Extension of the Limiting Quality Indexed Single Acceptance Sampling Plans for Attributes in High Precision Processes

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ABSTRACT

This study investigated the extension of existing single acceptance sampling plans indexed by lot tolerance percent defective (LTPD) or limiting quality (LQ) with respect to their applicability to high precision processes. LTPD/LQ indexed sampling plans were extended to cover the very low fraction defective levels of high precision processes. Plans based on the ISO 2859-2 LQ indexed plans, Dodge-Romig LTPD indexed plans, and lot sensitive plans (LSP) were used as bases for the extensions. Target levels for LTPD/LQ were set at defective parts per million levels ranging from 20 ppm to 5000 ppm. The performance of the extended plans was evaluated using measures relating to level of protection afforded by the plans and efficiency in terms of amount of required inspection. The extended Dodge-Romig LTPD plans showed best performance among the three plans generated. Three selected plans from the extended Dodge-Romig LTPD sampling scheme were then subjected to simulation studies. Comparison of the theoretical and simulation data indicated that the plans behave more strictly than expected from theoretical calculations. The specific demands of high precision processes for appropriate sampling inspection plans that cover lower fraction defective levels, and at the same time satisfy the requirement for economy and efficiency of inspection were shown to be provided by this plan. These results contributed significantly to the manufacturing industry as it continuously strives to improve process yields and decreases fraction defective levels to respond to customer demands of better quality and improved performance.

Key words: attributes acceptance sampling, lot tolerance percent defective, limiting quality, high precision processes

INTRODUCTION

The manufacturing industry has evolved into high volume and high precision processes using state of the art technology. Process yields have improved tremendously as a result of the use of highly efficient machines and methods, more suitable raw materials, and adequately trained human resources. Customers require products with very low fraction defectives from their suppliers, and demand an assurance that these products conform to specific quality requirements. For producers to be assured that customer specifications are met, inspection procedures are employed at specific points in the manufacturing process. Final sampling inspection is also carried out to determine quality levels of finished products prior to delivery.

Acceptance sampling plans have been used extensively to provide a system for inspecting production lots and

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making decisions concerning their disposition. The most prominent among these sampling plans is the Military Standard 105D (MIL STD 105D), which is comprised of a set of plans indexed by the Acceptable Quality Level (AQL). In recent years, there has been a growing interest in sampling plans that are indexed by Lot Tolerance Percent Defective (LTPD) or Limiting Quality (LQ). These plans aim to provide specified consumer protection in terms of defined LTPD/LQ level corresponding to the lot fraction defective with a probability of acceptance of 0.10. Thus, the LTPD/LQ indexed plans are oriented towards providing consumer protection against lots of poor quality and are used in high precision processes, such as in the manufacture of semiconductors, as an alternative to the AQL indexed plans given by the MIL STD 105D.

High precision processes refer to manufacturing processes operating at a high level of precision, resulting in high process yields, i.e., from 98% and above, and fraction defective levels within the two-digit defective parts per million (ppm) range. Leading microelectronic manufacturers have reported that they operate at an average of 7.5 to 30 defective parts per million (SAS Institute Inc., 1996). However, the limiting values provided for the existing LTPD/LQ plans are still far above the appropriate levels for high precision processes. The ISO 2859-2 1985 LQ indexed plans provide for LQ levels between 0.5% to 32.0%, or 5,000 to 320,000 ppm. Similarly, the Dodge-Romig LTPD indexed plans provide tables for LTPD levels between 0.5% to 10.0%, or 5,000 to 100,000 ppm. Hence, there is a need to establish sampling inspection plans for high precision processes based on appropriate levels of LQ/LTPD and intervals for process averages.

This study aimed to generate extensions of the existing LTPD/LQ indexed acceptance sampling plans and to evaluate the performance of these plans in terms of discriminatory power and efficiency. Extensions were generated from three existing sampling schemes, namely the ISO 2859-2:1985 LQ indexed sampling plans, the Dodge-Romig LTPD indexed sampling plans, and the lot sensitive plans with zero acceptance numbers. The following discussions present the derivation and procedure for the extension of these selected sampling plans.

