Mathematics and the Liberal Arts

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I would like to thank the College of Arts and Letters for this invitation to speak at this lecture series on General Education (GE) and converse with colleagues in other disciplines. I welcome this as an opportunity for us to step out of our specializations, communicate with each other, and explore the many ways our disciplines interact.

The University of the Philippines' (UP) faculty are usually caught up in the demands of teaching such as the endless checking of exams, research, committee work, and meeting all sorts of deadlines that we have little time to ask ourselves the fundamental questions: What is UP education? What type of graduate do we want to produce? This GE Lecture Series, as well as next week's two-day GE Conference, are occasions to revisit these questions and reexamine our mission, because our GE Program is at the core of UP education.

From Plato through medieval times, the core of higher education, as described in Plato's Republic, was the quadrivium, which consisted of arithmetic, geometry, music, and astronomy. These were taught after the trivium, comprised of grammar, rhetoric, and logic. All together, the seven made up the classical notion of the "liberal arts." Reincarnated in the American liberal arts, this notion emphasized the broadness of knowledge and the ideal "well-rounded student."

Our own liberal arts tradition, as embodied by our GE program, has gone a long way from its classical origins. It seeks to equip our students with the capacity to integrate different areas of knowledge and the critical faculties that will enable them to deal with complexity and change. By critical thinking, we refer to the habit of mind that enables one to analyze with rigor, read between lines and think out of the box, distinguish between substance and form/rhetoric, and appreciate and understand connections as well as differences between the many things we study, and how these impact on society.

Nationalist and secular in orientation, our GE program also seeks to impart values such as a sense of nationhood and concern for people and our environment. In his GE lecture[1] last month, National Artist Bienvenido Lumbera, spoke of the need for social conscience and social intelligence. In UP, we do not teach our students neutrality. They should look at all sides of an issue but must take a stand when the situation requires a stand. UP should offer choices but has to guide students as well and remind them of the ideals of excellence and of service to our people and our nation.

I first heard the term, "critical thinking," and about the liberal arts tradition of UP, in my freshman English class under my favorite teacher Prof. Dolores Feria. In my notebook, Prof. Feria wrote short notes, warning me of the narrow paths of specialization and encouraging me to explore ideas, whatever the consequences.

Her first note was a question: "What are you headed for in the College of Engineering?" I understood what she wanted to convey: not disapproval of the major I chose to write in my UPCAT application, but a reminder to examine and reexamine my choices and decisions.

I did, and so I shifted out of engineering to architecture, which I thought to be a nice course because it was a meeting place of the sciences and the humanities. But involvement in student activism eventually led me to consider the more abstract areas of study—philosophy and mathematics—which required no laboratories and drafting tables, giving

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[1] Bienvenido Lumbera. 2014. "Tatak UP: Social Conscience, Social Intelligence." CAL GE Lecture Series: Tatak UP.

me more time for the streets. Needless to say, I acquired my education from both inside and outside the classrooms of UP.

I entered UP during the politically repressive martial law years. My friends and I felt a deep hunger for books and ideas, especially those which were banned, and looked for every opportunity to read and discuss what we read—from the nationalist books of Renato Constantino and the humanist essays of the young Marx to the poetry of Yevgeny Yevtushenko and the novels of Thomas Hardy. Reading and learning was an intoxicating adventure and was thus liberating.

In the old AS Building and the Faculty Center during those years, there was constant interaction among faculty and students of different disciplines, who learned from each other in the spirit of thinking and learning beyond one's area of specialization. In this building [Faculty Center] one merely had to step out of his/her office to be able to mingle with colleagues from other disciplines. This sense of community was disrupted thirty-one years ago, when the former College of Arts and Sciences split into three colleges—the College of Arts and Letters (CAL), the College of Social Science and Philosophy (CSP), and the College of Science (CS).

Now I feel the physical separation more acutely because I hold office at the Institute of Mathematics, located at the fringes of the campus, where we have an excellent view not of UP Diliman's leafy acacia trees, but of the grotesquely over-towering SM Berkeley Condominium along Katipunan. I sometimes think that this physical separation reinforces the gap between the CS and the other colleges such as CAL—our own version of "Two Cultures," in reference to CP Snow's 1958 lecture in Cambridge where he lamented the fragmentation of learning in academe and drew attention to the widening wedge between scientists and humanists.

There are those who prefer the precision of numbers and rules over the ambiguity of the arts and humanities. And there are humanists who believe that in reducing everything to numbers, science and mathematics diminish the human experience and dull our sense of beauty and wonder.

