THE FORMULATION OF MATHEMATICAL MODELS AND THE DYNAMIC TESTING OF STRUCTURAL MODELS AND FULL-SCALE STRUCTURES

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In the conventional earthquake (lateral analysis of a structure, the frame is modelled as one consisting of line elements lying in one plane. The earthquake "forces" tributary to this frame are assumed to be a set of static lateral loads specified by codes and are acting at the floor levels only. It has been accepted that this arrangement would simulate dynamic forces which the structure experiences if subjected to the effects of a medium-size earthquake. Engineers have long wondered whether this universal acceptance of this practice is indeed a reasonable one for economy design considerations, or whether it is safe. Oftentimes, various codes suggest quite a varied acceleration range, and depending on whether one uses the lower or upper bounds, safety and economy cannot always be achieved to the optimum. The need for experimental research has always been emphasized because dynamic parameters of different types of structures greatly affect the response.

With the development of modern computers, the tedious process of manual computation is mostly eliminated. It is now possible to predict with a fairly good accuracy the effects of any given earthquake motion on almost any type of structure, or system, for which the dynamic properties are well-defined or could be determined accurately. Taking advantage of the tremendous computational capabilities of an electronic computer, the techniques of structural analysis can be made easier. As a matter of fact, the principal difficulty really now stems from lack of information concerning the earthquake ground motion itself, and with the minor problem of determining or assuming the physical properties of the system. Some of these properties can only be evaluated experimentally, and thereby justification of needed research is indicated.

The first concern in dynamic testing is the idealization of the structure into a mathematical model. Needless to say, if an electronic computer is to be most effective, most models must be one which is discrete. In other words, the digital computer is most advantageous if there are steps which utilize the counting procedure.

In this way, a model having an infinite degrees of freedom may not be the model we want. On the other hand, the more the degrees of freedom, the more accurate the solution. There must be a compromise between accuracy and economy. The utmost prerequisite of the model must be simplicity without omitting the features of the prototype which will affect the response.

In principle, if basic properties are measured for all basic structural materials, it should be possible by methods of structural dynamics to derive the corresponding properties of elements assemblages or complete systems made up of these materials. In practice, however, it will be necessary to carry out an integrated test program in which materials, elements, assemblages and complete systems are tested individually.

The elements which form a typical framed structure are the beams (or girders) and the columns, and for such elements, the behavior mechanisms are related to their properties determined by simple uniaxial tests. It follows, that the first step should be devoted to studies of stress-strain relationships and energy absorption which must be determined dynamically. The succeeding step is to perform cyclic tests on beam and column elements. Forces developed are determined as functions of displacements, velocities, including a simulation of deformation histories that would be expected in an earthquake. The overall objectives are to get strength, stiffness, ductility and energy-absorption properties.

The next series of tests are likely those which will give information as to the dynamic behavior of typical joints and the general behavior of the assembled structure.

For any form of dynamic testing, the objectives are to determine the physical properties among which the natural periods of vibration, determination of the mode shapes, measures of energy dissipation, any information with regards to the degree of damping, and the limit of linearity of the structure. Dynamic tests are classified according to the following:

1. Free-vibration tests
   a. initial displacement
   b. initial velocity
2. Forced-vibration tests
   a. Sinusoidal forcing function, study of the steady-state resonance by means of
      (1) Rotating eccentric weight vibrators
      (2) Man excited
   b. variable frequency sinusoidal vibration
   c. transient tests

(1) natural earthquakes
(2) blasts and explosions
(3) wind

3. Use of the Vibration Table ("Earthquake" table).
The above tests may be applied to models (with certain exceptions) or to full-scale structures.

Free vibration tests are simpler to assemble and to perform but they do not give complete information needed for a better understanding of the complicated dynamic behavior. An initial displacement can easily be imposed on a model or on a structure by means of cables, jacks, etc. When the holding force is suddenly released, the model will vibrate freely about its equilibrium axis position. The natural periods can be determined by plotting the vibration curve versus time. In this set-up, amplitudes are expected to decay because of energy dissipation and because no structure nor model is entirely undamped, which is an argument for determining damping characteristics. One main difficulty is the confinement of vibration in one plane only—that is to say, upon the quick release of the load, torsional modes may couple the vibration.