THE ISO 2859-2:1985 LQ INDEXED SAMPLING PLANS

These plans are compatible with the structure of MIL STD 105D and aim to ensure with known probability that the quality level of outgoing lots will not exceed specified LQ levels. Compatibility with MIL STD 105D was desired for administrative reasons since the present standard is widely used and accepted. The ISO 2859-2:1985 provides tables for single sampling plans indexed by LQ at the following percent levels: 0.50; 0.80; 1.25; 2.00; 3.15; 5.00; 8.00; 12.50; 20.00; and 32.00.

Derivation

The derivation of the extended plans was based on the procedure by Duncan et al. (1980). It was assumed that the plan was being applied to a series of lots and that the probability of lot acceptance at a process average equal to a specified LQ level would be given sufficiently well by the Poisson distribution. Lot size classes and sample size structure were made compatible with the MIL STD 105D resulting in a specific set of preferred LQ levels used as indexes for the table. These values were derived following an increasing series with a factor of 1.585 similar to MIL STD 105D. The product of sample size, n, and LQ were made constant down any diagonal of the table, leading to constant acceptance numbers at the diagonals.

PROCEDURE

The procedure used in the generation of the extended LQ indexed plans are shown below:

(1) The target LQ levels in percent were set following the LQ series of the existing plans. From the lowest LQ level of 0.50% in the existing table, the extended LQ levels were derived by extending the LQ indexes 12 steps backward following the ratio of 1.585.

(2) The lot size and sample size structure were extended by following the increasing series from the existing tables, also using a factor of 1.585.

(3) The following pertinent values were calculated to complete the information about the extended plans:

(a) the exact Binomial P_α when the plan is applied to a series of lots in which p=nominal LQ;

(b) the exact Poisson P_α when the acceptance number applies to the number of defectives per 100 units and the plan is applied to a series of lots in which the number of defectives per 100 units is equal to the nominal LQ;

(c) the $p_{0.95}$ point, that is, the process average p for which the Binomial P_α in a continuing series of lots is 0.95; and

(d) the $p_{0.95}$ point, that is, the process average for the number of defectives per 100 units for which the Poisson probability of lot acceptance in a continuing series of lots is 0.95.

The results of these derivations were tables of sampling plans for the extended LQ series (in percent): 0.002, 0.003, 0.005, 0.008, 0.013, 0.020, 0.030, 0.050, 0.080, 0.130, 0.200, and 0.320. Table 1 shows the summary of the extended sampling plans and their properties.

THE DODGE-ROMIG LTPD INDEXED SAMPLING PLANS

Harold F. Dodge and Harry G. Romig established the Dodge-Romig LTPD system of sampling plans in 1923. The system is highly suitable for a producer's final inspection if the production is normally in control with known process average and the protection against lots of high fraction defective is desired. The LTPD is considered as the poorest quality level in an individual lot that can be accepted by the consumer. The optimum LTPD plan is determined by the plan which maintains LTPD protection with a probability of acceptance of 0.10, and which satisfies the condition for minimum Average Total Inspection (ATI).

The LTPD sampling schemes involve screening or 100% inspection of rejected lots. Plans are selected based on lot size and process average. The set of tables is entered at the specific value of LTPD and cross-referenced by the specific process average and lot size values. If the process average is not known, the largest value of process average appearing in the table is used until adequate information is known about the specific process.

Derivation

The Dodge-Romig optimum sampling plan is determined by minimizing ATI with respect to (c, n) under the restriction that $H(c, n, Np_2, N)$=0.10. LTPD is calculated using Type A Hypergeometric probabilities because LTPD is based on individual lots. However, the Poisson distribution is used to simplify the calculations since it was found that the sufficiently good approximations to the exact solution result in $p_2 \leq 0.10$, $p_1 p_2 \leq 0.50$, and $n/N \leq 0.10$. Under Poisson conditions, the specified consumer's risk of 0.10 means that $P(p_2) = G(c, m) = G(c, np_2) = 0.10$, which leads to the relation $n = m_{0.10(c)}/p_2$. ATI is denoted by $I(p_2)$ and is calculated using the formula

$$I(p_2) = n + (N - n)\ [\ 1 - G(c, np_2)]. \qquad \textbf{(1)}$$

Hald (1981) described the derivation of a compact tabulation of the Poisson solution as a function of the acceptance number, c. The function below was first considered.

