AN OPTIONS ANALYSIS OF DEFERRED PAYMENT PLANS

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Deferred payment plans (DPPs) are quite common for high value items such as real property, cars, etc. Discounted cash flow (DCF) analysis has been the traditional technique for analyzing DPPs. This usually involves computing the effective interest rate implicit in the arrangement. This approach ignores the real options embedded in the arrangement. An options analysis of DPPs may yield useful insights that may not be evident in DCF analysis. As the paper will show, a DPP is equivalent to a call option with an extendible life and a declining exercise price. This approach is particularly useful when the DPP is being used to acquire an asset for investment or speculative purposes. The binomial option pricing model (BOPM) will be used to value the options in simple DPPs and to analyze how the option value reacts to changes in several variables. Overall, the results are consistent with option pricing theory.

I. REAL OPTIONS AND DEFERRED PAYMENT PLANS

The literature on real options developed from the dissatisfaction with traditional capital budgeting techniques such as net present value (NPV). Dixit and Pindyck (1995) report that executives were observed to override the signal given by NPV because it ignored the value of options inherent in a project and did not therefore value the project correctly. These options gave executives the flexibility to alter a project’s course depending on the actual conditions that materialized. These options enabled managers to defer undertaking projects until more information became available; expand the project if conditions were favorable; or abandon them under adverse conditions. Trigeorgis (2000) gives a longer list of common real options, although most of these can be considered variants of the aforementioned ones. He also matches these options with industries where they may be important, and with the articles which have studied these options. It is probably fair to say that real option analysis is the most significant advance in capital budgeting analysis since discounted cash flow (DCF) analysis. It is now a regular feature in corporate finance textbooks such as Principles of Corporate Finance (Brealey and Myers, 2000). Amran and Kulatilaka (1999) cite the circumstances for which real options analysis is mandatory. These mainly have to do with high uncertainty that makes more information or flexibility more critical. This implies that there may be contingent investment decisions or mid-course corrections which managers may decide on. Damodaran (2000) offers some questions which may serve as a test for situations where real options may be valuable.

Once real options are recognized, the question that naturally follows is how much are they worth? It is necessary to answer this question to advance the analysis of real
options from a subjective, qualitative appreciation of its importance to an objective measurement of its value. Fortunately, the option pricing models for financial options can easily be adapted to simple real options. The Black-Scholes model (BS) and the binomial option pricing model (BOPM) are often used to price simple real options. Amran and Kualilaka (1999) say that BS is appropriate for simple real options, i.e., an option with a single source of uncertainty and a single decision date. In more complex situations, numerical methods, of which BOPM is an example, may be required. They cite three advantages of BOPM. One, it can handle a wide range of real options applications, including complex cases. Two, it is “user friendly” since it retains the appearance of DCF analysis. Three, uncertainty and the consequences of contingent decisions are easy to visualize (Amran and Kualilaka, 1999, pp. 36-37). However it should be added that BOPM also has drawbacks, the most visible one being that price can only take one of two values. However this is also easily remedied by shortening the period so that the price movements will be a better approximation of reality.

While option pricing models for financial options lend themselves readily to real options, specifying the data inputs is more difficult for real options. Damodaran (2000) points out that the value of the underlying asset in a real option and its variance may be difficult to estimate since the asset is not traded. The exercise price and the life of the option are not precisely set as they are for financial options where they are set contractually. He suggests three ways of estimating variance. First, the variance of similar past projects may be used. Second, a simulation analysis can be done to estimate the variance. Third, a market based proxy such as the variance of the stock price of a company in a similar line of business may be used. The exercise price in a real option may also change over time as prices, technology and other factors change. The economic life of an option may be considerably shorter than its legal life. These are just some of the practical issues in pricing real options.

A real option that comes closest to the option discussed in this paper is discussed by Trigeorgis (2000). He calls this an option to default on planned cost “installments” during construction. He describes a situation where the investment is not made in a single lump sum but in a series of investments or installments. In the event of favorable conditions, the firm will continue to make the investments; otherwise, it ceases and abandons a half-finished project. The project can be viewed as a series of options; paying an installment is needed to acquire the subsequent option to continue with the project. In other words, the project can be viewed as a compound option. He says that this option to abandon is important for highly uncertain capital intensive projects with long development lead times.

What makes a DPP different from the Trigeorgis example is that a DPP is basically a financing arrangement and traditionally has been analyzed as such. However, this paper analyzes a DPP using options analysis. The Trigeorgis example involved making phased investments that gradually builds up a capital asset such as a factory building. The essential characteristic is the incompleteness of the asset in the development phase. In a DPP, the asset may be essentially complete but the payments, and therefore the ownership, are
incomplete. For example, imagine a firm acquiring machinery or equipment through a financing or leasing arrangement with a finance company. In the Trigeorgis example, the firm would abandon an incomplete asset. In a DPP, the firm would abandon an incomplete ownership over a complete asset. The difference may be subtle but it is this subtlety that makes the identification of real options challenging.

