AN INTRODUCTION
TO
HYDRAULIC MODEL STUDIES[1]

By

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FOREWORD

Simulation is a useful, and at times indispensable, tool for most planning and design activities of Engineers. It can be used to predict the behavior of a planned system, or to extract parameters to be used in the actual design.

In Fluid Mechanics, Hydraulic Engineering, and Hydrology, "Modelling" is a broad term that encompasses physical, mathematical, electrical analogue, and physical nalogue simulation systems. Physical hydraulic models themselves consist of a great variety. To name but a few, we have rigid bed, semi-rigid bed, movable bed, distorted and undistorted, tilted and non-tilted models.

This paper is intended to disseminate, at an introductory level, the techniques involved in hydraulic model studies. The word "technique" is used to emphasize that modelling is not an exact science. At present technology levels, whether local or elsewhere, hydraulic models are still products of both science and art. However, the paper shall show that modelling is a logical and systematic activity. It is also an attempt to stimulate the readers further investigation into the subject.

Notation

m used as a subscript to denote model quantity

p used as a subscript to denote prototype (actual size) quantity, also used as a pressure quantity

$\mathbb{F}$ Froude Number $= \dfrac{v}{\sqrt{gL}}$, dimensionless

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v	velocity
L	length quantity
g	acceleration due to gravity or used as a subscript to denote gravitational quantity
Q	volumetric discharge
$\mathbb{E}$	Euler Number $= \dfrac{(\Delta\rho)}{\rho v^2}$, dimensionless
(Δp)	pressure difference
ρ	mass density
l,k	length quantities
θ	angle
X Y Z	components of L in the longitudinal, lateral and vertical directions, respectively
F	force quantity
I	used as subscript to denote inertial quantity
P	used as subscript to denote pressure quantity
μ	dynamic viscosity
k	bulk modulus of elasticity
σ	surface tension
$\mathbb{R}$	Reynolds Number $= \dfrac{vL}{\nu}$, dimensionless
$\mathbb{M}$	Mach Number $= \dfrac{v}{\sqrt{k/\rho}}$, dimensionless
$\mathbb{W}$	Weber Number $= \dfrac{\rho v^2 L}{\sigma}$, dimensionless
ν	kinematic viscosity

I. *The Modelling Process*

The following discussion is applicable without distinction as to type of model. In the design of any hydraulic structure or water resources development measure the use of models implies a procedure shown in Figure I.1

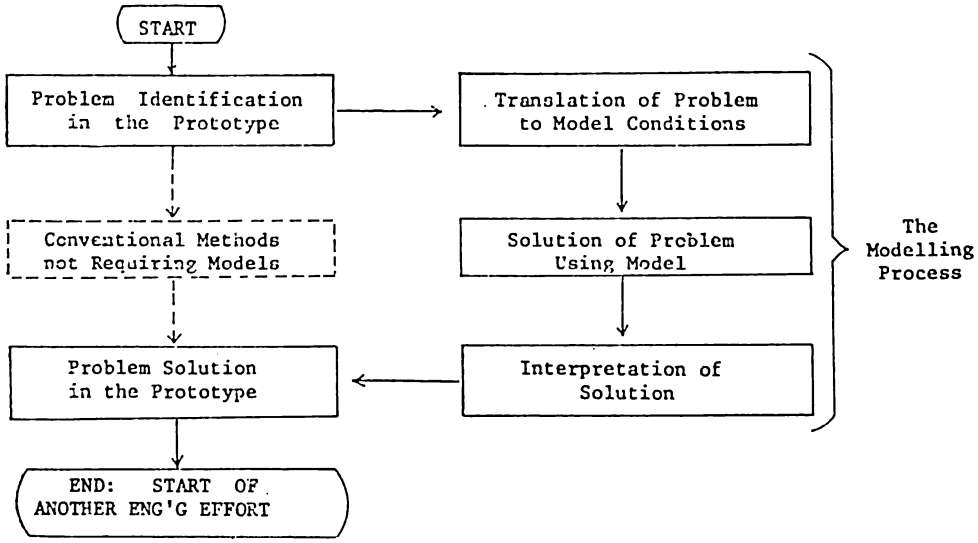

Figure I.1 Engineering in General

It is seen that not all engineering activities require the use of models. For these, the problems require methods of solution that have been established as theoretically sound and have been proven by extensive experience. The planning or design engineer relies on such methods with confidence.