To impart initial velocity, impact forces are used. These are usually done by falling weights and sometimes by means of a heavy pendulum capable of exerting horizontal blows. Still sometimes explosives or small rockets are used. The idea is that if impulsive forces act, the time duration during which these actions occur is very short compared to the natural periods which makes the response a function of the initial velocity rather than the force exerted. As a matter of fact, it must be recalled that this is essentially the same fact introduced in the derivation of Duhamels' Integral, that is, by the Principle of Impulse-Momentum, the initial velocity is expressed in terms of the impulse divided by the mass.

Forced vibration techniques are much more convenient and actually yield more complete and accurate information. The most popular forcing function is the sinusoidal variation and this is quite easily done by means of rotating eccentric-mass vibrators. The equipment consists of flat baskets rotating about a vertical axis. Counter-rotation by the loaded baskets causes unidirectional forces which are derived from the inertia of the unbalanced weights. The weights themselves may be adjusted and since they are rotating about a vertical axis, the effect of gravity is at once eliminated. The frequency of the forcing function can also be adjusted such that a typical combination may be an inertia force of 1000 lbs at 1 cps. By the use of synchronized units, the dynamics response of a full-sized dam may be studied.

The equation of motion of a viscously damped spring mass system excited by a harmonic force takes the form:

$$m\ddot{x} + c\dot{x} + Kx = F_o \sin \Omega t$$

where Ω is the frequency of the excitation. The solution of the equation may be broken up into two parts: the homogeneous solution representing the free vibration portion, and the particular solution defining the forced vibration part. The free vibration part soon disappears leaving only the forcing solution which is called the *steady-state* harmonic oscillation at the frequency of the exciting force.

The solution of the above equation is written:

$$X = e^{-\beta wt}(C_1 \cos w_d t + C_2 \sin w_d t$$

$$+ \frac{F_o}{K} \left[\frac{(1-r^2) \sin \Omega t - 2 r \cos \Omega t}{(1-r^2)^2 + (2\beta r)^2} \right]$$

where $r = \frac{\Omega}{w}$

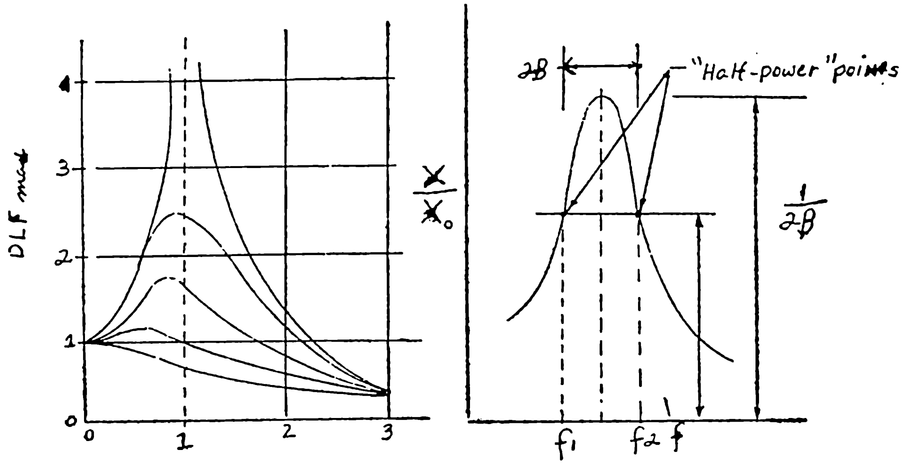
Since our interest is now confined to the steady state response only, the second term above may be re-written:

$$X = \frac{F_o}{K} \left[\frac{[(1-r^2)^2 + (2\beta r)^2]^{1/2} \sin (\Omega t + \Theta)}{(1-r^2)^2 + (2\beta r)^2} \right]$$

where Θ is the phase angle. It is apparent that the response is maximum when the sine is unity, thus:

$$X_{max} = \frac{F_o}{K} \frac{1}{\sqrt{(1-r^2)^2 + (2\beta r)^2}} = \frac{F_o}{K} \qquad DLF_{max}$$

