Numerical Investigation of Non-Homologous Collapse of the One-Dimensional Gravitational Gas

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ABSTRACT

In this paper, the one-dimensional gravitational gas is evolved numerically using an event-driven code. Two initial conditions are considered: (1) an initially uniform isolated system with no velocity dispersion and where the initial velocities are sine functions of the position, and (2) two "clusters" with initially constant phase-space densities in elliptical regions of phase space.

INTRODUCTION

There have been attempts to study the time evolution of the one-dimensional gravitational gas both analytically (Mineau et al., 1990; Muriel & Esguerra, 1996; Muriel et al., 1998) and numerically (Hohl & Feix, 1967; Severne et al., 1985). These attempts may be traced to two streams: the first, resulting from a long tradition of applying the kinetic theory to a gas of self-gravitating particles (Chandrasekhar, 1942; Landau & Lifshitz, 1980; Spitzer, 1987; Lightman & Shapiro, 1978; Saslaw, 1985) and the second, arising from Poincare's unsuccessful attempt to solve the three-body problem, the emergence of computers, and the development of numerical analysis. The motivation for these is the problem of structure formation in the universe–a challenging problem of classical physics, which is of great interest today because of recent and still improving data on the cosmic microwave background. In its full glory, the problem of structure formation is a complicated nonlinear problem, which is difficult to treat both analytically and numerically.

Because of the above reasons, the interest in simplified models of structure formation persists to this day. One model, that of collapsing globular clusters, involves concentric spherical shells representing groups of stars with equal radial velocities (Youngkins & Miller, 2000). A variation of the model gives each shell constant angular momentum (Barkov et al., 2002; Klinko et al., 2001).

A simpler model, that of self-gravitating parallel "infinite" sheets has been investigated for a longer time. It has been used in the past to model the behavior of halo stars (Prendergast, 1954; Camm, 1950; Schilt, 1950). It is sometimes referred to as the self-gravitating one-dimensional system (SOG) (Youngkins & Miller, 2000; Wright & Miller, 1984) or as the one-dimensional gravitational gas (1DGG) (Hohl & Feix, 1967; Mineau et al., 1990). Recently, a modification of the 1DGG, one that takes into account mass and energy loss due to the evaporation of stars, was introduced as a toy model of globular cluster (Fanelli et al., 2001).

In this paper, we study the non-homologous collapse of the 1DGG using numerical methods.

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METHOD

Consider a model of parallel self-gravitating "infinite" flat sheets, each with mass m_i and allowed to pass through each other. The Hamiltonian is

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + 2\pi G \sum_{i=1,\, j>i}^{N} m_i m_j \left(x_i - x_j \right) \qquad (1)$$

In the present study, every sheet has mass $m = 1 = N$, where N is the number of sheets in the system. To simplify further, we choose the scale such that $4\pi G$ is equal to one. In this system, the force acting on a sheet is constant as long as the sheet does not intersect (or "collide") with any other sheet–this makes it easy to implement numerically.With the choice of units, the acceleration of a sheet is

$$a = \frac{1}{2N}\left(N_R - N_L \right) \qquad (2)$$

where N_R and N_L are the number of sheets to the right and to the left of the sheet, respectively. If the sheets are ordered, the acceleration can be written as

$$a_i = \frac{1}{2N}\left(N - 2i + 1 \right) \qquad (3)$$

where i, a positive integer, is the order of the sheet defined as follows:

$$i < j \to x_i \le x_j \ . \qquad (4)$$

The evolution is then achieved using an event-driven algorithm described by Fanelli et al. (2001). In this algorithm, intersection times between adjacent sheets are calculated, and the system is evolved up to the minimum intersection time giving a solution that is exact to machine precision. (A more sophisticated heap-based algorithm (Noullez et al., 2001) which may give a faster running time is not used here.) The phase-space coordinates (x_i and v_i) at a later time t is then recorded.