On the other hand, some methods of solution cannot be relied upon completely. These arise in cases where the theories are not clearly established, or only limited experience have been gained in their use, or too much is at stake (lives and property, or large capital investments on structural measures), such that further assurance is required. It is in these instances that model studies proved their worth.

The right side of Figure I.1 is an extremely abridged presentation of what hydraulic modelling is all about. However, it implies that modelling is not arbitrarily building scaled-down versions of existing or proposed hydraulic structures. Firstly, prototype conditions must be properly converted to model conditions following a definite set of procedures and laws. After a solution is arrived at in the model, interpretation of results as applicable to prototype conditions requires both technical knowhow and engineering insight. The difficulty arises from the fact that the problem solved is actually a model problem, not a prototype problem.

Activities in the modelling process are outlined on Figure I.2.

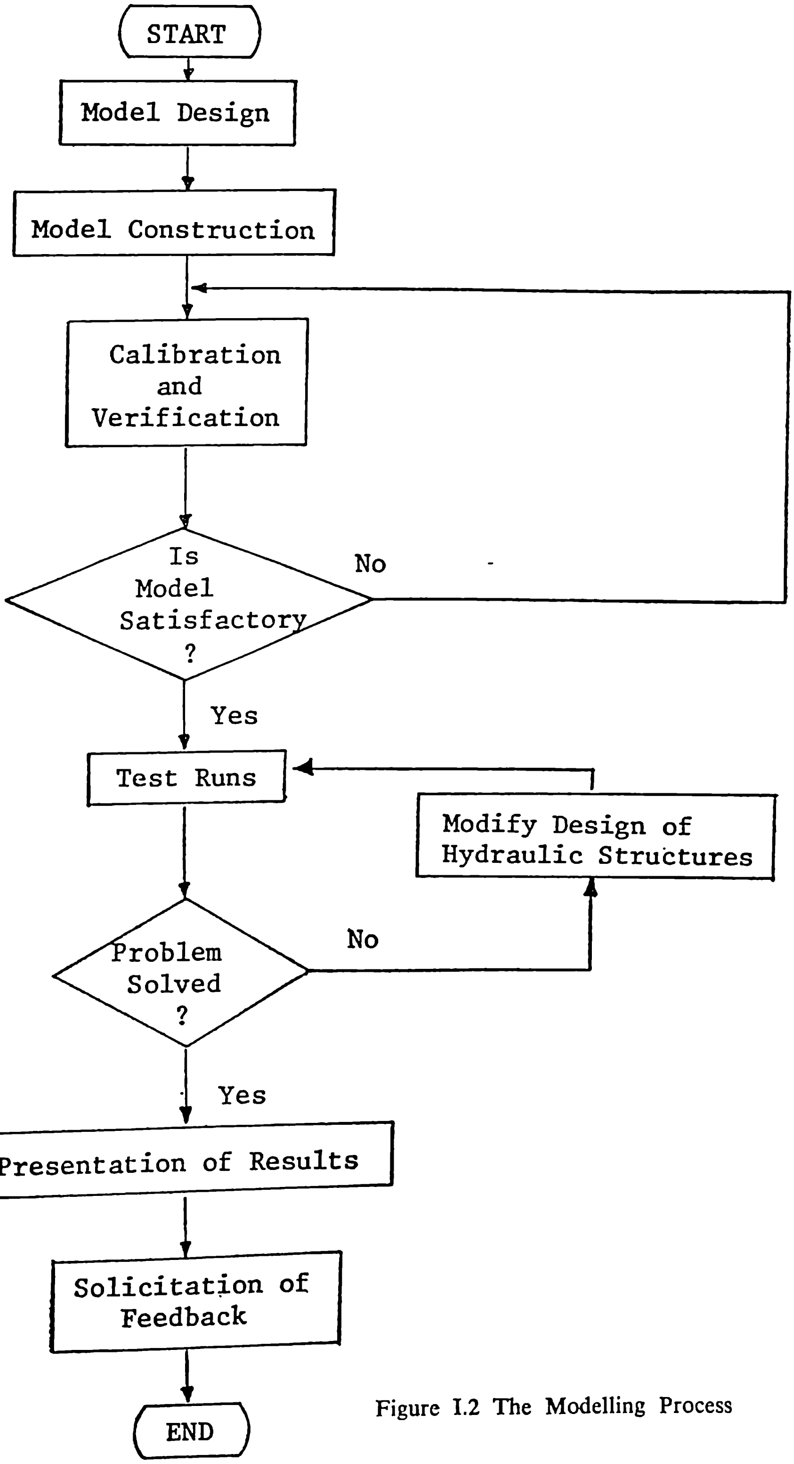

Figure I.2 The Modelling Process

Each step is discussed briefly as follows:

a. Model Design

For physical models, the most crucial and at times most difficult aspect in model design is the selection of scales. This is governed by the following criteria:

1) Governing Laws and Conditions — In short, this means that quantities in the prototype have corresponding values in the model and vice versa, derived in an orderly manner. These laws are detailed in the section on "Principles of Similarity".

