Full Gauge-Parameter-Independent Higgs-Boson Self-Energy

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ABSTRACT

We calculate the self-energy of the standard-model Higgs boson by means of the pinch technique. We work in the general R_ξ gauge and show that the final result is independent of the gauge parameters. This result is useful in order to demonstrate that the threshold singularities encountered in the description of Higgs-boson production and decay in the on-shell renormalization scheme only arise at physical thresholds and are independent of the chosen gauge.

Keywords: SM Higgs-boson self-energy, pinch technique, *S*-matrix PT framework, gauge invariance, quantum corrections, perturbation theory

INTRODUCTION

The standard model (SM) of Glashow, Salam and Weinberg is a non-abelian gauge field theory of the electroweak and strong interactions. It involves a Yang-Mills sector in its classical Lagrangian, which on the whole is invariant under a local SU(2) and U(1) gauge transformation. This gauge invariance property is compromised when one wishes to canonically quantize the theory. In order to achieve this, one needs to introduce gauge-fixing terms in the Lagrangian, which break the underlying gauge symmetry and introduce unphysical degrees of freedom. In order to compensate for this, one needs to add terms involving anti-commuting Faddeev-Popov ghost fields into the Lagrangian. Both these requirements for the quantization complicate matters concerning practical calculations, for example, of radiative corrections involving quantum-loop diagrams, in the vector- and scalar-boson portion of the theory (Papavassiliou & Pilaftsis, 1998). As for the Higgs-Boson, its full one-loop self-energy, when calculated from the assumption that the Higgs-Boson is an asymptotic state of the scattering (*S*) matrix, while gauge independent on-shell, turns out to depend on the gauge parameter ξ off-shell. This property, among others, is by no means an obstacle in conventional perturbation theory, which predicts meaningful observables independent of the gauge-fixing procedure. However, this is not the case where the conventional perturbation theory breaks down, like in the strongly coupled theory of quantum chromodynamics (QCD) and in the vicinity of resonances in a weakly coupled theory (Papavassiliou & Pilaftsis, 1996) such as the electroweak sector of the standard model. The search for a self-consistent scheme for constructing off-shell Green's functions resulted in a formalism based on what is called the pinch technique (PT).

As has been observed in Refs. (Kniehl, 1991; 1992; 1994), the one-loop corrected decay width of the SM Higgs-Boson as calculated in the conventional on-shell renormalization scheme exhibits singularities if the mass relations $m_H = 2m_W$ or $m_H = 2m_Z$ happen to be satisfied. These threshold singularities arise from the wave-function renormalization of the Higgs-Boson, which is given in terms of the derivative of the self-energy of the latter. One important application of the PT is to verify that these threshold singularities only occur at physical thresholds and are independent of the chosen gauge. This will be exhibited in a forthcoming paper (Kniehl et al., in preparation), which will also explain how the threshold singularities are removed by the consequent application of the pole definitions of mass and width. In this paper, we derive the full Higgs-boson self-energy in the PT framework. We work in the general renormalizable R_ξ gauge (Fujikawa et al., 1972) and explicitly show that the result obtained is independent of the gauge parameters. Our result generalizes a previous analysis (Papavassiliou & Pilaftsis, 1998).

Pinch Technique

The PT is an algorithm that renders one-loop gauge-boson self-energies gauge independent. It has rich applications in QCD (Cornwall & Papavassiliou, 1989) and in the electroweak sector of the SM (Degrassi & Sirlin, 1992; Papavassiliou, 1990; Papavassiliou & Sirlin, 1994). At the one-loop level, the technique unravels self-energy contributions from vertex and box diagrams that are otherwise excluded in the conventional manner of computing the self-energy. By themselves, these PT self-energy contributions are gauge dependent. When combined with the conventional self-energy, these contributions exactly cancel the gauge-dependent terms of the former rendering the combined self-energy gauge independent. The resulting PT self-energy satisfies desirable properties like being resummable and process independent and complying with the unitarity of the S matrix (Papavassiliou & Pilaftsis, 1995; 1996; 1998).