The poet John Keats accused Isaac Newton of "unweaving a rainbow" and "conquering its mystery by rule and line":

> Do not all charms fly
> At the mere touch of cold philosophy?
> There was an awful rainbow once in heaven:
> We know her woof, her texture; she is given
> In the dull catalogue of common things.
> Philosophy will clip an Angel's wings,
> Conquer all mysteries by rule and line,
> Empty the haunted air, and gnomèd mine—
> Unweave a rainbow …

In his book Unweaving the Rainbow, the scientist Richard Dawkins responds to this unfortunate image of science. Science, he says, does not diminish his appreciation of the beauty of nature. Rather, it has enhanced his sense of wonder, pleasure, and beauty.

Physics and geometry explain that rainbows are created when sunlight is refracted through tiny droplets of water. The spherical shape of the raindrops splits the light into different colors, sending off each color at a different angle. We detect only those colors that meet our eyes at particular angles. For this reason, when two people look at a rainbow, they see two different versions of it. Their eyes are in different places, and so they detect different sets of colored rays of light. Thus the rainbow you see is always a uniquely personal experience.

Geometry gives us new eyes to see and appreciate the beauty of rainbows. In his book, Dawkins argues that "science does not destroy, but rather discovers poetry" in the patterns and laws of nature.[2]

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[2] Richard Dawkins. 2006. Unweaving the Rainbow: Science, Delusion and the Appetite for Wonder. Penguin Books.

The Russian-American writer Vladimir Nabokov is famous as the author of the great 20th century novel, Lolita. Unknown to many, Nabokov was also a biologist; he was as passionate about his science as he was about his stories and the written word. He was, in particular, a lepidopterist—a taxonomist specializing in butterflies, classifying them into species by the peculiar method of examining their genitalia—six hours a day, seven days a week.

Nabokov loved his work, and he writes on the pleasures of science:

> The tactile delights of precise delineation, the silent paradise of the camera lucida, and the precision of poetry in taxonomic description represent the artistic side of the thrill which accumulation of new knowledge, absolutely useless to the layman, gives its first begetter. . . . There is no science without fancy, and no art without facts.[3]

In a letter to his sister Elena, Nabokov describes the delights of science, amidst his drab laboratory setting:

> My laboratory occupies half of the fourth floor. Most of it is taken up by rows of cabinets, containing sliding cases of butterflies. I am custodian of these absolutely fabulous collections. We have butterflies from all over the world. . . . Along the windows extend tables holding my microscopes, test tubes, acids, papers, pins, etc. I have an assistant, whose main task is spreading specimens sent by collectors. I work on my personal research … study of the classification of American "blues" based on the structure of their genitalia (minuscule sculpturesque hooks, teeth, spurs, etc., visible only under a microscope), which I sketch in with the aid of various marvelous devices, variants of the magic lantern. . . .

[3] Vladimir Nabokov. 1990. Strong Opinions. New York: Vintage International, 79.

> My work enraptures but utterly exhausts me. . . . To know that no one before you has seen an organ you are examining, to trace relationships that have occurred to no one before, to immerse yourself in the wondrous crystalline world of the microscope, where silence reigns, circumscribed by its own horizon, a blindingly white arena—all this is so enticing that I cannot describe it.[4]

Stories like these help dispel the misconception that science and the arts are at odds, as well as the false image of the scientist as an eccentric and dull personality who pursues truth and "conquers mysteries by rules and numbers." One can argue that Nabokov drew both literary and scientific inspiration from the same well. "Literature and science," the poet Gemino Abad says, "have in common one great human faculty or power of the mind—the imagination. All great scholars, scientists and artists are men and women of vibrant imagination."[5]

Nabokov's otherwise modest output of scientific papers had great literary value because he was Nabokov, master of both the Russian and English languages. Other scientists rely on another language, which wields a different type of power—mathematics.

A fundamental misconception, reproduced in both homes and schools, is that mathematics is the dull study of numbers and formulas, a useless obstacle course in university. Dreading a repeat of the drills of algebra, many students choose courses in college with the least amount of mathematics. Because of the public fear of mathematics, a magazine for kids once described the game of Sudoku as "numbers without math."

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[4] Vladimir Nabokov. 2000. Letter to his sister Elena Sikorski (1945). In Nabokov's Butterflies: Unpublished and Uncollected Writings, ed. and annot., Brian Boyd and Robert Michael Pyle. Beacon Press, 387.

[5] Gémino Abad. 2007. "Literature, Science, and the Imagination." The Philippine Star (Star Science).