In a DPP, the buyer makes an initial or down payment and commits to make a series of future payments. The buyer can only take title over the asset if he or she has paid in full. The buyer may discontinue making the payments at any time but this may result in a forfeiture of all or part of the payments the buyer has made. This may be the sensible thing to do if the value of the asset has fallen sharply and there is little chance of recovery within the time horizon of the DPP, or if more attractive investments arise. However, to extend the life of the option, the buyer has to continue making the payments. The unique feature of the arrangement is that the payments also reduce the exercise price of the option. In a sense, each payment is partly a payment to extend the option and partly a payment for the asset. Contrast this with DCF analysis that breaks down each payment into an interest component and a principal (asset) component.

The rest of the paper explores the implications of this approach by applying the BOPM to a simple example of a DPP.

II. OPTIONS ANALYSIS OF DPP

The assumptions for the base case are:

1. The underlying asset is land which is being acquired for speculative purposes and this land does not yield income. If the future value of the land is highly uncertain, it is obvious why it may be more advantageous to buy an option on the land rather than buy the land outright. It also requires less up front money. A DPP offers both advantages: it minimizes risk and the initial investment.

The current market value of the land is P80,000. Future values evolve in a binomial pattern, i.e., it either rises or falls. Values for this rise and fall must be set so that the resulting distribution corresponds to empirical reality. Under the risk neutral assumption of BOPM, the expected return on the asset is the risk-free rate but its volatility (variance) will be the same as that observed in the real world. In other words:

\[ pAu + (1 - p)Ad = Ae^{rt} \]  
\[ pu^2 + (1 - p)dt^2 - \left[ pu + (1 - p)dt \right]^2 = \sigma^2 \]

where

- \( A \) = asset value
- \( u \) = upward price adjustment factor
- \( d \) = downward price adjustment factor
- \( p \) and \( 1 - p \) are the risk neutral probabilities.
- \( r \) = risk-free rate per period
- \( t \) = time period between payments
- \( \sigma \) = volatility of asset value per period

One solution to the above equations that assumes symmetric up
and down movements \((u = 1/d)\) is given by:

\[
\begin{align*}
u &= e^{\sigma \sqrt{t}} \\
d &= e^{-\sigma \sqrt{t}} \\
p &= \left(e^{\sigma} - d\right)/(u - d)
\end{align*}
\]

In this paper, the rise is initially set at 25% and the fall is initially set at -20% in each period. This corresponds to an assumed standard deviation of 22%. This means the land value is either 125% or 80% of its value in the previous period. Subsequent examples will show the effect of having higher or lower volatility. Option pricing theory tells us that higher volatility should result in higher option values.

2. The DPP involves making the following payments:

\[
\begin{align*}
t=0 & & \text{Down payment}\quad \text{P}40,000 \\
t=1 & & \text{First installment}\quad 30,000 \\
t=2 & & \text{Second installment}\quad 30,000
\end{align*}
\]

Failure to pay any installment forfeits all the buyer's rights. At a 12% interest rate per period, these payments have a present value of P90,700. In the sensitivity analysis, the DPP will be modified by altering the payment schedule and extending the time horizon. However, the present value is preserved at P90,700. We should expect that the option values for longer DPPs should be higher than for shorter ones, everything else equal. A larger down payment reduces the future installments and is equivalent to a reduction in the exercise price. It should also result in a higher option value.

3. The risk-free rate is 5% per period in the base case. A higher risk-free rate should raise the option value.

Using the basic assumptions, the possible path of land values will be:

![Diagram](attachment:land_value_diagram.png)

The option values will then be:
The $t_2$ payoffs are just the possible land values at $t_2$ minus the final payment needed to take title over the land. Of course, the option will not be exercised if it will result in a loss. In this case, the payoff is zero. The BOPM will be used to work back to $C_u$ and $C_d$, and finally $C$. The interested reader may refer to Hull (1991) for a basic presentation on BOPM. The BOPM formula is:

$$C = [pC_u + (1 - p)C_d]e^{-rt}$$

The formula for $C_u$ and $C_d$ is analogous. Applying this formula yields the following results:

The initial value of the option, $C$, is greater than the required down payment of P40,000 and it makes sense for the buyer to purchase this option. If the land value rises in $t_1$, the option becomes more valuable at $C_u = P71,342$, much greater than the first installment of P30,000 to keep
it alive. Even if the land value falls however, it is still worthwhile keeping the option alive since \( C_d \) still exceeds P30,000. As subsequent cases will show, the DPP terms, the movement in land values, and the risk-free rate will have an effect on the value of the options and the decision to keep it alive.

The effects of changing key variables on the value of the option are presented in Table 1.