$$I(p_1)p_2 = m + (M - m)\ [1 - G(c, \rho m)] \qquad \textbf{(2)}$$

where
p_1 = process average
p_2 = lot tolerance percent defective (LTPD)
m = OC
fractile = np_2
M = critical number of defects (or defectives) per lot = Np_2
$\rho = p_1/p_2$

As p_2 is constant, the optimum plan was found by minimizing the above function under the restriction $G(c, m) = 0.10$, and the solution was expressed in terms of c, m and M. Since m is a function of c, that is $m = m_{0.10(c)} = m_c$, the solution then involved finding the optimum relation between c and M. The second function, shown below, was then considered:

Table 1a. Derived simple sampling plans for the extended LQ series using the ISO 2859-2 sample size and LQL structure

Lot Size	Sample Size	LQL (in percent)												
		0.002	0.003	0.005	0.008	0.013	0.02	0.03	0.05	0.08	0.13	0.20	0.32	0.50
500,001 to 750,000	1,250	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	3 0.13 0.13 0.11 0.11
750,001 to 1,125,000	2,000	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	3 0.12 0.12 0.07 0.07	5 0.07 0.07 0.13 0.13
1,125,001 to 1,700,000	3,150	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	3 0.13 0.13 0.044 1.044	5 0.06 0.06 0.081 0.081	10 0.09 0.09 0.20 0.20
1,700,000 to 2,500,000	5,000	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	3 0.11 0.11 0.028 0.028	5 0.07 0.07 0.051 0.051	10 0.08 0.08 0.125 0.125	18 0.09 0.09 0.25 0.25
2,500,000 to 3,800,000	8,000	⇓	⇓	⇓	⇓	⇓	⇓	⇓	⇓	3 0.12 0.12 0.017 0.017	5 0.05 0.05 0.032 0.032	10 0.08 0.08 0.078 0.078	18 0.07 0.07 0.155 0.155	31 0.09 0.09 0.29 0.29
3,800,001 to 5,700,000	12,500	⇓	⇓	⇓	⇓	⇓	⇓	⇓	3 0.13 0.13 0.011 0.011	5 0.07 0.07 0.021 0.021	10 0.07 0.07 0.049 0.049	18 0.09 0.09 0.10 0.10	31 0.09 0.09 0.185 0.185	52 0.10 0.10 0.335 0.335
5,700,001 to 8,500,000	20,000	⇓	⇓	⇓	⇓	⇓	⇓	3 0.15 0.15 0.007 0.007	5 0.07 0.07 0.013 0.013	10 0.08 0.08 0.031 0.031	18 0.06 0.06 0.062 0.062	31 0.09 0.09 0.116 0.116	52 0.07 0.07 0.207 0.207	87 0.10 0.10 0.365 0.365
8,500,001 to 12,800,000	31,500	⇓	⇓	⇓	⇓	⇓	3 0.13 0.13 0.0042 0.0042	5 0.09 0.09 0.0081 0.0081	10 0.09 0.09 0.0195 0.0195	18 0.06 0.06 0.039 0.039	31 0.07 0.07 0.074 0.074	52 0.09 0.09 0.133 0.133	87 0.09 0.09 0.233 0.233	144 0.15 0.05 0.40 0.40
12,800,001 to 19,300,000	50,000	⇓	⇓	⇓	⇓	3 0.11 0.11 0.0027 0.0027	5 0.07 0.07 0.0051 0.0051	10 0.12 0.12 0.0125 0.0125	18 0.09 0.09 0.025 0.025	31 0.09 0.09 0.047 0.047	52 0.06 0.06 0.083 0.083	87 0.10 0.10 0.147 0.147	144 0.11 0.11 0.252 0.252	229 0.10 0.10 0.41 0.41

Key Acc=acceptance number; a=binomial P_a for process average % nonconforming/defective=LQ; b=Poisson P_a for process average of noconformities/defects per 100 units=LQ; c=process average percent nonforming/defectives for which binomial P_a=0.95; d=process average number of nonconformities/defects per 100 units for which Poisson Pa=0.95.