Conventional Higgs-Boson Self-Energy in R_ξ Gauge

The Feynman diagrams contributing to the conventional self-energy of the SM Higgs-Boson are depicted in Fig. 1. In R_ξ gauge, we find

$$\begin{aligned}\Pi_{HH}(s) = \frac{G}{\pi}\Bigg\{&\left(\frac{s}{2}+3m_W^2\right)A_0\left(m_W^2\right)-\frac{1}{2}\left(s-m_H^2\right)A_0\left(\xi_W m_W^2\right)\\&+\left(\frac{s^2}{2}-sm_W^2+3m_W^4\right)B_0\left(s,m_W^2,m_W^2\right)\\&-\frac{1}{4}\left(s^2-m_H^4\right)B_0\left(s,\xi_W m_W^2,\xi_W m_W^2\right)+\frac{1}{2}(W\to Z)\\&+\frac{3}{4}m_H^2A_0\left(m_H^2\right)+\frac{9}{8}m_H^4B_0\left(s,m_H^2,m_H^2\right)\\&-\sum_f N_f m_f^2\left[2A_0\left(m_f^2\right)-\left(\frac{s}{2}-2m_f^2\right)B_0\left(s,m_f^2,m_f^2\right)\right]\Bigg\},\end{aligned} \tag{1}$$

where G is related to the Fermi constant by $G = G_F/(2\pi\sqrt{2})$, s is the Higgs-boson virtuality, m_H and m_W are the masses of the Higgs and W bosons, respectively, ξ_W is the gauge parameter associated with the W boson, the functions A_0 and B_0 are the so-called scalar one- and two-point functions (Kniehl, 1994), respectively, and the term $(W \to Z)$ signifies the contribution involving the Z boson, which is obtained from the one involving the W boson by replacing m_W^2 and ξ_W with m_Z^2 and ξ_Z, respectively. In the 't Hooft-Feynman gauge, with $\xi_W = \xi_Z = 1$, Eq. (1) agrees with the corresponding result given in Eqs. (B.2) and (B.3) of Ref. (Kniehl, 1991).