Two different configurations are analyzed: (1) an initially uniform isolated system with no velocity dispersion, and where the initial velocities are negative sine functions of the position (note that the choice of a negative sine dependence of initial velocity on the position is not essential–any other functional dependence of initial velocity on position, which assures that all sheets go towards the origin initially could have been chosen); and (2) two "clusters" with initially constant phase-space densities in elliptical regions of phase-space.

RESULTS

The results are shown in Figs. 1 and 2. One may think of the results illustrated in both figures as metaphors: Fig. 1 for the evolution of a collapsing galaxy, and Fig. 2 for the evolution of two neighbor galaxies for different initial separation distances.

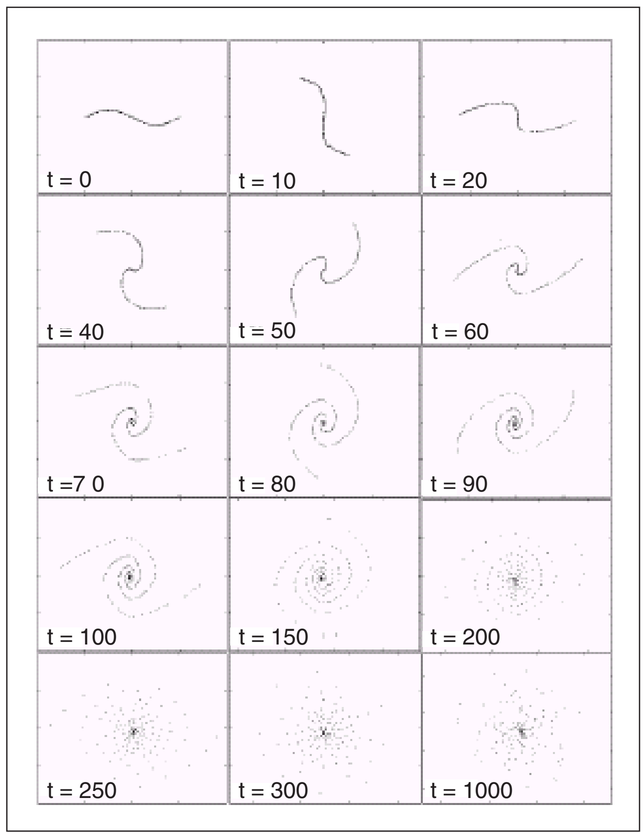

Fig. 1. Evolution in phase-space of an initially uniform isolated system with no velocity dispersion and where the initial velocities are negative sine functions of the position. The sheets at t = 0, 40, 70, 100, and 250 are moving to the right while the sheets at t = 20, 60, 90, 200, and 1,000 are moving to the left–a collapse.

The following features in Fig. 1 should be noted. At t = 0, the system is uniform and there is no velocity dispersion. At t = 10, the density at the origin is high. In the region near the origin are positions in which sheets moving with different velocities and in opposite directions can be found–this is called multistreaming. The region exhibiting multistreaming gradually expands. The formation of a high-density core region surrounded by a low-density halo region is apparent, beginning at t = 250.

The following features in Fig. 2 should be noted. In the time interval shown, the shape of the 300-sheet cluster in phase-space hardly changes, while that of the 100-sheet cluster changes a lot (e.g., at t = 7, the sheets from the 100-sheet cluster occupy a length that is about twice the length occupied at t = 0). It is tempting to conclude that the sheets from the 100-sheet cluster will eventually form a halo around the 300-sheet cluster, but the numerical simulations have not gone that far to make the conclusion definitive.

