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Characterizing the Application of Mathematical Thinking in Citizen Science

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ABSTRACT

Mathematical thinking is important for citizens to develop not only for academic purposes but also more importantly for participating responsibly in society. This paper describes the features of the application of mathematical thinking processes in citizen science. Mathematical thinking involves processes such as problem-solving, reasoning and proof, communication, connections, and representation. Citizen science, on the other hand, is an emerging research activity in which the public can participate. In contrast to conventional science, individuals in citizen science participate in research activities rather than as subjects of study. Purposive sampling and a grounded theory approach were used in this investigation. In this study, 15 citizen science projects were considered as the sample. The findings suggest that problem-solving and connection are the most commonly practiced mathematical thinking processes in citizen science. Furthermore, the study reveals that individuals involved in citizen science demonstrate problem-solving, reasoning and proof, communication, and representation as primary tasks in the various research activities. Connection, on the other hand, is mostly used as a subprocess. These findings suggest that persons engaged in citizen science can execute all mathematical thinking processes. However, among these processes, problem-solving is the most widely implemented and hence most developed through citizen science.

Keywords: ***Mathematical thinking, citizen science, grounded theory, problem-solving***

Introduction

Mathematical thinking is not just important and applicable in an academic setting. As argued by Goos et al. (2020), mathematical thinking centers on mathematical processes rather than content. For this reason, mathematical thinking can also be useful and practiced in various real-life experiences, fields, and circumstances. Hence, the development of mathematical thinking is essential not just for individuals who work in the fields of mathematics. In fact, it is now vital for every citizen to have well-developed mathematical thinking.

In this research, mathematical thinking is defined as the ability of an individual to practice standard mathematical processes and apply them to different activities and/or fields. The National Council of Teachers of Mathematics (NCTM) (2000) published process standards for school mathematics which characterized how learners or individuals learned and applied mathematical content knowledge. These process standards are the following: *problem-solving*, *reasoning and proof*, *communication*, *mathematical connections, and representations*. These five process standards were used as the framework of mathematical thinking in this study.

The need to open mathematical thinking to the public can be solved by the opportunities offered by an emerging scientific activity called citizen science. Citizen science is differentiated from traditional science by its two distinct characteristics: (1) its openness for the public to participate; and (2) its openness to sharing the intermediate results with the public (Koch & Stisen, 2017; National Geographic, 2012). However, only a few studies investigated the potential of utilizing citizen science in developing the mathematical thinking of individuals. As a consequence, this paper studied the application of mathematical thinking in citizen science projects, specifically the characteristics of the application in different citizen science projects. This article is part of a wider study, which investigated the convergence of mathematical thinking and citizen science including the opportunities and challenges that emerged from it. This wider study also explored the differences between citizen science projects from other countries and those from the Philippines.

Mathematical Thinking

Regardless of the wide range of experts who recognize the importance and role of mathematical thinking, there is no one unified definition of the concept found in the literature (Drijvers et al., 2019). Notwithstanding this, some researchers found two descriptions that could summarize the most frequent definitions of mathematical thinking. Watson (2001) and Isoda (2012) identified two popular descriptions of mathematical thinking: (1) centers on problem-solving heuristics and mathematical processes, and (2) focuses on conceptual development. It is worth noting, however, that the descriptions presented are not exclusive of each other.

Some of the experts who define mathematical thinking using the first description are Burton (1984), Stacey (2006), and Bal et al. (2020). Burton (1984) defines mathematical thinking as “operations, processes, and dynamics of mathematical thinking” (p. 36). Stacey (2006), on the other hand, stated that mathematical thinking was centered on processes. Likewise, Bal et al. (2020) said that mathematical thinking was associated with mathematical processes. Among the various mathematical thinking frameworks, the process standards of NCTM encompassed the different mathematical processes from other frameworks. The process standards of NCTM comprehensively listed the processes that were part of mathematical thinking.