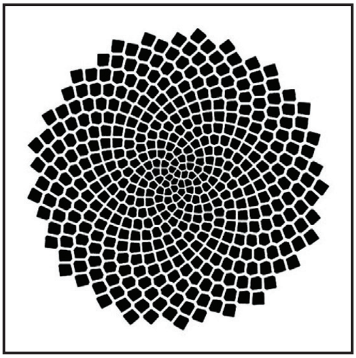
However, if one knows how the game is played, he/she realizes that the 'numbers' in the game are not the numbers we know, for they do not represent quantity. Each 'number' is an empty and meaningless symbol, and can therefore be replaced by the corresponding letter in the alphabet, or any symbol for that matter. To play the game, we use not numbers but logic and our ability to see pattern. Sudoku is math without numbers.

Just as musical notes are symbols that encode musical patterns and ideas, the symbols of mathematics are representations of mathematical ideas. Like ordinary language, mathematics has its own alphabet of symbols, governed by a special grammar and the rules of logic. Like language, mathematics allows one to represent and communicate ideas and shared meanings. It has been described as the language for the study of patterns about quantity, space, shape and symmetry, and structure and order. There are even new mathematical ideas (like fractals and chaos theory) that explain how order and disorder arise from one another. In his poem "Connoisseur of Chaos," Wallace Stevens describes this unity: "A violent order is disorder, and a great disorder is an order. These two things are one."

Mathematics is about ideas, and not formulas. It is a way of looking at the world, a means of helping us understand and make sense of it. Mathematics is abstract, but because of its precision, it is the language of science, helping us model and understand the world, providing the ideas that power technology.

Let me provide an example from everyday technology. Aoccdrnig to rscheearch, you can raed a taotl mses wouthit a porbelm. This is bcuseae the human mnid deos not raed ervey lteter by istlef, but the word as a wlohe. We are able to read and make sense of corrupted sentences such as this one because the human mind has "error-correction" capability. Just like the word-processing programs in our laptops, we are able to detect misspelled words and correct them.

Many of us are unaware that the gadgets we use every day such as our mobile phones, CD players, and digital cameras, rely on mathematics to ensure the integrity of data and the clarity of sound or images. Errors (or 'noise', in engineering parlance) occur whenever data is transmitted, but our gadgets are able to detect these errors and correct them. The design of 'error-correction codes' requires algebra, matrices, probability theory and, as in the case of sophisticated codes, geometry.

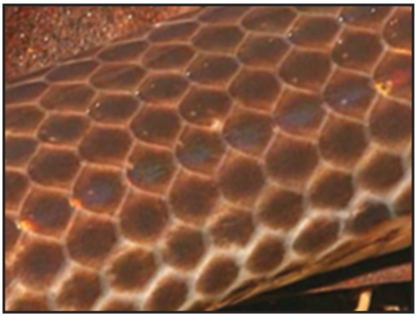


Spiral patterns in sunflower.
Source: momath.org

Nature is full of mathematical patterns which not only please the eye but also provide scientists clues to understanding the laws that govern it. A beloved example used in the classroom is the sunflower. The seeds on the head of the sunflower are arranged in two sets of spiral rows—one that swirls clockwise and another that goes in the opposite direction. One may even notice a third set of rows with smaller slant. The numbers of rows in the three sets are 21, 34, and 55. These belong to the sequence of numbers called the Fibonacci sequence (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,…) where each number in sequence is the sum of the two previous numbers. Such number patterns are ubiquitous in nature.

Another pattern commonly found in nature is the hexagon, or the polygon with 6 sides. Bubbles, when clumped together, usually transform from spheres into hexagons. The scales on snake skin are hexagons. Bees optimize the design of their beehives by creating hexagonal cells. With their choice of the six-sided shape, they maximize storage space with the minimal amount of material.

The process of putting together shapes without gaps or overlaps—as in a jigsaw puzzle—is called a 'tiling' or tessellation. While hexagons are

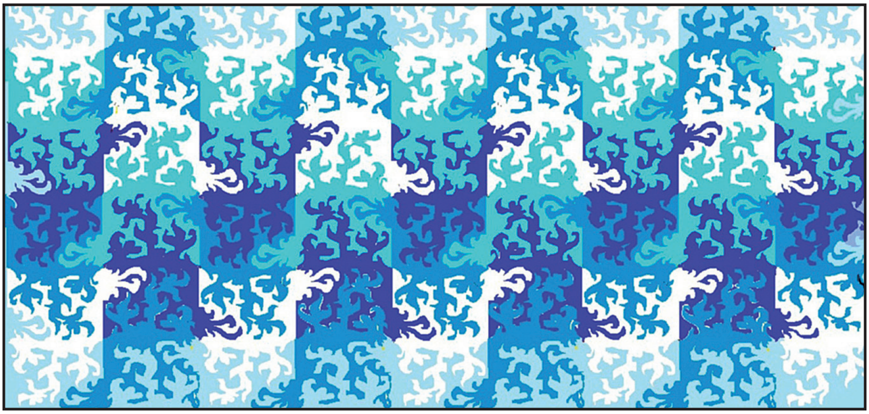


Hexagonal scales on snake skin.
Source: www.wildsingapore.per.sg/discovery/factsheet/snakesunbeam.htm