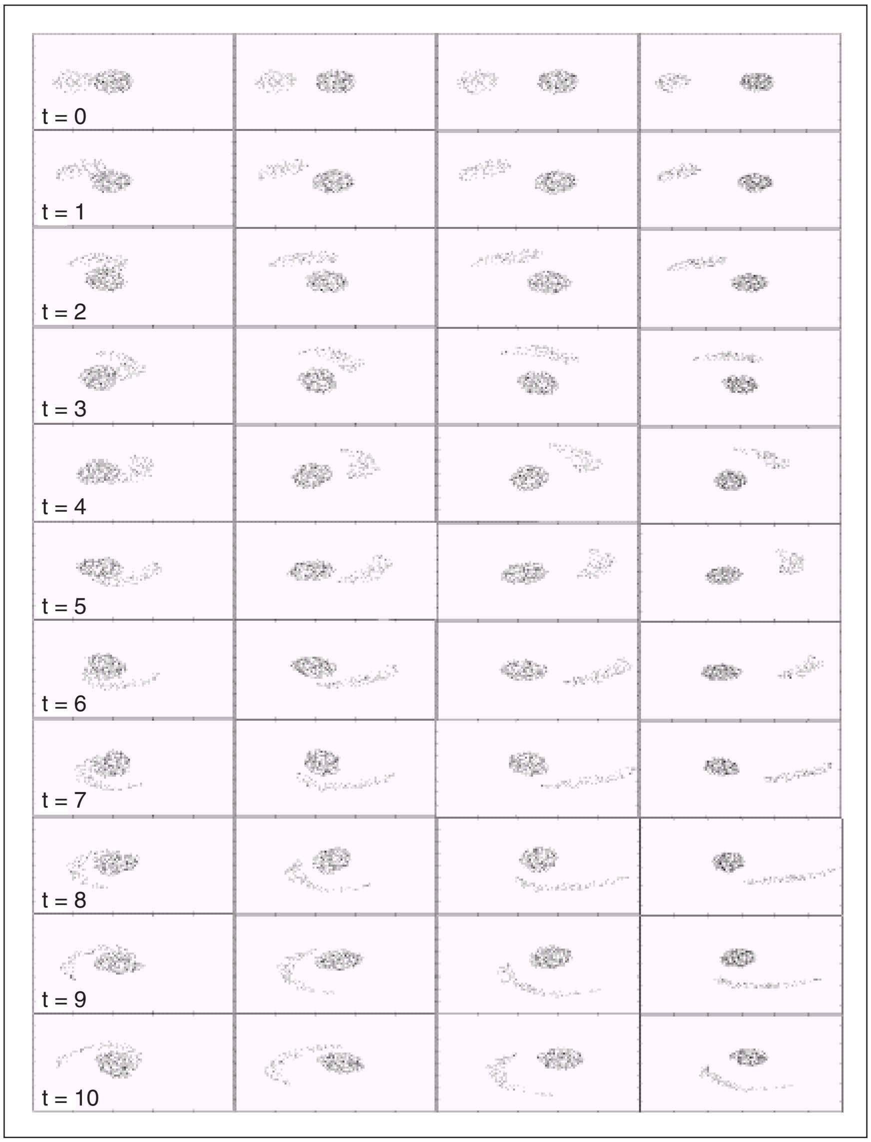

Fig. 2. Evolution of two clusters with initially constant phase densities in elliptical regions of phase-space. Four initial conditions, which differ in the separation distance of the clusters, are evolved. The more massive cluster has 300 sheets while the other has 100.

FINAL REMARKS

In this paper, the one-dimensional gravitational gas has been evolved numerically using an event-driven code for two initial conditions. Features such as the onset of multistreaming and the formation of a core-halo structure have been exhibited. The event-driven code gives results that are exact to machine precision, but is rather slow compared to numerical integration, especially when the system being evolved has regions with high sheet density. However, one should not interpret this statement as an endorsement of faster routines employing straight numerical integration, such as the Runge-Kutta method–as a naive implementation yields non-physical, non-energy conserving results fast. From here, one may pursue several research directions: (1) one may run the existing code on a faster computer; (2) implement a faster, more sophisticated (and still event driven) heap-based algorithm (Noullez et al., 2001); (3) develop and optimize algorithms based on numerical integration using the existing event-driven code as a benchmark for accuracy; and, (4) perform analytical work. Analytical efforts dealing with essentially the same initial conditions are now under way (Gargar, 2002).

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