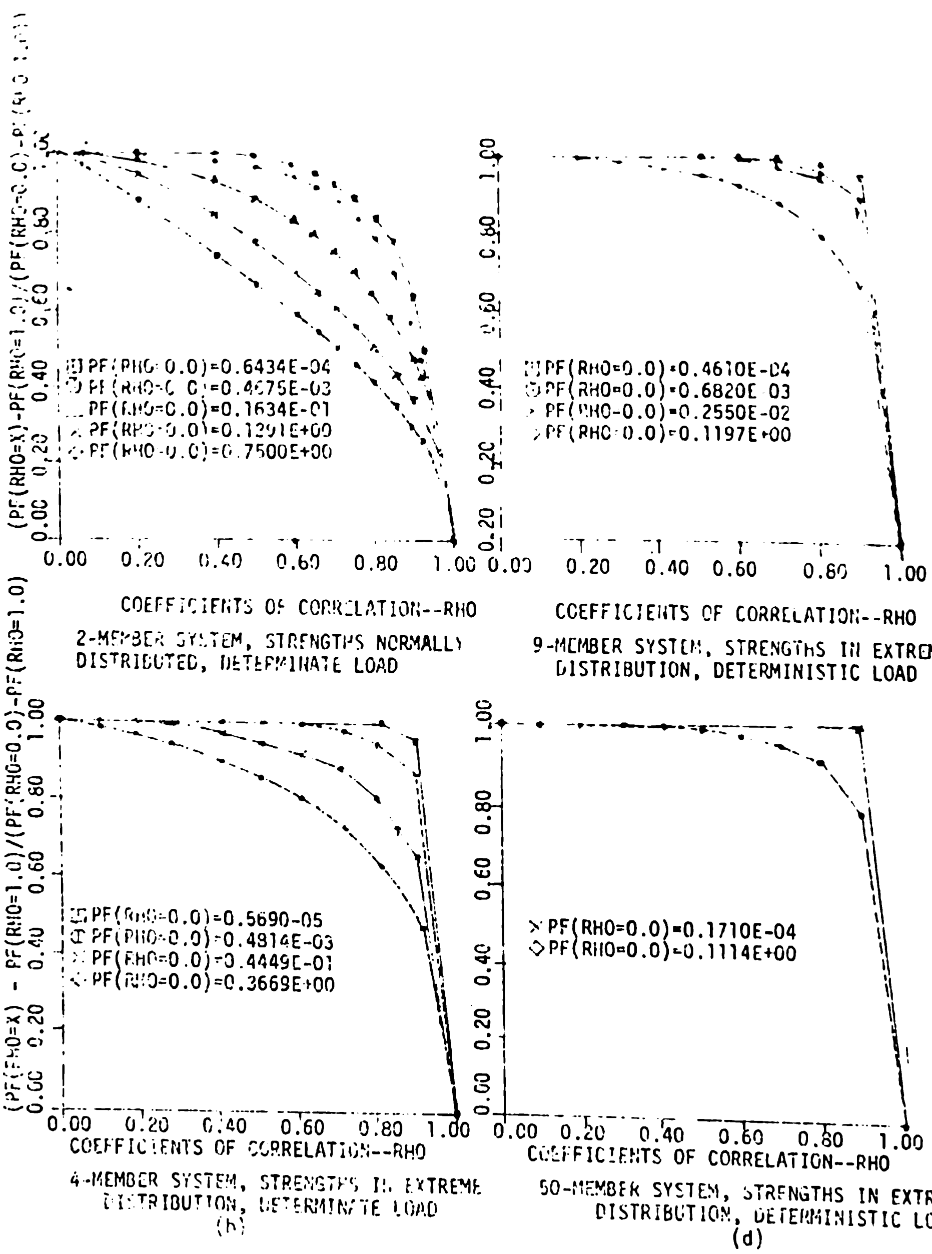


FIGURE 4.1
NORMALIZED CURVES SHOWING
THE EFFECT OF STRENGTH CORRELATIONS

Weakest Link Approximation for Parallel Link Systems

It was demonstrated in the study that at high probabilities of failure, the parallel link systems with brittle failure do not exhibit noticeable reserve strength over the weakest link systems. Whereas, for the case of ductile failure, the system failure probability is significantly lowered as compared to the weakest link even at high failure ranges. It is further observed that failure probabilities increase at increasing member population or increasing correlation coefficients.

Conclusions

The conclusions will be presented in two main categories: weakest link systems and parallel link systems.

For the weakest link systems, the conclusions are the following:

1. The assumption of zero member correlation in system failure probability analysis does not yield results with errors greater than 10% of the true probabilities when the failure probabilities are less than 10^{-3} and the member correlation coefficients are less than 0.7 for normal distribution.

2. The range of applicability of assuming zero member correlation in reliability analysis may be increased to include lower failure probabilities and higher member correlation coefficients for the extremal distribution.

3. The range of applicability of assuming zero member correlation in system failure probability analysis may be broadened to include higher failure probabilities and higher correlation coefficients when the member population in the system increases. This is true when the failure probabilities of the individual members are approximately equal.

4. The assumption of full member correlation in system failure probability analysis resulted in grossly erroneous and unconservative answers except when the member correlation coefficients are extremely close to or equal to unity.

5. The probability of failure is very sensitive to strength correlation when the coefficients of correlation is beyond 0.8. The Ordering Survivability Method as developed in this study is the most efficient and the most accurate approximate solution that includes correlation in the analysis.

For the parallel link system, the conclusions are the following:

1. When failures are defined by yielding of all members forming the failure mode, the probability of failure obtained by parallel link approach is much less than that obtained by the weakest link method.

2. The probability of failure of parallel link systems failed by yielding generally increases at increasing member correlation coefficients.

3. When the failure of a parallel link system is defined by the brittle failure (failed members carry no load) of all members leading to a failure mode, the weakest link solution may be used to approximate the failure probability All conclusions for weakest link systems also apply to this type of structures.

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