DLF $\equiv$ the Dynamic Load Factor. The plot of DLF_{max} versus values of r is given in the preceeding sheet. It becomes clear, the even with small amount of damping, theoretically infinite amplitudes do occur at resonance. For the extreme case of critical damping, the maximum resonant deflection is only one-half of the static deflection from 0.5 to 10 cps, a resonant effect is attained. Actually the exciting frequency is slowly increased until the acceleration trace on a recording chart is large enough for measurement. Then a few more cycles are allowed to damp out the free vibration part, leaving only the steady state response. With regard to the mode shape, the number of points required to determine the plot depends upon the mode itself and the number of degrees of freedom. For example in a case of 20-storey building, only a few points, say 5, are necessary to plot the 1st mode, the general shape of which is known.



However, if the plot of the 5th mode is desired, maybe 10 points at least are needed, and if the 15th mode is needed, then all the floor points must be included. For economic reasons, an accelerogram at every floor of a building is not reasonable—preference must be to spread out the instruments to other typical structures in an area where a probable earthquake is likely to happen. It would be ironic if all instruments are concentrated on one building only to find out that the most damage occurred somewhere else. The best arrangement is to have one strong-motion accelerograph at the basement which will record about the ground acceleration itself, and one or two more at the upper stories and preferrably another one at the roof level.

In the kind of testing just described, the natural earthquake is utilized as the dynamic vibrator and the "model" the actual structure itself. At this instance, it is of utmost importance that *solid* preparations *before* and *sound* investigations *after* the earthquake be carried out in a very systematic and orderly manner. Interpretation is meaningful only if the input is accurately known. Unfortunately in the past, no ground motion measurements were taken of many large and major earthquakes.

Large earthquakes are rather rare and so explosives have been used to simulate transient ground motions. Large quarry blasts, for example, are known to generate ground accelerations in their immediate vicinity comparable to earthquakes.

In addition to accelerograms, other measurements which will measure structural strains and relative displacements are useful. In any event, simultaneous readings versus time are taken of each of these instruments. Electrical transducers have been extremely efficient especially if interconnected to switching channels and corresponding oscillographs. Strain meters and SR-4 electrical strain gages have been used although care must be exercised for the latter so that the correct type (for dynamic analyses) must be used.

Damping factors may be found from the resonance curves similar to that shown page 5. β is equal to $\Delta f/sf$ where Δf is the band width and f is the resonant frequency. Strictly, the above expression for is only applicable for a linear SDF system with small amount of viscous damping. However, this procedure of making use of the band width has been accepted only as a "reasonable" amount of damping. In the case of a full-size structure, the percentage of damping does not have to be very accurately known. It is sufficient if the *range* is specified.

A full scale test of a 14-storey residential apartment building was conducted. The frame is made of reinforced concrete with hollow block walls. The building which is 22.2 m x 22.2 m x 47.0 m high is shown if Fig. 2.4.1 and Fig. 2.4.2. Eccentric mass type vibration generators were bolted to the top floor. The frequency-response curves of the first, second and third modes are shown in Figs. 2.4.3, 2.4.4 and 2.4.5. The frequency response curves for the first two modes for three different loads are shown in Fig. 2.4.5 and Fig. 2.4.7. The vertical mode shapes are shown in Fig. 2.4.8.

An earth-filled dam 60.0 m high was also tested. The cross-section and downstreams elevation are shown in Fig. 2.5.1. and Fig. 2.5.2. Note the location of the vibration generators on the crest. Curves are shown in Figs. 2.5.3 thru 2.5.6 inclusive. The symmetrical modes were excited by operating the machines *in phase* and anti-symmetrical modes by operating the machines *180° out of phase*. The rest of the results are shown in Figs. 2.5.8 thru 2.5.10.

As conclusions, we have learned that the dynamic parameters of a structure can be determined experimentally. Although the structural behavior is completely defined in the elastic range, it must be borned in mind that during an earthquake elastoplastic deformations do occur. The reduction of the protoype into a mathematical model further modify the various parameters. If the study begins with a model, the investigation may point out design changes in the full-scale structure—which in turn is *not* the one represented by the model. Constant modifications are required if the study has to be successful. Further efforts must be made to simplify the model and to define suitable models for complex structures as well.