The first mathematical process from NCTM's process standards is problem-solving. According to NCTM (2000), problem-solving is considered both a goal and a method of learning mathematics. This process allows the integration and application of various mathematical concepts and ideas (Tarim & Öktem, 2014). The next process standard is reasoning and proof. In the application of this process, NCTM (2000) said that certain structures and principles in a mathematical context and daily life were identified. Individuals execute this process as they compare and contrast, generalize and conjecture (Vale et al., 2017), and justify and explain (Mata-Pereira & Ponte, 2017). The third process standard is communication. In this process, learners share their ideas and prove their understanding (NCTM, 2000). Connection is the fourth process. This is crucial as the different strands and standards of mathematics are integrated and connected, and not separated from each other (NCTM, 2000). According to Eli et al. (2013), connections are the various links among mathematical ideas and concepts. Finally, representation completes the five process standards. Bal (2014) and Cai (2005) characterized this process as the formation of mathematical concepts and their relationships through visualization employing pictures, physical objects, tables, graphs, and charts.

Citizen Science

Citizen science, despite being considered an emerging way of conducting research, is not something new. Levasseu (2021) argued that the public already had the opportunity to help the scientists in research, specifically in collecting data. She explained that citizens gathered data during the locust outbreaks in ancient China. Ferran-Ferrer (2015) also mentioned that certain individuals who were interested in biodiversity participated in scientific research related to this during the 19th century. Moreover, Silvertown (2009) explained that, for a century now, citizens had been participating in collecting data from nature in different fields such as archaeology, astronomy, and natural history.

However, the term "citizen science" was only coined in the 1990s (Vohland et al., 2021). According to Vohland et al., the Oxford English Dictionary (OED) reported that the term "citizen science" was first recorded in an issue of MIT Technology Review in January 1989. The first official use of the term "citizen science" referred to the production of scientific knowledge, bringing in the general public to participate, and addressing societal matters.

One of the main features of citizen science that defines and separates it from traditional science is the participation of the public. According to National Geographic (2012), the general public in citizen science collaborates with experts or scientists. Further, Wehn et al. (2020) explained that the participation of the general public in citizen science entailed being involved in different processes and parts of scientific research. In this scientific activity, citizens are no longer subject to scientific study. Instead, they are now collaborators who also perform the data gathering and analysis (Koch & Stisen, 2017).

The last feature of citizen science that differentiates it from traditional science is its "openness in participation and disclosure of intermediate results" (Koch & Stisen, 2017, p. 1). According to Franzoni and Sauermann (2013), in citizen science projects, the "images, videos, audio tracks, and arrays and intermediate solutions or data produced by project participants" (p. 9) are shared and disclosed.

Clustering of Citizen Science

Wiggins and Crowston (2011) organized current citizen science projects into five clusters, namely, action, conservation, investigation, virtual, and education. The first cluster, the Action Cluster, is characterized as a group of projects that addresses local community problems and promotes civic agenda. The Conservation Cluster, on the other hand, is defined as a cluster of projects focused on natural resource management. Next is the Investigation Cluster which, according to Wiggins and Crowston, is the collection of data in any physical environment to explain and predict different phenomena. Comparable to the Investigation Cluster is the Virtual cluster. However, instead of using a physical environment, Virtual Cluster requires the investigation of phenomena using ICT tools and relies on algorithms and a large number of evaluations. Finally, the Education Cluster aims to educate citizens and connect with other academic institutions for the engagement of teachers and students.

The majority of research, such as those featured in this study, exclusively considers mathematical thinking and citizen science as separate entities. There is a little literature on the application of mathematical thinking to citizen science, specifically studies that discuss the nature and breadth of mathematical thinking and citizen science convergence. Thus, this paper aims to describe the features of the application of the five mathematical thinking processes in the participation of individuals in citizen science.

Methods

Research Design

This paper utilized a qualitative method, specifically the grounded theory method. This method is the most appropriate for studies that still have little knowledge generated (Hutchinson, 1986). In this study, the specific grounded theory approach used was the constructivist approach. Charmaz (2006) determined the features of this approach. Some of these characteristics are adopted in the methodology of this study, specifically: (1) Sampling's goal is to construct theory; (2) data collection and analysis are executed concurrently; (3) comparison analysis is employed at all stages of analysis; and (4) additional sampling is employed to further develop ideas.