Hexagonal structure of beehive

nature's favorite tiles, there are many other tilings in art. Wonderful examples of these are found in the works of the Dutch mathematician-artist M.C. Escher, the walls of the Alhambra Palace in Spain, the intricate banig designs by the weavers of Samar and Mindanao, or the nice exercises in tessellation created by my Math 1 students.

The ideas in the study of these patterns help form our understanding of chemistry, physics, and geology, as well as the complicated symmetries of Rubik's cube.



Tessellation by Math 1 student

While mathematics is the language of science and a tool for understanding and modeling natural and social phenomena, it can also be described as an abstract axiomatic system, governed by internal rules and developed by pure logic. It only has to comply with its own requirements of logic and consistency, like the game of chess. Just as every configuration on the chessboard is the result of a sequence of legal moves, a mathematical statement can be obtained from the axioms of the system via a finite chain of statements, each one a logical consequence of previous statements.

The abstract nature of mathematics was sealed by the work of 18th and 19th century mathematicians on foundations. Freed from the world and the constraints of empirical reality, pure mathematics developed along with other cultural movements which produced abstraction in art such as cubism and impressionism, Bauhaus architecture, and atonal music. The history of cultures and ideas is linked with the development of mathematics and science.

A significant outcome of this freedom is non-Euclidean geometry, which describes the surreal world of warped space. Viewed at first by many as an exercise in pure thought, this strange geometry found surprising applications many decades later. Albert Einstein used it to describe the fabric of space. Brain scientists found out that human vision is mapped in non-Euclidean geometry. Advances in computer technology combined with knowledge of how visual images are produced inside our brains have led to the development of 3D movies. A 3D movie is actually a clever deception on the brain. Animators use algorithms based on complex ideas of geometry, partial differential equations, and matrices that trick our minds into seeing an extra dimension in images projected onto a two-dimensional screen.

I have been describing the power of mathematics through its applications. We are surrounded by the creations of mathematics: the cars that we ride, the phones that enable us to communicate through invisible channels, the cameras that we use to take selfies, and the internet which connects us all. But mathematics is also used for destruction and other nefarious purposes: it powers the technology that directs smart bombs into homes and school buildings with deadly precision.

Science and the language of mathematics have helped us understand ourselves and our world and create new technologies, but this power also compels us to continuously examine the consequences of our decisions and actions. Our GE program should also impart in us the ability to discern between good and bad; our discernment guided by social conscience and a sense of responsibility to the environment, to peoples and communities, and to the public good. In this age of increasing specialization and complexity, our GE program takes on a more important role as it provides the unifying thread that helps us see the bigger picture. Furthermore, the GE program helps provide us with the necessary cultural and ethical moorings in this age of globalization and the internet.

Let me return to my main points. The gap among science, mathematics, and the arts comes from a failure of imagination and our ability to build bridges and communicate beyond the borders of our disciplines.

Feeding on a fear of mathematics, there are humanists who feel that math is merely a tool for technology, a collection of formulas and symbols with no connection to the great themes of our culture. The genuine liberal arts tradition argues the opposite: that math has been and is linked in fundamental ways to the development of culture and our ways of thought.

On the other hand, there are teachers of mathematics who reproduce this fear, by teaching mathematics as nothing but an endless series of drills in arithmetic, algebra, and calculus. Our GE program, guided by the spirit of the liberal arts, is an opportunity to bridge the different disciplines and teach math and science not as an academic obstacle course, but as an adventure in ideas that is exciting and relevant to understanding the world.

The GE program should be able to make students understand that the numbers and formulas of the classroom are merely the scaffolding for more powerful ideas, in the same way that a student needs to master language and grammar in order to appreciate literature. It is not the students' mastery of mathematics, but an appreciation of it as a language, a tool, and way of seeing, that should be among the goals of GE. An educated person need not have a grasp of equations and formulas, but should understand the role of mathematics and science in shaping our world.

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