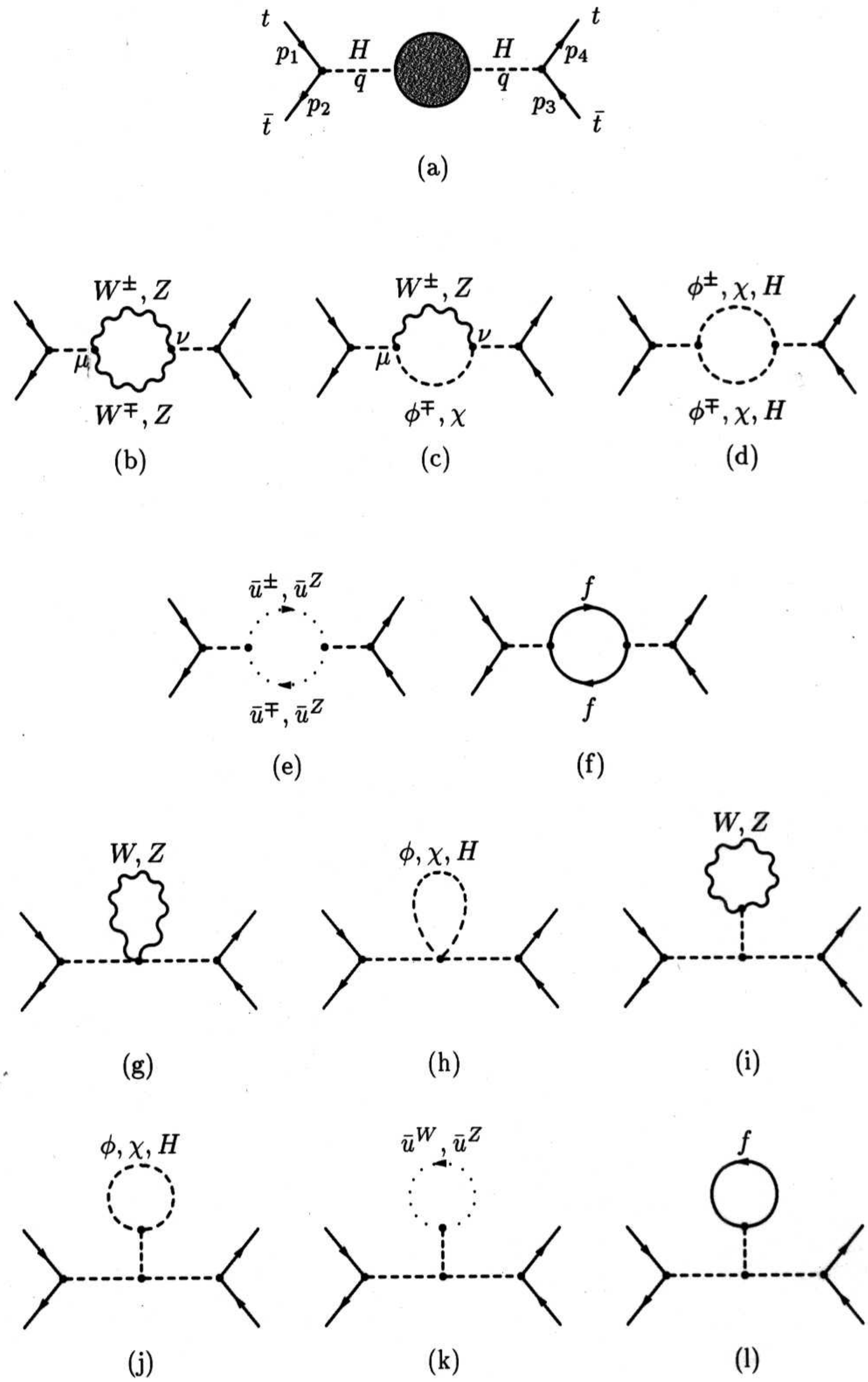

Figure 1. Feynman diagrams pertinent to the conventional self-energy of the SM Higgs boson in R_ξ gauge.

Full PT Contribution in R_ξ Gauge

In calculating the full PT contribution to the Higgs-boson self-energy in R_ξ gauge, we use the *S*-matrix PT framework elaborated in Ref. (Degrassi & Sirlin, 1992). The scattering process considered is a four-fermion process with the Higgs-Boson as intermediate state. In the formulation of Ref. (Degrassi & Sirlin, 1992), the relevant amplitudes reflecting the gauge-boson and external-fermion interactions are described in terms of matrix elements of Fourier transforms of time-ordered products of current operators. Through successive current contractions with the longitudinal four-momenta found in the propagators of the massive vector bosons, Ward identities are triggered, after which the relevant pinch contribution is identified upon application of appropriate equal-time commutators of currents. The relevant Feynman diagrams are depicted in Fig. 2. Setting aside the details, the full PT contribution is found to have the following form,

$$\Delta\Pi_{HH}(s) = \frac{G}{\pi}\left\{\frac{1}{2}\left(s - m_H^2\right)\left[A_0\left(\xi_W m_W^2\right) - A_0\left(m_W^2\right)\right]\right.$$
$$- \left(s - m_H^2\right)\left[\frac{1}{4}\left(s + m_H^2\right) + m_W^2\right]B_0\left(s, m_W^2, m_W^2\right)$$
$$\left.+ \frac{1}{4}\left(s^2 - m_H^4\right)B_0\left(s, \xi_W m_W^2, \xi_W m_W^2\right) + \frac{1}{2}\left(W \to Z\right)\right\}. \qquad (2)$$

DISCUSSION OF THE RESULTS

Some comments on Eqs. (1) and (2) are in order. As expected (Papavassiliou & Pilaftsis, 1998), their sum,

$$\Pi_{HH}^{PT}(s) = \Pi_{HH}(s) + \Delta\Pi_{HH}(s) \qquad (3)$$
$$= \frac{G}{\pi}\left\{\left(\frac{m_H^2}{2} + 3m_W^2\right)A_0\left(m_W^2\right)\right.$$
$$+ \left[m_H^2\left(\frac{m_H^2}{4} + m_W^2\right) - 2sm_W^2 + 3m_W^4\right]B_0\left(s, m_W^2, m_W^2\right) + \frac{1}{2}\left(W \to Z\right)$$
$$+ \frac{3}{4}m_H^2 A_0\left(m_H^2\right) + \frac{9}{8}m_H^4 B_0\left(s, m_H^2, m_H^2\right)$$
$$\left.- \sum_f N_f m_f^2\left[2A_0\left(m_f^2\right) - \left(\frac{s}{2} - 2m_f^2\right)B_0\left(s, m_f^2, m_f^2\right)\right]\right\},$$

which represents the full PT self-energy of the Higgs-Boson, is manifestly independent of ξ_W and ξ_Z for all values of *s*. In the special case of $s = m_H^2$, $\Pi_{HH}(s)$ is separately gauge independent, while $\Delta\Pi_{HH}(s)$ vanishes. In fact, this ensures that the total decay width of the Higgs-Boson, which is proportional to the absorptive part of its self-energy, is gauge independent and does not receive additional contribution due to the application of the PT.