Table 1b. Derived simple sampling plans for the extended LQ series using the ISO 2859-2 sample size and LQL structure

Lot Size	Sample Size	LQL (in percent) 0.002	0.003	0.005	0.008	0.013	0.02	0.03	0.05	0.08	0.13	0.20	0.32	0.50
19,300,001 to 29,000,000	80,000	⇓	⇓	⇓	3 0.12 0.12 0.0017 0.0017	5 0.05 0.05 0.0033 0.0033	10 0.08 0.08 0.0076 0.0076	18 0.13 0.13 0.0155 0.0155	31 0.09 0.09 0.029 0.029	52 0.07 0.07 0.052 0.052	87 0.05 0.05 0.091 0.091	144 0.11 0.11 0.157 0.157	229 0.05 0.05 0.257 0.257	374 0.10 0.10 0.43 0.43
29,000,001 to 43,300,000	125,000	⇓	⇓	3 0.13 0.13 0.0011 0.0011	5 0.07 0.07 0.0021 0.0021	10 0.07 0.07 0.0049 0.0049	18 0.09 0.09 0.0099 0.0099	31 0.16 0.16 0.0185 0.0185	52 0.10 0.10 0.0335 0.0335	87 0.10 0.10 0.0585 0.0585	144 0.08 0.08 0.101 0.101	229 0.10 0.10 0.165 0.165	374 0.10 0.10 0.275 0.275	598 0.14 0.14 0.448 0.448
43,300,001 to 65,000,000	200,000	⇓	3 0.15 0.15 0.0007 0.0007	5 0.07 0.07 0.0013 0.0013	10 0.08 0.08 0.0031 0.0031	18 0.06 0.06 0.0062 0.0062	31 0.09 0.09 0.0016 0.0016	52 0.17 0.17 0.0207 0.0207	87 0.10 0.10 0.0367 0.0367	144 0.11 0.11 0.063 0.063	229 0.03 0.03 0.103 0.103	374 0.10 0.10 0.172 0.172	598 0.05 0.05 0.28 0.28	960 0.11 0.11 0.455 0.455
65,000,001 to 98,000,000	315,000	3 0.13 0.13 0.00044 0.00044	5 0.09 0.09 0.00085 0.00085	10 0.09 0.09 0.00195 0.00195	18 0.09 0.09 0.0039 0.0039	31 0.07 0.07 0.0074 0.0074	52 0.09 0.09 0.0132 0.0132	87 0.24 0.24 0.0233 0.0233	144 0.015 0.15 0.040 0.040	229 0.08 0.08 0.0651 0.0651	374 0.04 0.04 0.108 0.108	598 0.10 0.10 0.1775 0.1775	960 0.07 07 0.289 0.289	1549 0.26 0.26 0.472 0.472
98,000,001 to 145,000,000	500,000	5 0.07 0.07 0.00053 0.00053	10 0.12 0.12 0.00125 0.00125	18 0.09 0.09 0.0025 0.0025	31 0.09 0.09 0.0047 0.0047	52 0.06 0.06 0.0083 0.0083	87 0.10 0.10 0.0146 0.0146	144 0.33 0.33 0.0251 0.0251	229 0.10 0.10 0.041 0.041	374 0.10 0.10 0.0689 0.0689	598 0.02 0.02 0.112 0.112	960 0.11 0.11 0.182 0.182	1549 0.10 0.10 0.297 0.297	2436 0.10 0.10 0.471 0.471
>145,000,000	800,000	10 0.08 0.08 0.0076 0.0076	18 0.13 0.13 0.00155 0.00155	31 0.09 0.09 0.0029 0.0029	52 0.07 0.07 0.0052 0.0052	87 0.05 0.05 0.0091 0.0091	144 0.11 0.11 0.0157 0.0157	229 0.25 0.25 0.0257 0.0257	374 0.10 0.10 0.043 0.043	598 0.05 0.05 0.070 0.070	960 0.01 0.01 0.114 0.114	1549 0.10 0.10 0.1855 0.1855	2436 0.01 0.01 0.2945 0.2945	3919 0.10 0.10 0.477 0.477

Key Acc=acceptance number; a=binomial Pa for process average % nonconforming/defective=LQ; b=Poisson P_α for process average of noconformities/defects per 100 units=LQ; c=process average percent nonforming/defectives for which binomial P_α=0.95; d=process average number of nonconformities/defects per 100 units for which Poisson P_α=0.95

$$R(c, M) = M_c + (M - m_c)\, Q_c \quad (3)$$

where Q_c = producer's average risk = 1 - $G(c, rm_c)$.

