THE USE OF MODELS IN HYDRAULIC DESIGN

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INTRODUCTION

The use of hydraulic models as a tool in hydraulic design is fairly widespread. Many hydraulic phenomena which occur in nature are too complex to be described by rigorous mathematical techniques and models are used as an alternative means of obtaining the information necessary to complete an efficient and satisfactory design. Even in relatively simple situations, such as the design of spillways or river barrages, it is often impossible to predict the exact nature of the flow patterns without conducting a model study. Theoretical analyses can be used to estimate flow depths but, if the geometry is in any way unusual it is difficult to predict the occurrence of adverse eddies or unwanted features such as reversals of flow. In more complex cases, such as those involving the release of pollutants from industrial or thermal outfalls, model studies have become mandatory in many countries because of the necessity of satisfying regulatory boards that there will be no harmful effects on the water body.

A model may be thought of as a prediction device in which the full sized phenomenon, the prototype, is reproduced at a small scale. Geometric models are quite familiar. In these, every length of the prototype is scaled down by the same amount so that in architectural models, for example, the public may gain an appreciation of the "look" of a building and may understand how it will fit into its surroundings. Hydraulic models, however, although often geometrically similar to the prototype, are intended to reproduce physical phenomena and can therefore only be useful if the important physical aspects of the prototype are reproduced at certain scales which can be calculated. An incorrectly designed model cannot be used as a prediction device and the determination of the criteria which must be satisfied to ensure the validity of the model is therefore of prime importance. These criteria are represented by various models laws which must be used in the design of the model. In turn, the model laws give rise to scale ratios so that measurements of velocity and force, etc., can be made in the model and scaled up to give the appropriate prototype value.

Model laws and scales may be generated in a number of different ways. If the mathematical equations which govern the phenomena are known in

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sufficient detail it is possible to use these equations and develop modelling criteria without great difficulty. Unfortunately, however, this is not often practicable because most model studies are undertaken precisely to deal with conditions which are not amenable to rigorous mathematical analysis. Under these circumstances the modelling criteria must be developed from incomplete methods of analyses.

These methods, generally known as Partial Analyses, are incomplete only in so far as they fail to provide a complete mathematical description of the phenomenon under investigation. However, based largely on dimensional considerations, a Partial Analysis does take the investigator a good way along the road to complete understanding and outlines criteria which can be used as a guide to the organization of an experimental study. This, in essence, is the basis of model testing in which result sfrom one particular set of experiments, conducted in a laboratory, are scaled up and applied to similar phenomena in the field.

Methods of Partial Analysis are described in many texts dealing with fluid mechanics or models(1,2,3). Basically, they fall into three categories; (1) dimensional analysis(4), based on the principle of dimensional homogeneity; (2) similitude and similarity(5), based on the principles of geometric, kinematic and dynamic similarity; and (3) recently developed combinations of the two primary methods(6,7).

Irrespective of which method is used, the scaling laws developed depend on the physical characteristics of the fluid system. In many cases, these laws are incompatible and it is necessary to design the model so that the dominant force actions are reproduced correctly. Other force actions are then out of scale but the scale errors which arise because of this are generally small provided the dominant force action is chosen correctly. For example, similarity of viscous effects would require equal Reynolds numbers in model and prototype. However, in fully developed rough turbulent flow, viscous forces are negligible relative to inertial or gravitational forces and it is known that variation of Reynolds number over a fairly wide range has no significant effect on the flow patterns. Thus, it is possible to model one rough turbulent flow by another at a lower Reynolds number even though scale effects prohibit the obtaining of precious dynamic similarity.

The flows with which civil engineers are concerned are generally large, turbulent, free surface flows dominated by gravity. In such cases, it can be shown that similarity will be achieved only if the Froude number of the model is equal to the Froude number of the prototype.

$$\frac{V}{(gL\ )^{\frac{1}{2}}}_{m} = \frac{V}{(gL\ )^{\frac{1}{2}}}_{p} \qquad (1)$$

where V = velocity, g = gravitational acceleration, L ≐ length and subscripts m and p refer to model and prototype, respectively.