2) Precision of Measurement — This means that variables measured in the model should have magnitudes much larger than the standard error of measuring instruments. For example, when water depths are measured, the rule of thumb commonly used is that the average depth throughout the model should not be less than 10 cm.

3) Logistical Limitations — Aside from availability of funds, this includes consideration of space and water supply capacity available at the research facility.

A simple hypothetical example is presented here to illustrate the above considerations: It is proposed to test a new design of an ogee spillway with prototype conditions shown on Figure I.3. In addition to the usual determination of the rating curve, pressure measurements on the surface of the ogee shall also be conducted. Considering available funds and space, as well as precision of available instruments, a length scale of 1/50 was deemed satisfactory. From the nature of the problem, it is evilent that Froude Condition predominates,

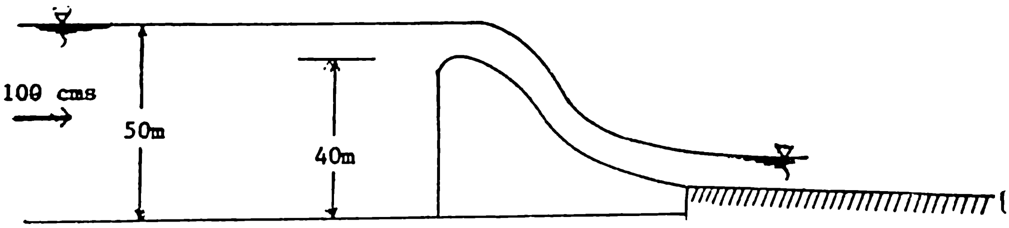

Figure I.3 Hypothetical Spillway

i.e. that, $\mathbb{F}_m = \mathbb{F}_p$ (I.1) or,

$$\frac{\nu_m}{\sqrt{gLm}} = \frac{\nu_p}{\sqrt{gLp}} \qquad (I.2)$$

From which,

$$\frac{\nu_m L_m^2}{L_m^2\sqrt{gLm}} = \frac{\nu_p L_p^2}{L_p^2\sqrt{gLp}}$$

which gives

$$\frac{Q_m}{L_m^{5/2}} = \frac{Q_p}{L_p^{5/2}} \qquad (I.3)$$

Since

$$\frac{L_m}{L_p} = \frac{1}{50}$$

then

$$\frac{Q_m}{Q_p} = \left(\frac{1}{50}\right)^{5/2} = \frac{1}{17678} .$$

For the maximum prototype discharge of 100 CMS, we get the required model discharge of 5.66 liters per second. This is easily attainable.