**Table 1**

Binomial Option Pricing Model Analysis of Deferred Payment Plans

<table>
<thead>
<tr>
<th>Case</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tr>
<td>Variables</td>
<td>Low down</td>
<td>Base case</td>
<td>High down</td>
<td>Very high down</td>
<td>Low volatility</td>
<td>Very high volatility</td>
<td>Higher risk-free rate</td>
<td>Extension of DPP</td>
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<td>Land values:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Up</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.125</td>
<td>0.650</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Down</td>
<td>-0.20</td>
<td>-0.20</td>
<td>-0.20</td>
<td>-0.20</td>
<td>-0.111</td>
<td>-0.390</td>
<td>-0.20</td>
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<tr>
<td>Risk-free rate</td>
<td>0.05</td>
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<td>0.10</td>
<td>0.05</td>
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<tr>
<td>Risk neutral Probability:</td>
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<td></td>
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<td></td>
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<td></td>
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<tr>
<td>( P )</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.68</td>
<td>0.43</td>
<td>0.67</td>
<td>0.56</td>
</tr>
<tr>
<td>( 1-P )</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.44</td>
<td>0.32</td>
<td>0.57</td>
<td>0.33</td>
<td>0.44</td>
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<td>DPP:</td>
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<td></td>
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<td></td>
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<tr>
<td>Down</td>
<td>20,000</td>
<td>40,000</td>
<td>60,000</td>
<td>70,000</td>
<td>40,000</td>
<td>40,000</td>
<td>40,000</td>
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<tr>
<td>First</td>
<td>41,834</td>
<td>30,000</td>
<td>18,165</td>
<td>12,248</td>
<td>30,000</td>
<td>30,000</td>
<td>30,000</td>
<td>21,116</td>
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<tr>
<td>Second</td>
<td>41,834</td>
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<td>18,165</td>
<td>12,248</td>
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<td>30,000</td>
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<tr>
<td>Third</td>
<td>21,116</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Option values:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>41,954</td>
<td>52,662</td>
<td>63,370</td>
<td>68,724</td>
<td>52,662</td>
<td>52,833</td>
<td>53,068</td>
<td>61,535</td>
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<tr>
<td>( C_u )</td>
<td>60,085</td>
<td>71,342</td>
<td>82,600</td>
<td>88,228</td>
<td>61,354</td>
<td>103,200</td>
<td>70,593</td>
<td>80,652</td>
</tr>
<tr>
<td>( C_d )</td>
<td>24,129</td>
<td>35,386</td>
<td>46,644</td>
<td>52,272</td>
<td>42,497</td>
<td>20,237</td>
<td>34,761</td>
<td>44,739</td>
</tr>
<tr>
<td>( C_{ua} )</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>104,763</td>
</tr>
<tr>
<td>( C_{ud} )</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>59,817</td>
</tr>
<tr>
<td>( C_{dd} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>31,052</td>
</tr>
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</table>

Cases 0, 2 and 3 modify the base case by assuming different down payment levels. As mentioned earlier, the present value of the payments is preserved. It shows that the value of the options rise with the down payment. This is consistent with expectations. However the increase in option value is less than the increase in down payment. A 50% increase in the down payment (from the base of P40,000) increases the option value from P52,662 to P63,370, or by 20%. Thus,
a point is reached when the down payment exceeds the option value and it will not pay to purchase the option. This happens in case 3 when the P70,000 down payment exceeds the option value of P68,724. The increase in option value with an increase in down payment seems counterintuitive. The “typical” view is that a large down payment has the effect of “binding” the buyer into completing the deal, i.e., it reduces his options. In this analysis, the larger down payment enhances the value of the call option since it reduces the future exercise price. Of course, a lower exercise price also induces the buyer to complete the deal.

Cases 4 and 5 analyze the situations of lower and higher volatility in land values. The results in case 5 are consistent with option pricing theory: higher volatility has resulted in a higher option value compared to the base case. What is a little surprising is the case 4 result. Case 4 has lower volatility but has the same option value as the base case. A possible explanation is that in case 5, there is a possibility that the option would be out of the money and would not be exercised. This would happen if there are two successive falls in land values leading to $t_2$ land value of P29,427. Since the final payment is still P30,000, it would not pay to exercise the option. The truncation of the payoff to zero instead of a loss while retaining the higher upside accounts for the higher value of the option in case 5. It is this loss limiting feature of options that give it a higher value in the face of higher volatility. In the base case and in case 4, the option still ends in the money despite two successive falls in land values.

Case 6 assumes a higher risk-free rate of 10%. The interest rate in the DPP is set at 17% to maintain a constant spread over the risk-free rate. The payments are adjusted to yield the same present value of P90,700. The result is consistent with option pricing theory: a higher risk-free rate leads to a higher value for call options. In our example, C rises to P53,068 from P52,662 in the base case. In a high interest rate environment, the buyer would rather not part with his money and the DPP becomes more attractive since it requires a smaller outlay. The insight here is that investors or speculators may not stop their activities just because interest rates are high. They will simply find a more efficient way to do it and options embedded in DPPs allow them to do it. Finally, case 7 extends the DPP by one period. As expected, the option value rises with the extended term.

III. CONCLUSION

The paper has applied a new way of looking at deferred payment plans by analyzing the options implicit in such plans. The approach emphasizes the fact that a buyer in a DPP has choices that depend on the outcome of asset values. This is different from the traditional DCF analysis which assumes that all payments are made and valuation is just a matter of computing present values or effective interest rates. This approach may yield more insights than the traditional approach. Of course, the options approach is not a new idea. Its application to DPPs is just an extension of the idea that corporate securities such as equity and debt can be analyzed as options.
REFERENCES