Various models of this type, but of differing degrees of complexity will be discussed.

MODELS OF HYDRAULIC STRUCTURES

Models of simple hydraulic structures are probably the most common type of hydraulic model to be studied. They are, compared to river models for example, relatively cheap, and, being concerned primarily with the determination of flow patterns, they are generally easy to operate and easy to interpret.

Flow over, and around, hydraulic structures involves significant vertical components and it is therefore necessary to construct the model to a natural, undistorted scale. Any exaggeration of the vertical scale would cause an unacceptable exaggeration of vertical flow components and must be avoided for this reason. Incorporation of a model weir into a distorted river model poses no significant problems. Despite the exaggeration of river profiles, the weir itself must be undistorted. Similarity will still be achieved provided the effects of the side walls are negligible.

Because the flow is dominated by gravitational effects, models of free surface flow around hydraulic structures must be scaled according to the Froudian criterion. To ensure freedom from scale effects the model must be fairly large and scales ranging from about 1/15 to 1/60 are common. In achieving geometric similarity it is generally not necessary to attempt to scale the roughness because the roughness of the prototype is usually small so that the model should be as smooth as possible.

Tests on the performance of spillways and stilling basins represent an important example of the use of structural hydraulic models. Rapid flow leaving the toe of a spillways could cause considerable erosion problems in the downstream channel and stilling basins are designed to ensure that this does not occur. Design criteria include good energy dissipation, smooth distribution of flow entering the stilling basin and a transition from rapid to tranquil flow without undue vibration or oscillation of the high velocity jet. It is important to ensure that any hydraulic jump is not swept downstream at high flows and that the submerged jet does not penetrate through the basin to cause erosion at the downstream end. Typical stilling basin layouts are available in many design manuals but models studies are advantageous particularly where the configuration deviates from the norm. Testing generally involves determination of energy disssipation and is concerned with obtaining satisfactory flow patterns and freedom from scour. The use of chute blocks, basin blocks and end sills is fairly standard and these may, of course, be studied in the model to determine the most satisfactory arrangement.

Operating and prediction scales are obtained directly from equation (1). Assuming the same gravitational acceleration in model and prototype, rearrangement of equation (1) leads to the velocity scale.

$$\frac{Vm}{Vp} = \left(\frac{Lm}{Lp}\right)^{\frac{1}{2}} \qquad (2)$$

indicating that if, for example, the linear scale were 1/25, the velocities in the model would be one-fifth of those in the prototype. Because discharge is given by the product of velocity and cross sectional area (ie. $Q \alpha VL^2$) the discharge scale may be similarly calculated:

$$\frac{Qm}{Qp} = \left(\frac{Lm}{Lp}\right)^{5/2} \qquad (3)$$

In most cases, equation (3) provides the scale needed to operate the model whereas, equation (2) would be a predictive scale used to predict prototype velocities from measured model velocities.

Although most structural models will be free from scale effects by virtue of their size it is important to realize the dangers associated with small models. As smaller scales and smaller models are used, the effects of viscous influence and surface tension forces increase. These may have a significant effect on the flow if model depths are very small and some studies have been undertaken to determine the smallest scales which could be employed. In a study of surface tension effects, Maxwell(8) found that the lower scale limit was not determined by surface tension effects but, "by one or more of the following: roughness of the model; viscous effects in the model; accuracy of instrumentation used in the model; accuracy of determination of the model crest datum; and the accuracy with which the prototype geometry is reproduced in the model". Nevertheless, surface tension effects can become significant in models of sharp crested weirs if the jet fails to spring clear of the weir. It has then been suggested(9) that the head over the weir should generally exceed 50mm to avoid the slinging nappe condition.