For the pressure scale, Euler Condition holds, i.e.,

$$E_m = E_p \qquad (I.4)$$

or

$$\frac{(\Delta p)m}{\rho_m \nu_m^2} = \frac{(\Delta p)p}{\rho_p \nu_p^2} \qquad (I.5)$$

Equation 11.2 indicates that $\nu_m/\nu_\rho = (L_m/L_\rho)^{1/2}$, then

$$\frac{(\Delta\rho)_m}{(\Delta\rho)_p} = \frac{L_m}{L_p} \qquad (1.6)$$

or that $(\Delta\rho)_p = 50(\Delta\rho)_m$

Another aspect of model design involves the detailed design of the water recirculation system. This includes the supply pipes, head tank, return channels and occasionally sump and pump selection. Figure I.4 shows schematically the integral parts of a self-contained (it has its own pump and sump) model.

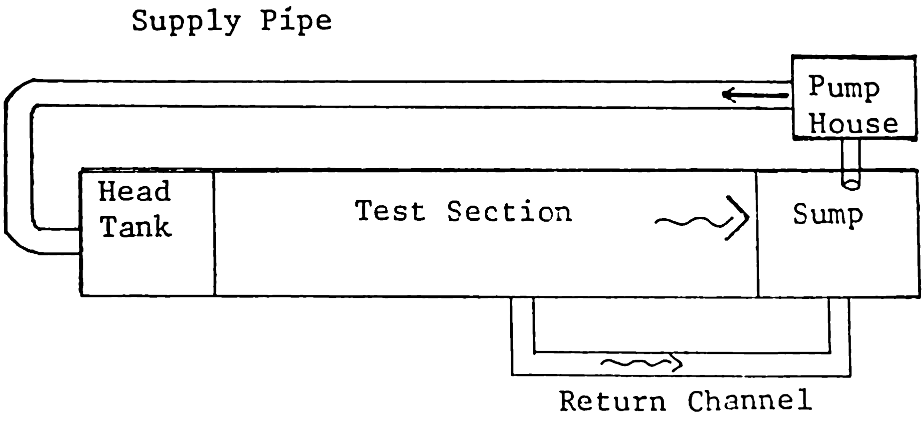

Figure I.4 Model and Appurtenant Structures

b. Model Construction

One may look at a model as a precision instrument for the study of a specific problem. As such, its construction should be viewed exactly likewise, that of the fabrication of a precision instrument. Any systematic fabrication effort necessitates the use of construction drawings. However, model construction differs from prototype construction in terms of scale or size of construction. Thus, at an early stage, the model designer decides on which details of the prototype cannot be reproduced and will have negligible effects on the model tests. On the one hand, therefore, model construction drawings contain less detail.

On the other hand because of its size and the precision required, the construction drawings for the model are made to show exactly the materials to be used for the fabrication of the various structures to be included. For this, the model designer has wide variety of materials to select from. These range from conventional mortar cement, reinforced concrete and lumber, to exotic materials like molded acrylic sheets and fiberglass. Choices are made depending on the precision required, material cost and difficulty of fabrication.

c. Model Calibration and Verification

After the model is completed, it should be "proven". It should do what it was designed for, that of simulating prototype conditions. Thus, on its initial runs, measurements are made and compared against a set of data gathered from the prototype. Model features, e.g., bed roughness, are adjusted until close agreement is achieved.

If available, a second set of prototype data is used to verify that the model indeed properly simulates the prototype. In some instances, prototype data may not be available due to lack of field measurements, or at times, the problem calls for the simulation of proposed structures. In these cases, calibration and verification is done using measurements on similar structures. Herein lies the value of modelling, as well as field experience.

d. Test Runs and Modifications

A test program is prepared for each model investigation project. This tool is used to minimize the number of runs to be done by grouping the required measurements and observations in a logical and efficient sequence.

Models of proposed structures are monitored to detect flaws on events not foreseen by the designer of the prototype such as unwanted vortices or undesirable pressure distributions. Since a perfect design is seldom achieved, which attests to the value of hydraulic models, close examination is a must. The design of the structure or parts of it are modified towards the elimination or minimization of negative effects. This may requires tests on a number of schemes.