Sampling

A purposive sampling method was employed in this research. This sampling method was chosen because of the principle that sampling in grounded theory focused on the development of a theory and not on the statistical representation of the population. Thus, in selecting the sample for this study, purposive sampling assisted to make sure that the characteristics of the sample were able to generate appropriate and applicable data for theory construction.

The sample in this study were citizen science projects that (1) provided opportunities for citizen scientists, the general public, to participate not as subjects of the study but as contributors in different parts of the research (e.g., in data collection and analysis), (2) had available documents or information that determined the participation of the citizen scientists (e.g., field guides, participation manuals, and tutorials), and (3) shared the intermediate data or result of the project as open access.

In selecting the sample for this study, the research added a criterion to the characteristics mentioned above. The criterion required a citizen science project to have an application of two of the five mathematical process standards in the participation of the citizen scientists. There are a total of fifteen citizen science projects chosen as part of the sample of this study.

Data Collection

After selecting the fifteen citizen science projects as the sample of this research, data collection was employed. On the website of each of the citizen science projects, documents, such as volunteer guides, manuals, videos, tutorials, educator's resources, and other information were downloaded and/or extracted. Additionally, interviews with the project leaders and citizen science coordinators were conducted. A total of seven project leaders or citizen science coordinators were interviewed. All interviewees signed the consent form which indicated the recording of the interview and the process of handling and analyzing the data collected from the interview.

Since this paper employed a grounded theory approach, data collection, and analysis were concurrently done. Adopting Charmaz (2001)'s approach, additional data were gathered. After initial coding and focused coding, additional sampling was executed by going back to the websites of the citizen science projects or the original documents collected.

Data Analysis

This research employed three levels of coding for the first phase of analysis based on Charmaz (2006). The first level of coding was called qualitative coding. Charmaz explained that this level of coding aimed to "separate, sort and synthesize the data"

(Charmaz, 2006 p. 3). Specifically, this coding was executed by sorting the texts from the documents according to what mathematical thinking processes were applied. The sorting was based on the open-ended questions in one of the instruments, the Mathematical Thinking Process Checklist. This instrument was used in capturing the absence or presence and the scope of mathematical thinking processes in the sample citizen science projects.

The next level of coding was initial coding, which entailed the construction of codes. The codes at this level were in gerund form and were executed by answering the questions included in the Mathematical Thinking Process Checklist. For example, one of the parts in the tutorial of CrowdWater states, "Take a picture of the site and mark the location your observations refers to using the arrow and/or the circle. This way, others can see where exactly you did the observation." To answer the question, "How are mathematical concepts, ideas, or relationships represented?", the initial codes assigned to the text were "Taking an image and marking it using arrow or circle to represent the exact location of observation" and "Taking a picture of the site and mark the location your observations refers to using the arrow and/or the circle. This way, others can see where exactly you did the observation."

The last level of the first part of the coding was focused coding. At this level of coding, the codes summarized and described the chunk of the parts of the data (Charmaz, 2006). Moreover, various comparisons were conducted, data to other data, data to codes, and focused codes to data.

After the three levels of coding, the data and the codes were interpreted and analyzed. This part of the grounded theory method is called "memo-writing" (Charmaz, 2006). Comparisons among the codes and data were employed. Codes were analyzed within each cluster and among the five different clusters. Memos (see Table 1) were written in both paragraph form and bullet form. These memos were initial interpretations, written in an informal format, and were drafts of the initial interpretations.

Because of the concurrency of data collection and data analysis in the grounded theory approach, theoretical sampling was also employed. Theoretical sampling was conducted to fill the gaps and questions that arose from the memo writing. Additional texts were extracted from documents or the websites of the citizen science projects and from the interview responses. Texts, tables, graphs, and images attached to the texts were also collected.