Various, rather complex, methods have been developed for calculating the minimum scale which can be used but, to some extent, these calculations are somewhat academic and are needed only where a precise calibration is required and where very small models are involved. Otherwise, as suggested by Allen(10), "with any scale likely to be adopted in practice, a model of a river weir, even of complex shape, will yield coefficients which may be applied to the full size weir with an error of only a few percent; in general, about 5%, provided that the head in the model is not less than 0.25 in. (6.5mm), and that a sufficient length of the upstream portion of the river is reproduced to ensure that the flow conditions in the region of the weir are similar to those found in nature".

The foregoing has discussed the most simple types of structural models where interest is centered largely on velocity, discharge and flow patterns. More complex situations arise where there is air entrainment. This is a true surface tension effect which cannot be adequately modelled in a Froudian model because there is no way to scale down bubble size and bubble velocity. In a spillway model, this has the relatively insignificant effiect of causing less air bulking in the model than in the prototype. However, water intakes, which may be subject to vortex action, and siphons, which rely on air entrainment for their correct operation, represent a category of model in which the effects of air entrainment are significant and cannot be neglected. Various experimental criteria(11-13) are then used in addition to equations 1-3 to ensure reasonable reproduction of prototype performance.

Another most complex class of hydraulic structure model is that in which the individual parts of the structure vibrate because of the fluid action. These models are termed hydroelastic and have become particularly important because of developing interest in modelling the behavior of large structures at the various offshore oil fields around the world. Then, in addition to modelling the fluid system, it is necessary to model the response of the structure to that system. It can be shown(14) that this requires a Young modulus scaled down according to the length scale, similar damping in model and prototype, and a structural density in the model equal to that in the prototype. Once again these requirements are incompatible and various compromises(14) are made to ensure realistic performance.

Other aspects of hydraulic structure models relate to the effects of scour and sediment transport. These topics have been analyzed in great depth and considerable attention has been paid to the determination of scaling laws(15) which will permit accurate quantitative data to be obtained from a model. In many cases, however, such data are not required and qualitative results will be sufficient. A Froudian model may not indicate depths of scour holes correctly but it may conveniently be used to determine if one structural configuration is more or less susceptible to scour than another. Often such information may be adequate to make decisions regarding the "best" prototype design. Another approach, again qualitative, would be to measure model velocities and scale these up to determine whether the expected prototype velocities will cause erosion in the known prototype bed materials.

RIVER MODELS

When interest is centered in the river itself, rather than in some structure placed in it, the model must be more complex because friction is of obvious importance in determining the movement of water along the channel. Thus, in addition to the previous requirements that gravitational forces be properly simulated and that the flow be turbulent, it is necessary also to ensure similarity of the frictional forces. Further complications exist with respect to the general shape of open channels. Whereas river structures are fairly compact, so that vertical and horizontal dimensions are not unduly different, the opposite is true of the river in which these structures are constructed. Commonly a river model might be designed to investigate some problem over a reach of several kilometers in which depths are no more than a few meters. Thus, a factor of 10^3, or more, may be involved in a comparison of horizontal and vertical distances. If the vertical dimensions are to be represented by a reasonable length in the model, it follows that the horizontal dimensions of the model (at the same scale) must be very large indeed. This may not be unduly difficult when the model is concerned with short river reaches, but in many cases it is not feasible to simulate both dimensions to the same scale without constructing an enormous, hopelessly uneconomic, model or a model in which the vertical dimensions, the depths are ridiculously small.

In fixed bed models, that is models in which the bed material is solid or does not move under the action of the flow, three conditions are necessary for similarity. The Froude number must be the same in model and prototype, the flow in the model must be turbulent and the roughness in the model must be scaled in accordance with a resistance equation. This would show, for example, that the scale of Manning's roughness, n, would be given by

$$\frac{nm}{np} = \left(\frac{Lm}{Lp}\right)^{1/6} \qquad (4)$$

These three conditions cause immediate problems unless the geometric scale is large. For example, consider the scales for a model of 1800 m of a river having a roughness typified by n = 0.02. The model is to be constructed in a tank with a working length of 15.52m and it will be assumed that the Reynolds number, based on a prototype mid stream depth of 2.2m and a mean velocity of 0.3m/sec, must be greater than 800 to ensure rough turbulent flow.