At times, various schemes are tried out in the model for the purpose of arriving at more economical designs, such as reduction of height or length of training walls or retention dikes. It is a moot point to say that it is much cheaper to try schemes in the model than to try the same in the prototype, not counting the possibility of grave economic and social implications of a design error.

e. Presentation of Results

Aside from submission of reports, modern audio-visual equipment are utilized. By documenting the study on film or video-tape, the model investigator can communicate his methods and their results results in a manner easily understood by designers, decision makers, or non-model experts.

f. Solicitation of Feedback

The modelling process should not stop with the submission of the project report. It would be of great advantage for both the designer and the model investigator to check on the performance of the prototype during and after the proposed structures are built. Both will benefit with greater insight on the limitations of hydraulic design and modelling. The public in general will benefit when both improve their techniques in view of the feedback they receive.

II. *Principles of Similarity*

A model is said to be similar to a prototype if for any quantity or event in the prototype, a corresponding quantity or event is or can be reproduced or law. In fluid mechanics, similarity may be conceptually divided into three categories: Geometric, Dynamic and Kinematic similarity (or similitude).

a. Geometric Similarity specifies that there be a one-to-one correspondence between each point, line or angle in the prototype and model, following a definite scale. Referring to Figure II.1, two lines in the prototype, lp and kp, meeting at a point is reproduced in the model following the rule $L_m/L_p = K_m/K_p$ and that $\Theta_m = \Theta_p$. In general complete geometric similarity may be stated as:

Figure II.1 Geometric Similarity

$$\frac{Lm}{Lp} = \frac{Xm}{Xp} = \frac{Ym}{Yp} = \frac{Zm}{Zp} \qquad \text{(II.1)}$$

b. Dynamic Similarity is established through a consideration of the forces involved in fluid flow. This will be discussed further below. At this point, the principle may be stated simply as: Force polygons in the model and the prototype are similar. This is illustrated in Figure II. 2. Shown there are force vectors acting on fluid particles

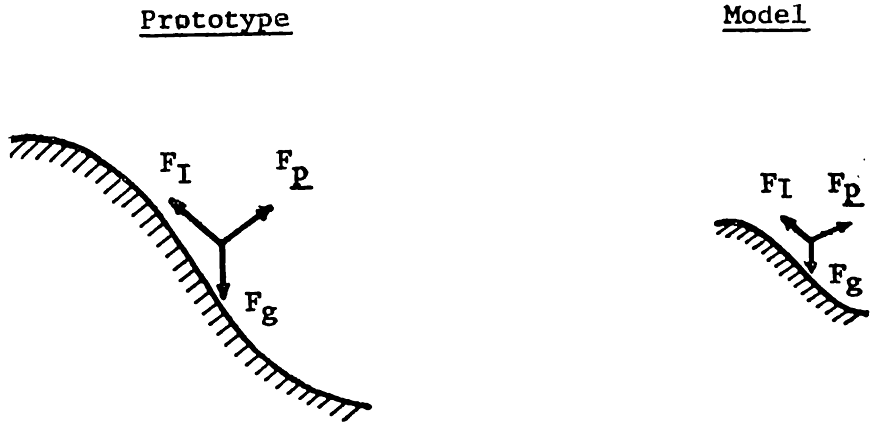

Figure II.2 Dynamic Similarity

sliding down the face of a spillway. Being a simple example, not all forces are represented. Those shown are that of inertia, F_I, pressure, F_p, and gravity F_g. Dynamic Similarity specifies that corresponding ratios of forces in model and prototype are equal, i.e., that

$$\left.\frac{F_I}{F_g}\right|_m = \left.\frac{F_I}{F_g}\right|_p \;;\; \left.\frac{F_p}{F_I}\right|_m = \left.\frac{F_p}{F_I}\right|_p \qquad \text{(II.2)}$$

as with that o fany ratio of forces.

c. Kinematic Similarity states that streamline systems in the model and prototype are similar. It is a consequence of the presence of both geometric and dynamic similarity. In fluid mechanics, the necessary and sufficient condition for exact simulation of the prototype by a model is the presence of complete geometric and dynamic similarity. It shall be shown below that complete similarity is not achieved in practice, since it implies a 1:1 scale model.

