A NOTE ON THE ANALYSIS OF PILE GROUPS

by

SALVADOR F. REYES, Ph.D.*

A group of piles which receives structural loads from a pile cap, can be analyzed by assuming the latter as rigid. In turn, the forces developed in the individual piles due to the imposed displacements at their butts can be quantified. The five independent forces are shown in Fig. 1 (twisting moment is neglected). Note that the shearing forces and bending moments are resolved on the principal axes of the pile section.

Now the complete force displacement relationship at the butt can be expressed in terms of seven independent constants. For example, if a point of fixity is assumed at a distance of L below the butt, the stiffness factors can be computed as

$$S = \left\{ \frac{AE}{L}, \frac{12EIy}{L^3}, \frac{12EIz}{L^3}, \frac{6EIz}{L^2}, \frac{6EIy}{L^2}, \frac{4EIy}{L}, \frac{4EIz}{L} \right\} \tag{1}$$

if the pile is idealized as an elastic structural line element rigidly attached to the pile cap. If the pile is assumed to be pinned at the cap the stiffness factors are

$$S = \left\{ \frac{AE}{L}, \frac{3EIy}{L^3}, \frac{3EIz}{L^3}, 0, 0, \frac{3EIy}{L}, \frac{3EIz}{L} \right\} \tag{2}$$

where E is Young's modulus and (A, Iy, Iz) are the section area and moments of inertia, respectively.

A more realistic idealization can be prescribed by modifying the first term of S, e.g., on the basis of the actual load-deflection behavior as obtained from a field load test.

The coordinates of the pile butt may be expressed relative to an assumed reference or global frame at the point of application of the result and superstructure load on the pile cap, as shown in Fig. 2. The x-axis is assumed to be directed vertically downward and the y-axis directed due south.

The coordinates of the butt of any pile can now be specified. As shown in Fig. 3, the azimuth, a (of the pile projection on the horizontal plane) and batter, b, can also be specified relative to the global frame orientation. These can be transformed to Euler angles as follows (refer to Fig. 3)

*Professor of Civil Engineering

$$\theta = \tan^{-1}\left(\frac{\sin b \ \sin a}{\cos b}\right) \tag{3}$$

$$\phi = \tan^{-1}\left(\frac{\cos a \ \sin b}{\sqrt{1-(\cos a \ \sin b)^2}}\right) \tag{4}$$

For practical purposes it is sufficient to limit the pile section orientation to the four cases illustrated in Fig. 4, of Euler angle ζ are expressed in terms of ψ and η defined as

$$\psi = \tan^{-1}\left(\frac{\cos^2\phi \ \sin\phi \ \sin\theta}{\sqrt{1-(\sin\phi \ \sin\theta)^2}}\right) \tag{5}$$

$$\eta = \tan^{-1}\left(\frac{\sin\phi \ \sin\theta}{\sqrt{1-(\sin\phi \ \sin\theta)^2}}\right) \tag{6}$$

Note that for vertical piles only orientation types 3 and 4 are meaningful. In the latter case, the proper azimuth must be specified.

The analysis may now be formulated according to the stiffness method. The stiffness matrix of a pile at its load axis is expressed in terms of S as follows

$$K_p = \begin{bmatrix} S_1 & 0 & 0 & 0 & 0 & 0 \\ & S_2 & 0 & 0 & 0 & S_6 \\ & & S_3 & 0 & -S_5 & 0 \\ & & & 0 & 0 & 0 \\ & & & & S_4 & 0 \\ & & & & & S_5 \end{bmatrix} \tag{7}$$

Then, the stiffness matrix of the assembly of np piles is

$$K = \sum_{p=1}^{np} T_p K_p T_p^t \tag{8}$$

where T_p is the force transformation matrix from the local (pile) axes to the global frame (see Hall & Woodhead, *Frame Analysis*).

Given the applied for a vector P (Fig. 2) the associated displacements υ are determined by solving