The memo-writing and the theoretical sampling led to the last phase of coding in the data analysis, which is the theoretical coding. Theoretical codes were developed from the pattern that emerged in the focused codes and the memos. In the theoretical codes, each mathematical thinking process was assigned a two-letter code as follows: PS or ps for Problem-solving; RP or rp for Reasoning and Proof; CM or cm for Communication; RE or re for Representation; and CO or co for Connection. The categories developed determined if the two-letter code was written in uppercase or lowercase. Uppercase letters were assigned to a mathematical thinking process that was applied as the primary task performed by the citizen scientists in this phase of their participation. Lowercase letters, meanwhile, were used for a mathematical process that was only a step or a procedure done to apply the primary task. Most of the phases of the citizen scientists' participation required them to apply more than one mathematical thinking process. Thus, two-letter codes of the combined mathematical thinking processes were also combined using hyphen/s. After the theoretical coding, all the data were compared and analyzed again.

Table 1

Excerpt from an Example Memo

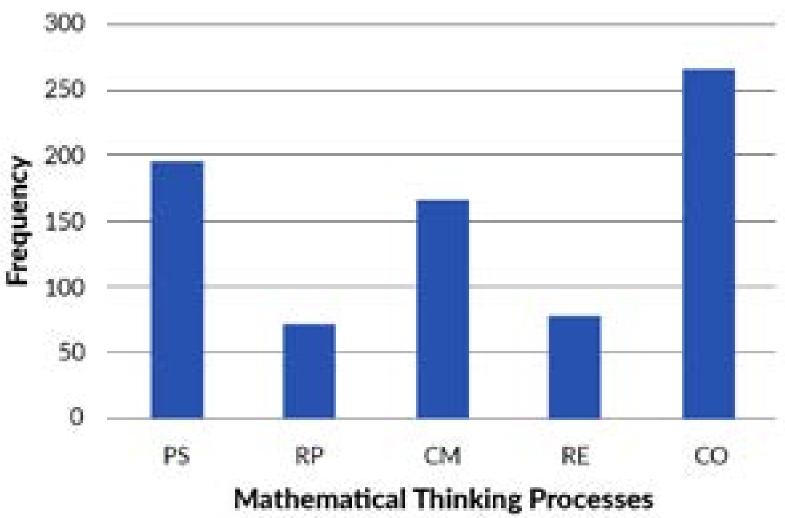
Title of Memo	Excerpt from the Memo
The Four Phases of Participations that Applies Problem-Solving and Connection [Conservation cluster]	There are four phases of participation where citizen scientists are applying mathematical problem-solving while also applying connection. These four phases are the following: • Sampling • Collecting data and measurements using tools, equipment, and/or instruments • Calculating measurements • Interpreting data While calculating measurements, one of the main tasks the participants do is to solve sets of operations which can also include averaging, solving for rate of change, and solving for proportion to the standard criteria. These processes provide opportunities for the participants to connect mathematical concepts like average, ratio and proportion, order of operations, logarithm, radius and volume to real-life contexts. The mathematical content strands these concepts and the other concepts used in this phase of participation belong are number and number sense, measurement, patterns and algebra and geometry.

Results

The findings of this study showed that the five mathematical thinking processes, namely, Problem-Solving, Reasoning and Proof, Communication, Representation, and Connection, were all executed by citizen scientists as they performed different research activities. Of the tasks assigned to the citizen scientists, Connection was the most frequent process that they were required to perform. Ninety-two percent (92%) of their tasks expected them to link mathematical concepts either to real-life contexts, to concepts from another field, or to another mathematical concept. Secondly, 68.06% of the participation of citizen scientists provided an opportunity for them to apply Problem-Solving. Then, this was followed by Communication which was executed in 57.64% of their total tasks. Finally, citizen scientists used Representation and Reasoning and Proof to a relatively similar amount of activities. Representation was applied in 27.08% of all the tasks while 24.65% of all the tasks practice Reasoning and Proof. Figure 1 shows a summary of the number of times each mathematical thinking process has been completed in the tasks provided to citizen scientists.