Forces acting on a moving body of fluid may be categorized and expressed as in Table II.1

Table II.1 Forces in Fluid Mechanics

Force Type	Expression	Explanation
Inertia	$\rho v^2 L^2$	unit momentum x volumetric discharge
Pressure	$(\Delta p)\ L^2$	pressure x area
Gravity	$\rho g L^3$	unit weight x volume
Shear	$\mu v L$	average shear stress x area
Elastic	kL^2	modulus of elasticity x area
Surface ~~tension~~	σL	surface tension x length

If we consider certain ratios of forces we get:

$$\frac{\text{Pressure Force}}{\text{Inertial Force}} = \frac{(\Delta p)L^2}{\rho v^2 L^2} = \frac{(\Delta p)}{\rho v^2} = \mathbb{E} = \text{Euler Number}$$

$$\frac{\text{Inertia Force}}{\text{Gravity Force}} = \frac{\rho v^2 L^2}{\rho g L^3} = \frac{v^2}{gL} = \mathbb{F}^2 = (\text{Fronde Number})^2$$

$$\frac{\text{Inertia Force}}{\text{Shear Force}} = \frac{\rho\nu^2L^2}{\mu\nu\ L} = \frac{\nu L}{\mu/\rho} = \mathbb{R} = \text{Reynolds Number}$$

$$\frac{\text{Inertia Force}}{\text{Elastic Force}} = \frac{\rho\nu^2L^2}{kL^2} = \frac{\rho\nu^2}{k} = \mathbb{M}^2 = (\text{Mach Number})^2$$

$$\frac{\text{Inertia Force}}{\text{Surface Tension Force}} = \frac{\rho\nu^2L^2}{\sigma L} = \frac{\rho\nu^2L}{\sigma} = \mathbb{W} = \text{Weber Number}$$

A statement of dynamic similarity may be expressed as:

$$\mathbb{E}_m = \mathbb{E}_p \ ; \ \mathbb{F}_m = \mathbb{F}_p \ ; \ \mathbb{R}_m = \mathbb{R}_p \ ; \ \mathbb{M}_m = \mathbb{M}_p \ ; \ \mathbb{W}_m = \mathbb{W}_p \qquad \text{(II.3)}$$

If all the equalities (II.3) are satisfied, then we say that the model has dynamically complete similarity to the prototype. However, this is difficult to achieve in practice, as will be shown. If we aspire to make the model satisfy the Froude and Reynolds conditions simultaneously, we get for Froude similarity; $F_m = F_p$

$$\frac{\nu_m}{\sqrt{g_mL_m}} = \frac{\nu_p}{\sqrt{g_pL_p}} \qquad \text{(II.4)}$$

and since both model and prototype are on the same planet, $g_m = g_p$ and we get:

$$\frac{\nu_m}{\nu_p} = \sqrt{\frac{L_m}{L_p}} \qquad \text{(II.5)}$$

For Reynolds similarity; $\mathbb{R}_m \quad \mathbb{R}_p$:

$$\frac{\nu_mL_m}{\nu_m} = \frac{\nu_pL_p}{\nu_p} \qquad \text{(II.6)}$$

or

$$\frac{\nu_m}{\nu_p} = \frac{L_p}{L_m} \cdot \frac{\nu_m}{\nu_p} \qquad \text{(II.7)}$$

Combining (II.5) and (II.7) we get:

$$\frac{\nu_m}{\nu_p} = \left(\frac{L_m}{L_p}\right)^{3/2} \qquad \text{(II.8)}$$

It is evident that it is highly impractical to satisfy equation (II.8).

If we want to use the most economical model fluid, i.e., water, then $V_m = V_p$, and we have to use a 1:1 scale model to satisfy equation (II.8). Or we may find a different fluid, choose a scale to satisfy (II.8), and raise the cost of modelling one hundred fold.

The point is that in any model, some trade-offs are made and complete similarity is never achieved. This does not mean that models are not satisfactorily accurate. In practice, when the problem involves simulation of flow systems with a free surface (open channel flow), Froude and Euler similarity is sufficiently accurate. The effects of viscous, elastic, and surface tension forces are very small compared to that of inertia pressure and gravity forces. Use of this simplification was illustrated in the example on Section I.

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