In the 't Hooft-Feynman gauge, our full PT result agrees with that reported in Ref. (Papavassiliou & Pilaftsis, 1998). For gauges other than the 't Hooft-Feynman gauge, our result only differs from the one in Ref. (Papavassiliou & Pilaftsis, 1998) with respect to the first term in Eq. (2). However, consistency between the two results remains and has not been compromised, since the authors of Ref. (Papavassiliou & Pilaftsis, 1998) have explicitly omitted contributions from tadpole and seagull graphs, which are included in our consideration and are the ones responsible for the presence of the first term in Eq. (2). On the other hand, their significance must not be underestimated, since they render the dispersive part of the Higgs-Boson self-energy gauge independent as well.

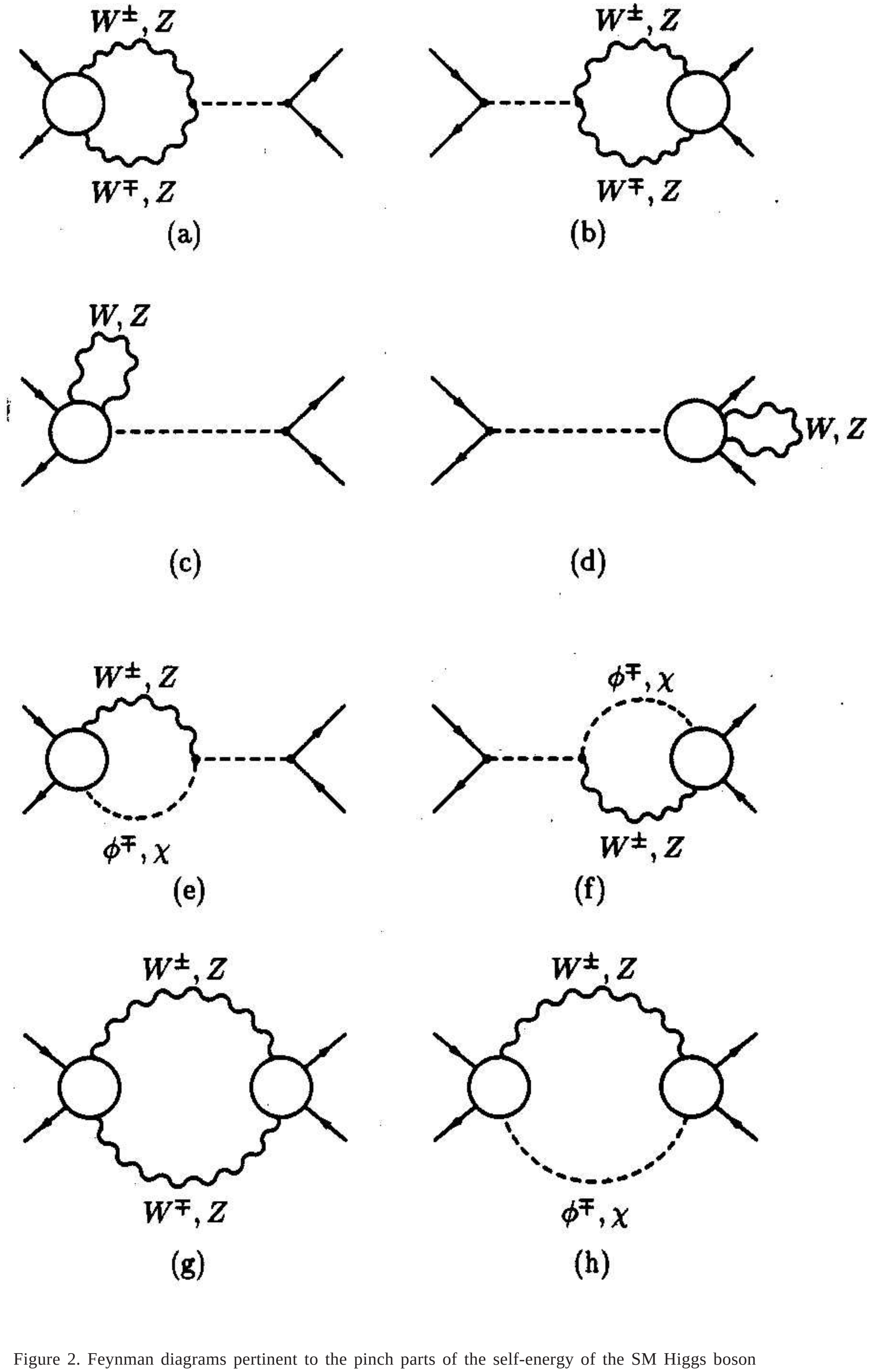

Figure 2. Feynman diagrams pertinent to the pinch parts of the self-energy of the SM Higgs boson

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