Figure 1

Frequency of the Mathematical Thinking Processes that Emerged in the Tasks of Citizen Scientists



Legends:
PS= Problem-Solving
RP= Reasoning and Proof
CM= Communication
RE= Representation
CO= Connection

Connection

Research activities performed by citizen scientists are always contextualized in real life. Hence, the majority of these tasks require the application of a mathematical idea or concept. As a result, the bulk of the activities performed by citizen scientists required individuals to use Connection as a mathematical thinking process. They practiced linking their knowledge of mathematical concepts to the real-life situations of the tasks citizen science projects demanded them to perform. Some of the mathematical concepts that were utilized by citizen scientists were function, number sense, ratio and proportion, scale, area, percent, angles, measurements, average, the four basic operations such as addition, subtraction, multiplication, and division, and some geometric figures namely points and lines. Table 2 shows examples of some tasks citizen scientists in which they apply Connection and the focused code for each task.

Notably, Connection was not just applied in citizen science as citizen scientists linked mathematical concepts to real-life situations. It was also executed as they related a mathematical concept to another mathematical concept. Some of the concepts that were linked together were variation to the graphs of a function, ratio, and proportion

Table 2

Examples of Tasks of Citizen Scientists Applying Connection

Text	Focused codes	Mathematical concept
Zooplankton data are reported as µg L-1 which is calculated as follows: Volume of water filtered: V1 (liters) = (π x r^2 x h) x 0.001 Where: π = 3.14 r = radius of plankton net (cms) h = height of plankton tow (cms) Volume conversion factor: Con.V = V2 / V1 Where: V2 = volume of ZAPPR (60 mls) V1 = volume of plankton tow (529.87 mls) *(U.S. Environmental Protection Agency, 2021, p. 61)*	Calculating a certain measurement of real-life objects, that is the final biomass, by performing a set of operations and applying the concept of radius and volume.	Radius and Volume
7. Give the frame time (eg 20:18 or 2018) and x and y values for each position for at least two observations in LASCO C2, and three ob servations in LASCO C3. Click the “Add Another Item” button to add more frames. Please note that a comet must ultimately be visible in at least 6 or 7 consecutive images for us to be able to confirm it! *(The Sungrazer Project, n.d., Introduction Guide section)*	Reporting the positions of real-life objects, those are positions of comet, using the concept of x and y coordinates of a point of a rectangular coordinate system.	Rectangular Coordinate System and Coordinates of a point

to the concept of conversion of measurement, fraction, and percentage to quartile and interquartile range, and law of exponents when multiplying or dividing scientific notations. Moreover, the result of the theoretical coding of this study revealed that Connection was dominantly executed as a Subprocess, suggesting it was used as the only technique for another mathematical thinking process.

Problem–Solving

The second most frequent mathematical thinking process practiced in citizen science was Problem-Solving. Citizen scientists engaged with a wide variety of tasks in relation to this mathematical thinking process. Some of the key action words related to Problem-Solving that were observed in this study were calculating, estimating, investigating, sampling, interpreting, measuring, identifying, averaging and brainstorming.

Furthermore, citizen scientists participated in several types of research activities that allowed them to create, evaluate, and execute strategies in solving problems. Initially, they had the opportunity to develop and identify specific strategies to solve the problems of the citizen science project. For example, in Chesapeake Monitoring Cooperative, the citizen scientists could propose their study design by filling out a template in which they were to determine the research question/s and the monitoring methods to answer the questions and accomplish their objective. The tasks in other citizen science projects where they employed problem-solving strategies were defining methods to clean data, designing equipment to be used in data collection, and brainstorming to develop a plan for communicating ideas and solutions.

Not as extensive as creating strategies, citizen scientists had fewer opportunities to evaluate problem-solving strategies. Some of the few examples for this were evaluating the draft of the communication strategy to be used for sharing the project's results and information and assessing the auditing protocol of the citizen science project.

Fundamentally, among the subcomponents of Problem-Solving, executing strategies was the most common application. There was a wide range of tasks the citizen scientists performed. Take for example the following type of activities, selecting sample or sampling area, installing instruments, calibrating instruments, averaging values, estimating measurements, calculating values or measurements, measuring or collecting data from the physical environment, investigating the site of study and objects of the study while applying mathematical concept, evaluating data, and interpreting data. Table 3 contains examples of activities completed by citizen scientists using Problem-Solving and the encoded task's focused code.