$$P = K^{\upsilon} \qquad (9)$$

whence the forces at any pile can be computed by

$$R_p = K_p T_p{}^{t\upsilon} \qquad (10)$$

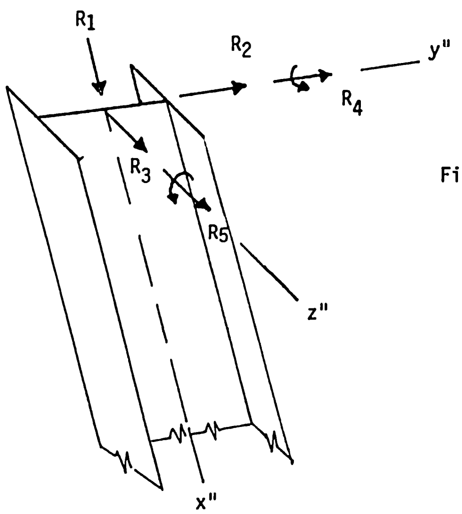

Fig. 1 FORCES AT PILE BUTT

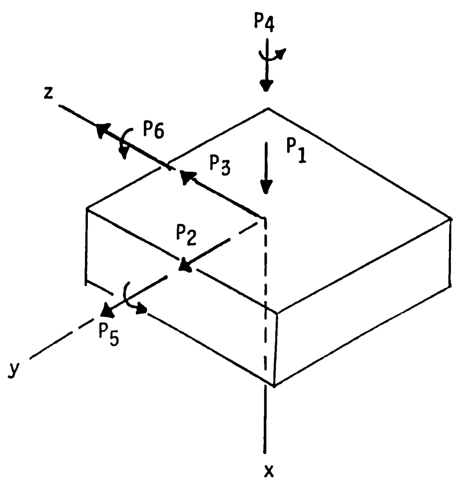

Fig. 2 GLOBAL REFERENCE FRAME AND APPLIED LOADS

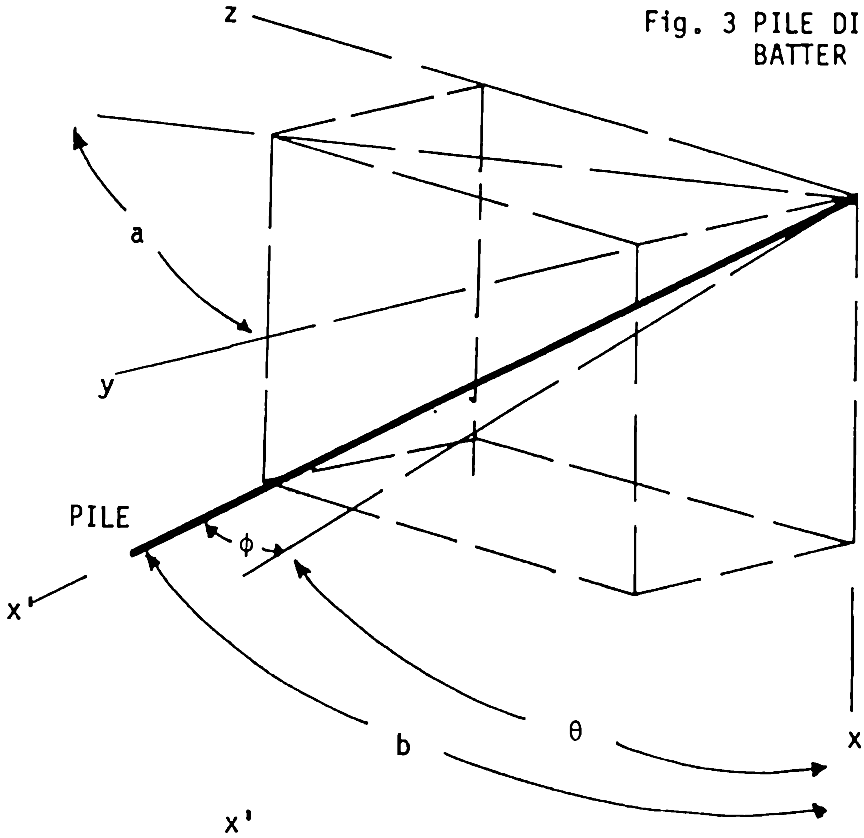

Fig. 3 PILE DIRECTION & BATTER (angles a & b)

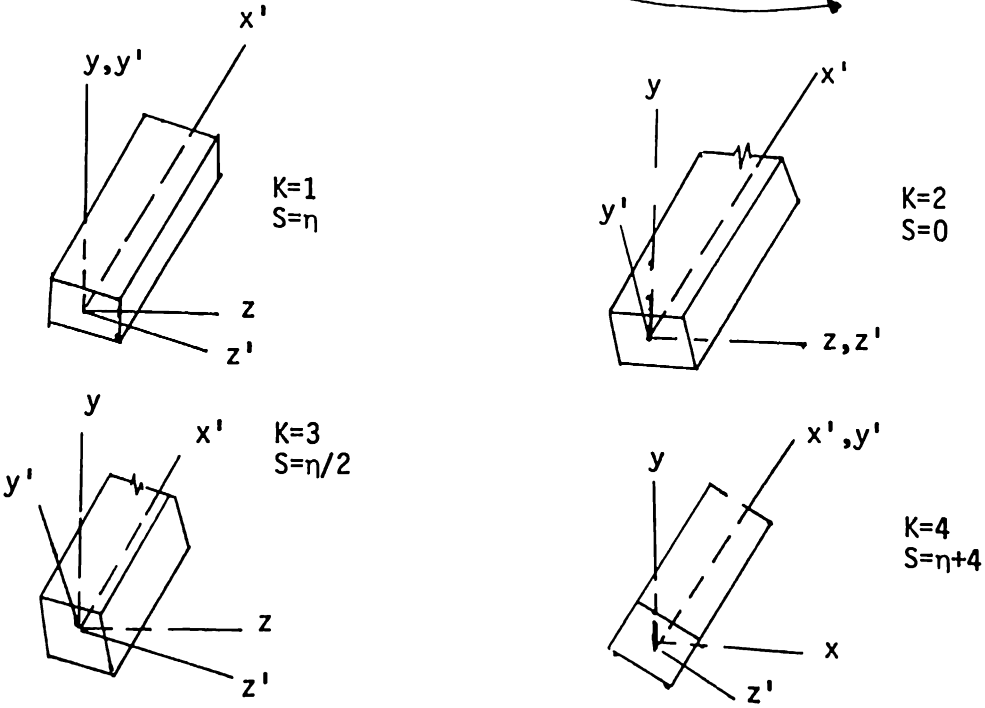

Fig. 4 PILE SECTION ORIENTATION