The maximum length of the model is limited to 15.52m and, on this basis alone, the geometric scale can be no larger than

$$\frac{L_m}{L_p} = \frac{15.52}{1800} = \frac{1}{116} \qquad (5)$$

For Froudian similarity,

$$\frac{V_m}{V_p} = \left(\frac{1}{116}\right)^{\frac{1}{2}} = \frac{1}{10.77} \qquad (6)$$

The mean velocity in the model will then be

$$V_m = 0.3 \times \frac{1}{10.77} = 0.028\text{m/sec.} \qquad (7)$$

and the mid stream depth, h, will be

$$h_m = 2.2 \times \frac{1}{116} = 18.97\text{mm.} \qquad (8)$$

From equation 4, the roughness in the model should be typified by

$$n_m = .02 \times \left(\frac{1}{116}\right)^{1/6} = .009 \qquad (9)$$

and, assuming a kinematic viscosity of $10^{-6}\text{m}^2/\text{sec}$, the Reynolds number is given by

$$R_e = \frac{.028 \times .01897}{10^{-6}} = 531.2 \qquad (10)$$

Three difficulties are obvious from this simple example.

Firstly, as mentioned earlier, there is the problem of measurement. Depths and velocities in the model will be very small and it must be remembered that in some parts of the model the magnitudes may be considerably smaller than the mid stream values given in equations 7 and 8. Variations in depth and velocity will be even smaller.

The second difficulty relates to the type of flow experienced in the model because equation 10 shows that the model flow will not be rough turbulent. A similar calculation with prototype values yields a Reynolds number of 660,000 which indicates that the prototype, as would be expected, is rough turbulent. Clearly the flow in the model will be quite dissimilar to that of the full size river.

Thirdly, there is the problem of roughness. Equation 9 suggests that the model should have a value of Manning's n = .009 which is almost impossibly smooth. However, this is not the whole picture and the model roughness must be considered in conjunction with the reduction in Reynold's number when it can be seen tha tthe overall effect is to move the flow from the rough turbulent zone towards the smooth zone of flow from the rough turbulent zones towards the smooth zone of flow and perhaps, with sufficient reduction in size, into the laminar region. Not only does the type of flow change radically but the type of resistance experienced also changes. From resistance dominates in the prototype but viscous resistance becomes much more significant in the model.

One method of overcoming all these difficulties is to enlarge the model and use a much bigger scale. In many cases, however, this is unacceptable and a commonly used alternative is to exaggerate the model depths and to use a distorted model with different scales for horizontal and vertical dimensions. This has been found to be practically effective and there are fundamental reasons which suggest the possibility of distortion. Rivers and channels show a natural distortion when the cross sections of large naturally occurring channels are compared with small natural channels. Width to depth ratios decrease as the overall size decreases and, although the reasons have never been fully explained, there seems to be a natural tendency towards deeper, narrower cross sections as the overall size of the channel decreases.

This technique solves all the problems mentioned earlier. In the previous example, with a horizontal scale of 1/116, as before, but with a vertical scale of 1/15, it can be shown that mean model velocity should be .077m/sec. and that the value of Manning's n should be 0.035. The flow in the model will be turbulent and there is now no problem in obtaining the necessary model roughness. However, roughness calculations here are only approximate because the roughness of a natural river cannot be adequately described by a single value of n and an empirical approach to the design of model roughness must be used.

If the model is run to satisfy Froudian scaling conditions the surface slopes will only be correctly represented if the model resistance is correctly scaled. Too little resistance will result in slopes which are too flat and too much resistance will have the opposite effect. With model cross sections moulded in cement plaster, or some similar smooth material, it is likely that the model will be too smooth and this raises the possibility of post construction adjustment of the model roughness, it being known that the correct resistance will be obtained when the model correctly reproduces prototype surface profiles.