Most crucially, problem-solving was used not only in many types of citizen science research activities but also as a primary task for citizen scientists. In addition to being the second most common mathematical thinking process, much of the participation of the citizen scientists directly required them to employ Problem-Solving rather than as a subprocess of another mathematical thinking process.

Communication

Problem-Solving was followed by Communication as the third most commonly applied mathematical thinking process in citizen science. This process was often carried out by citizen scientists that participated in data-gathering activities. Some of the action words that emerged as keywords in Communication were filling out, representing, reporting, sketching, drawing, uploading, explaining, recording, submitting, and storytelling.

Table 3

Examples of Tasks of Citizen Scientists Applying Problem-Solving

Text	Focused codes
4. Gross Area: Perimeter of entire area of invasive species present may contain significant parcels of land not occupied by invasive species. You can enter Gross Area for plants, insects, diseases, and wildlife. You can either draw the gross area by clicking on the little blue marker next to "Gross Area" and drawing a polygon of the area. This will automatically add the sq. feet or acres to the form. You can also manually enter the estimated area and choose the unit area from the drop-down menu as acres, hectares, square feet, or square meters *(Rawlins et al., 2018, p. 28)*	Estimating and communicating the area of extent of the object of study with proper unit of measurement
Analyze data Compile and analyze results. Data analysis tools are freely available. Some project websites offer specific ways to compare your results with other citizen science data. Mapping the project location helps place the results in context. The How to Participate section of project listings indicates which data and mapping tools are available. *(EcoSpark 2018, p. 13)*	Compiling and analyzing data using free data analysis tools.

citizen scientists that participated in data-gathering activities. Some of the action words that emerged as keywords in Communication were filling out, representing, reporting, sketching, drawing, uploading, explaining, recording, submitting, and storytelling.

Critically, although this mathematical thinking process was observed in more than 50% of the tasks in citizen science, it was scarcely used extensively. In most of the cases, Communication was only practiced in filling out data sheet templates or recording data gathered. To illustrate the practice of Communication, Table 4 shows examples of activities of citizen scientists that employ Communication and the focused code for each activity.

In addition to the aforementioned result, it was worth noting that Communication was applied in citizen science as a primary task but was commonly integrated with other mathematical thinking processes. Take for example the second task included in Table 4. The citizen scientists participating in Community Collaborative Rain, Hail, and Snow Network were required to perform both Problem-Solving and Communication in this specific activity. Similarly, in the third example, citizen scientists that were involved in The Sungrazer Project integrated Reasoning and Proof with Communication as they justified their observation.

Table 4

Examples of Tasks of Citizen Scientists Applying Communication

Text	Focused codes
Measuring Total Snow ! on the Ground! • Snow is rarely uniform in coverage, so take several measurements and average them to obtain your total depth of snow. • Slide snow ruler through all layers of snow (new and old). • Read value on snow ruler and record (values are to the nearest ! *(Colorado Climate Center, n.d., p. 45)*	Measuring, calculating and communicating the measurement while applying mathematical concepts
STEP 5: Measure the comet positions To submit a comet report, we need to know the dates and times of the images you're looking at, and the x and y pixel positions of the object you are reporting. This is where the photo editing software is particularly useful. You'll recall that in "STEP 3" (above) we opened the images as "Layers". We now need to view those Layers and selectively turn them on/off - that is, toggle their visibility - so we can measure the comet positions. *(The Sungrazer Project, n.d., Cosmet Measuring Tutorial section)*	Justifying the observation by Identifying and communicating the coordinates of the positions of the object

Reasoning and Proof

There were a fairly adequate number of types of tasks that applied Reasoning and Proof in the participation of citizen scientists. Some of the action words that emerged in the execution of Reasoning and Proof were comparing, generalizing, justifying, explaining, analyzing, evaluating, conjecturing and interpreting. Among these action words, comparing was the most prevalent form of activity in Reasoning and Proof. More opportunities for data comparison were being provided to citizen scientists. They either compared their obtained data with their other data sets or their data to data collected by other citizen scientists participating in the same project. Table 5 shows examples of tasks where citizen scientists apply Reasoning and Proof and the focused code for each task. Importantly, Reasoning and Proof was a mathematical thinking process that was generally executed both as a primary task and as a subprocess.