The solution to the inequality $R(c, M) \leq R(c + 1, M)$ resulted in a formula for the derivation of M_c representing the critical number of defects or defectives per lot:

$$M \leq m_{c+1} + (1 - Q_c)(m_{c+1} - m_c)/(Q_c - Q_{c+1}) = M_c \quad (4)$$

This means that for $M < M_c$, the plan $(c, m_c/p_2)$ gives smaller ATI than the plan $(c + 1, m_c/p_2)$, whereas for $M > M_c$, the opposite is true. It was seen that $(c, m_c/p_2)$ is the optimum plan for all $M = Np_2$ in the interval (M_{c-1}, M_c).

Procedure

The procedure used in the generation of the extended LTPD plans followed the same procedure described by Hald (1981). Process averages and lot size ranges were set based on the expected levels or high precision processes. The steps involved in the determination of the extended LTPD plans are outlined below:

(1) For each lot size, $M_c = Np_2$ was determined.

(2) Using tables for LTPD sampling plans with consumer's risk at 10%, the optimum sampling plan was determined, in terms of c and m, corresponding to the computed M_c from step 1.

(3) Sample size, $n = m_c\ p_2$, was computed accordingly. The optimum sampling plan was defined in terms of (c, n) corresponding to a specified lot size interval and process average.

(4) ATI values for each plan were calculated, where ATI = n + (N - n)Q_c, and Q_c is the producer's average risk, 1 - $G(c, m_c)$ in percent. Q_c values corresponding to c and p are given by tables containing m_c and the producer's average risk.

The derivation procedure resulted in 14 single sampling tables for the target extended LTPD values, in percent 0.002, 0.003, 0.005, 0.008, 0.010, 0.050, 0.100, 0.150, 0.200, 0.250, 0.300, 0.350, 0.400, and 0.500. Table 2 shows the single sampling table for LTPD = 0.05%, together with the computed ATI values corresponding to the lower and upper range of process averages for each column. The complete set of tables can be obtained from the author's thesis report.

THE LOT SENSITIVE PLANS

These plans, as described by Schilling (1978), are based on the concept of acceptance with zero defects in the sample. It is applicable in general acceptance sampling and is particularly useful in compliance testing and safety-related testing. LSPs relate the sample size to lot size in a straightforward way and provides a minimum sample size for sampling applications. LSP plans provide LTPD protection to the consumer at the limiting fraction defective specified. The probability of acceptance for a lot having fraction defective equal to or worse than the specified limiting value is 0.10. Since acceptance is only when no defectives are found in the sample, the producer must produce at a quality level that is less than about 5% of the limiting level protected against by the plan. This is to assure that a reasonable small proportion of good lots will be rejected. Single sampling plans that require no defectives in the sample for lot acceptance should be used only when the state of manufacturing technology permits near perfect quality levels to be economically produced. Otherwise, the severity of the acceptance criteria would lead to a significant number of good lots being rejected.

Derivation

The values of $D = Np_1$, corresponding to the value of f, the fraction of the lot to be inspected were derived as described below. When $c = 0$, the Hypergeometric distribution is given as

$$p(0) = \frac{\binom{k}{0}\binom{N-k}{n-0}}{\binom{N}{0}} = \frac{(N-n)(N-n-1).....(N-n-k+1)}{N(N-1)...(N-k+1)} \leq \left(\frac{N-n}{0}\right)^k \quad (5)$$

for k defective pieces in the lot. This is equivalent to the f binomial approximation to the hypergeometric for an argument of zero. But $k = Np$, and since, $c =$