The procedure is fairly simple. Following construction the model is calibrated by running it at a discharge which simulates, under Froudian conditions, a prototype discharge for which surface profiles have previously been recorded. If, in a particular reach of the model, the downstream surface elevations are correct and the upstream depths are too small the model is overly smooth and some roughness must be added. Various methods of roughening the model are available. Angular pebbles, large enough to remain stationary at the maximum velocity to be experienced, may be scattered on the bed or, if necessary, glued in place. Wire mesh may be attached to the bed or can be suspended from the banks to simulate resistance caused by overhanging bushes or trees. If more roughness is required, vertical pegs, extending above the surface, may be fixed to the bed.

The location of the individual roughness elements is chosen in a fairly random manner but attention must be given to the probability that some parts of the model will require more added roughness than others. Generally, areas of low depth and low velocity, such as tidal and flats, will require less roughening than the main channels.

When the model has been adequately tested and it is known that it can reproduce phenomena already measured in the prototype, it is then possible to use it for prediction purposes. The type of flow and the information required from the model to a large extent determine the distortion to be used(16). Natural scale models should be used for studies of high velocity flow or studies where the velocity distribution at rapid bends is important. Transitions from rapid to tranquil flow, which involve significant vertical velocity components must also be constructed with no distortion. Minimal distortion up to about 6.0, is permissable for models involving flow distribution in branching channels or velocity distributions at locations remote from bends or rapid changes in cross section. When interest is centered primarily on mean cross sectional flow characteristics and surface profiles, the distortion may be increased but it is suggested that an upper limit for reasonable similarity is in the vicinity of 20.0.

When the bed of a model is free to move under the action of the flowing water, the scaling requirements become more exacting and the problem of modelling becomes more complex. In addition to modelling the hydrodynamic phenomena, attention must be given to the morphological processes involved and, indeed, these become of primary importance because mobile bed models are used particularly to study sediment, or bed movements. In this respect hydrodynamic effects are of somewhat lesser importance. The difficulties encountered in the attempt to achieve reasonable similarity are, however, caused by more than increasingly complex scaling requirements because the modeler can no longer control frictional resistance to the extent which is possible in a fixed bed model. Neither is it possible to control the slope of the bed and so the choice of material used to simulate the model bed becomes of primary importance. Many modelling institutions have developed considerable experience with a few different materials and only relatively recently have formal scaling methods been developed(15) to permit calculation of such things as specific weight and particle size, etc.

Further complications may exist if the model incorporates part of a river estuary, or bay, because tidal action, and possibly wave action, must then be taken into account. In such models the criteria for simulating the different processes involved may conflict to a considerable extent and extensive experience is required to obtain a reasonably useful model.

OUTFALL MODELS

One important and complex class of hydraulic model is that which deals with ocean outfalls. When effluent or heated cooling water from a thermal power station is discharged into the marine environment, density currents develop and move gradually away from the outfall site. Designers are interested in the location of the density current and in the dilution which occurs in it. For example, it is unacceptable for domestic sewage to be washed back onto beaches or for heated cooling water to be recirculated between outfall and intake. Dilution is important because this affects the rate of natural purification or, in the case of a heated effluent, the temperature of the mixture. Models are of value in such a study because the mixing process is particularly difficult to describe in mathematical terms unless the geometry and the hydraulics of the receiving water are fairly simple.