Table 5

Examples of Tasks of Citizen Scientists Applying Reasoning and Proof

Text	Focused codes
Work in Small Groups Classrooms are busy places. Having 24-31 students makes garden observation difficult. It is best to divide the class into small groups of 6-12 people. They would then work in pairs to record data on three to six plants. When one group is finished taking observations, another group could follow. Having several groups collect data on the same plants allows the class to compare data collected and can help validate the accuracy of the observations. They will likely not be exactly the same, but a trend will be obvious and that's what is most important. *(CleanAIRE NC, 2019, p. 9)*	Comparing data collected by different groups of participants, then later on generalize the pattern/s on the data collected to help in validating the accuracy of observations
The Night You Hatched Behavior Overtime How has the sea turtle population changed overtime? As the cities grow… The number of lights increase, As the light pollution increases… The turtle population decreases As the… What other changes overtime occur because of the LP? *(National Optical Astronomy Observatory, n.d., p. 8)*	Investigating, generalizing and communicating the pattern in the change of measurement overtime, using the concept of mathematical concepts

Representation

Representation is the least observed of the five mathematical thinking processes. The majority of the tasks in which they used Representation were connected to capturing pictures, sketching, drawing, and graphing. Table 6 contains examples of activities of citizen scientists that employ Representation and the focused code for each activity. Furthermore, it was notable that Representation was usually applied with another mathematical thinking process. In addition to that, there were also observations in which it was applied as a subprocess of other mathematical thinking processes.

Table 6

Examples of Tasks of Citizen Scientists Applying Representation

Text	Focused codes
Take a picture of the site and mark the location your observations refers to using the arrow and/or the circle. This way, others can see where exactly you did the observation. *(CrowdWater, n.d, Temporary stream: New Spot section)*	Communicating and representing the exact location of the site of the study by drawing a geometric figure
Site drawings A hand-drawn map illustrates major stream and landscape features, entry to the site, landmarks and the sampling area. This map is used as a guide for site revisits, indicating how to get to the site and the sampling location and for interpretation based on habitat features. 1. Draw a map (aerial view) of your sample reach on the field sheet in the appropriate space. 2. If the channel is irregular, sketch a profile of the channel in addition to the aerial view. *(Her Majesty the Queen in Right of Canada, represented by the Minister of the Environment, 2011, p. 20)*	Communicating pertinent features and sampling area and sketching a map using a prescribed view or perspective and adding some notes on the sketched map

Discussion

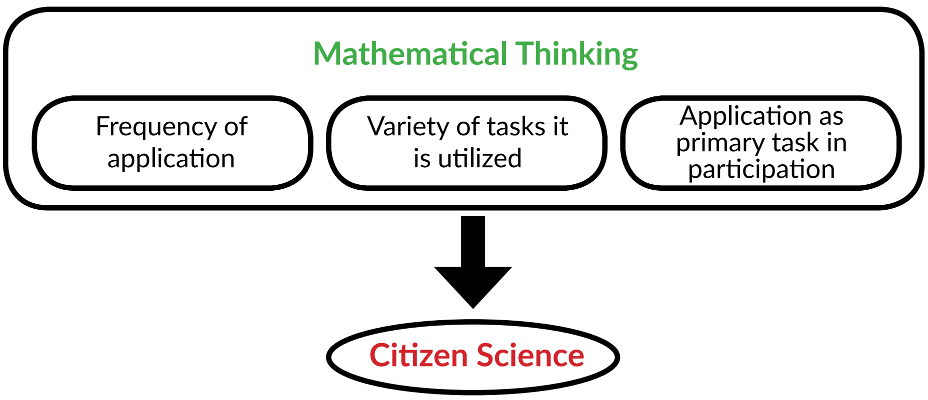
The result of this study revealed that Connection is the most frequently applied in citizen science. However, the result also showed that it was applied only as a Subprocess. This process was not the primary task the citizen scientists were required to perform in their participation, which indicates that the quality of the application of Connection was less extensive than the other mathematical thinking process applied as Main Processes. Problem-Solving, on the other hand, was seen to be used as a Main Process in almost all of the tasks of citizen scientists, while being just the second most commonly used process. This finding suggests that this mathematical thinking process may be used more broadly in citizen science. It indicates that, when compared to the other mathematical thinking processes, citizen scientists have the most opportunity to practice Problem-Solving. On the contrary, despite being comparable to Problem-Solving in terms of frequency, Communication was only applied in a limited depth. Citizen scientists generally only communicated measurements or values, and not explanations, interpretations, or narratives.

For the fourth process, the number of tasks that executed Reasoning and Proof was comparable to the number of tasks that applied Representation, but this number was almost only half or fewer than for those of Connection, Problem-Solving, and Communication. Moreover, the result of this study revealed that Reasoning and Proof can be applied either as a Main Process or a Subprocess. The citizen scientists were given tasks that directly required them to perform Reasoning and Proof, but they were also given tasks that required them to apply another mathematical thinking process, specifically Problem-Solving, that utilized Reasoning and Proof as a procedure to complete the primary tasks.

Finally, this study revealed that Representation was not the focus of most of the tasks citizen scientists executed. It was the least employed mathematical thinking process in citizen science, not only in terms of frequency of application in citizen scientists' assignments but also in terms of the characteristics of its application in their involvement.

Figure 2

Framework in Characterizing Mathematical Thinking in Citizen Science



Framework in Characterizing Mathematical Thinking in Citizen Science

The results and findings about the application of mathematical thinking in different citizen science projects can be synthesized in a framework. This framework, as shown in Figure 2, highlights the features that determine how mathematical thinking processes are utilized in citizen science. The application of mathematical thinking processes in different citizen science projects can be evaluated and described by three features: the frequency of application, the variety of tasks utilized, and the application as the primary task in participation of citizen scientists. These features primarily show how extensive the application of a mathematical thinking process in a citizen science project is. Hence, it can be used as a lens for individuals who want to evaluate the possibility of developing mathematical thinking through participating in a citizen science project.

Conclusion

Based on the results and analysis of the data, this study concluded that the five mathematical thinking processes were applied in the participation of citizen scientists. It may also be stated that the extent to which mathematical thinking processes were applied varied. In the discussion, three attributes are used to characterize the application of each mathematical thinking process, namely, the frequency of process execution in citizen scientist tasks, the variety of tasks that apply a process, and the classification of a process based on whether it is a primary process or a subprocess. Considering these attributes, it could be inferred that Problem-Solving was the most extensively used in citizen science. Problem-Solving was second to the most executed by citizen scientists, but citizen scientists engaged in diverse kinds of tasks that applied Problem-Solving. Moreover, this mathematical thinking process was applied as the primary process in most of the activities citizen scientists performed. Each of the next two mathematical thinking processes, Connection and Communication, had a distinct benefit over the other. Connection had an advantage over Communication since it was the most frequently used in citizen scientist assignments. Communication, on the other hand, was more valuable than Connection since it was used as a fundamental process. However, it should be highlighted that the extensiveness of its applications was still not comparable to that of Problem-Solving because it was generally used in conjunction with other mathematical thinking processes and was seldom used alone. Lastly, Reasoning and Proof, and Representation were relatively comparable only with regard to their frequency of application in the tasks of citizen scientists. However, Reasoning and Proof emerged with an advantage over Representation in terms of the variety of the tasks where it was performed. Given these points, the ranking of the extensiveness of application of mathematical thinking processes in the participation of citizen scientists was as follows: 1) Problem-Solving, 2) Connection and Communication, 3) Reasoning and Proof, and 4) Representation. Given the opportunities to develop mathematical thinking through citizen science despite its slow development and emergence in the Philippines, it opens new possibilities for an integrative pedagogy and implications for curriculum development, assessment, teacher professional development, and community engagement.

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