In such a case, a number of different processes are involved and each must be handed so as to be similar to the prototype process. Very close to the outfall the jet of effluent is dominated by its initial momentum. Density differences have relatively little effect here but become significant as the momentum falls off due to turbulent mixing. When the outfall is submerged density differences cause the jet to bend and rise towards the surface. There it turns and spreads horizontally away from the outfall site. Three different phenomena are important throughout this stage of the dispersal process. First the effluent will spread because of the density difference between the effluent-receiving water mixture and the surface water. This is essentially a convective, or densimetric spread. Some dispersion will also be caused by prevailing ambient currents. These may be constant, as in the case of some fresh water lakes or may vary periodically as would be normal in tidal areas and, in particular, near the mouth of river estuaries. Another process which might be considered is the dispersion resulting from eddy diffusion. However, this is usually very small and is generally neglected. The final stage is either surface heat transfer, in the case of a warm effluent, or natural decay in the case of a non-conservative substance (e.g., bacterial concentration or biochemical oxygen demand). These are difficult to model and are usually treated in a conservative fashion. For example, heat dissipation models would be operated in an enclosed building in which surface heat transfer rates would be less than in the field. It is then possible to be sure that temperatures in the field will be lower than those measured in the model.

The processes which must be modelled accurately are thus reduced to (a) turbulent diffusion and buoyant rise between the outfall and the surface, (b) densimetric or convective spread away from the outfall site and (c) mass transport by ambient currents.

Gravitational forces dominate each process but in cases (a) and (b), the gravitational effects are reduced by the buoyancy of the effluent. Furthermore, during the buoyant rise there are significant vertical flow components so any model must be undistorted. In general then, the requirement is for a turbulent Froudian model, to obtain overall flow similarity, with the same density difference as that which exists in the prototype, to ensure similarity of buyoncy forces, and with the same vertical and horizontal scales to obtain similarity of vertical flow components. Such a model will provide accurate information on spread characteristics but may prove to be rather large and uneconomic if an extensive area is to be modelled.

Then it is common to model the dispersion process in two stages(17). First a small undistorted model is built and operated. This would cover only the outfall and its immediate vicinity. The purpose of the model would be to establish the optimum outfall geometry and to obtain information on the nature of the flow as it leaves the outfall. A second, larger and distorted, model covering the entire area of interest, would then be built to determine the gross flow characteristics. The input to this model would be based on the output data of the previous smaller models. Even though two models are required the costs are still less because of the smaller scales which can be used. For example, in one investigation of heat dissipation in 50 km. of a river estuary, calculation(18) showed that an undistorted model would require to be operated at a scale no smaller than 1/14. This would have required a model over 3 kms. long and would obviously be uneconomic. The use of distortion however, made it possible to reduce the overall size(19) and the actual model investigation was run with a horizontal scale of 1/400 and a vertical scale of 1/60. This model, operated in the 1960's, cost in the region of four million pesos but was estimated to have saved approximately ten million pesos by showing that special mixing devices were unnecessary.

CONCLUSIONS

Only three of the many different types of hydraulic model have been discussed in this paper. Other models which are of importance to Civil Engineers include wave models to investigate harbours breakwaters and ship moorings, tidal models for studies of river estuaries, surge models for flow visualization in complex hydro-electric surge chambers and, the most complicated of all, mobile bed coastal models involving wave and tidal action for studies of sediment movement along beaches and coasts. Recent developments, of increasing importance to countries in northern latitudes, have led to models involving movement of ice in lakes and rivers or drifting snow around buildings, along roads and across railways.

Models invariably save money for the client by providing answers to problems which could not otherwise be solved prior to construction of the full size prototype. They highlight any deficiencies in design, and in the hands of a skilled operator, suggest alternative configurations which eliminate the possibility of having to make costly, post construction alterations to the finished plan. Models are, therefore, an integral part of the design process and should be treated as such. With impressive accuracy, depending on the degree of complexity, they provide information on such things as: flow patterns, surface elevations, velocities, tidal heights and times, wave or current forces, salinities, mixing and dispersion, sediment movement and depth of scour holes, etc. Nevertheless, some caution is necessary because, as the hydraulic environment becomes more complex, it is increasingly difficult, and in many cases impossible, to simultaneously satisfy all the scaling laws for each of the various processes which are involved.

Under these circumstances successful modelling relies considerably on the expertise of the modeller and science must be effectively reinforced, if not replaced, by art and experience.

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