0, $P_a = p(0)$, if we let $f = n\,N$, then

$$P_a \cong [\,1 - n/N\,]^{Np} \quad (6)$$

$$\log P_a \cong (Np) \log (1 - f); \text{ or} \quad (7)$$

$$D = Np \cong \frac{\log P_a}{\log(1-f)} \quad (8)$$

which gives rise to the values on Table 3. Since $N = n\,f$, we have $D = (n/f)p$. Thus, $Df = np$ and for large lot sizes and relatively small values of p, sampling is characterized by the Poisson model. So for $f < 0.01$:

$np \sim 2.303$
$Df \sim 2.303$
$f \sim 2.303 / D$

where $np = 2.303$ has probability of acceptance 0.10 for the Poisson model at $c = 0$. Because of the nature of application in conformance and safety testing, i.e., large N and small p, the Poisson model is adequate for practical purposes, particularly when $np < 5$.

When rectifying inspection is applied to a series of successive lots, the quality outgoing from this procedure will be improved over the initial quality of the lots. The estimated maximum long-run average fraction defective that could result from rectifying inspection is given by the Average Outgoing Quality Limit (AOQL). This can be approximated for LSP plans by:

$$AOQL = \frac{0.3679}{n}\left(\frac{1}{f} - 1\right) \quad (9)$$

Procedure

The preferred LTPD values and lot size structure were obtained from the same extension series used in the Dodge-Romig LTPD indexed plans. The procedure for deriving the single sampling plans for the extended LTPD values followed the procedure outlined below:

(1) The lot size series and preferred LTPD values were specified.

(2) $D = N_{pL}$ was calculated.

(3) Table 3 is used to determine the value of f corresponding to the nearest value of D. f is the fraction of lot to be inspected. If $f < 0.01$, f is given by $2.303/D$.

(4) The sampling plan is given by
n = sample size = fN
c = acceptance number = 0

Fourteen tables containing the sampling plans based on LSP for LQ levels equal to the extended series of the Dodge-Romig LTPD indexed plans were generated. The tables also provide values of lot quality corresponding to different probabilities of acceptance used in the construction of the operating characteristic curves for the individual sampling plans. Table 4 shows the single sampling table for LQ = 0.05%. The complete set of tables can be likewise obtained from the final report of the study.

PERFORMANCE EVALUATION OF THE EXTENDED PLANS

The validity of the extensions made on the existing LTPD/LQ indexed sampling plans was evaluated using measures of performance that indicate level of protection offered by the sampling plan and the cost of inspection expressed in terms of number of samples or inspections required by the sampling plans. The level of protection was given by the Operating Characteristic (OC) curves and the Average Outgoing Quality (AOQ) curves. Cost of inspection was given by the Average Total Inspection (ATI) which was then used to evaluate the efficiency of the plan. The ensuing discussion follows that of Montgomery (1985).

Operating Characteristic (OC) curves

OC curves provide a measure of discriminatory power of the sampling plan by presenting the probability of acceptance of the lot versus the lot fraction defective. Generally, there are three specific points of interest on the OC curves, $p_{0.95}$, $p_{0.50}$, and $p_{0.05}$. These were calculated to illustrate the OC curves of the plans.

Table 2. Single Sampling Table for LTPD = 0.05%. Computed Average Total Inspection (ATI) for upper and lower process average for each column

Lot Size	Process Average 0 to 0.002% r = 0.040			Process Average 0.003% to 0.005% r = 0.060 r = 0.100			Process Average 0.006% to 0.008% r = 0.120 r = 0.150			Process Average 0.009% to 0.011% r = 0.180 r = 0.220		
	(c,n)	AT_L	ATL_U	(c,n)	ATI_L	ATI_U	(c,n)	ATI_L	ATI_U	(c,n)	ATI_L	ATI_U
5,000 - 10,000	.	.	.	.	.	.	.	.	.	.	.	.
10,000 - 20,000	.	.	.	(0,4606)	6,593	7,773	(0,4606)	8,323	9,103	(0,4406)	9,697	10,605
20,001 - 30,000	(0,4606)	.	6,840	(0,4606)	7,884	9,830	(0,4606)	10,738	12,024	(1,7780)	1,243	12,477
30,001 - 40,000	(0,4606)	.	7,720	(0,4606)	9,174	11,887	(0,4606)	13,153	14,946	(1,7780)	2,802	14,591
40,001 - 50,000	(0,4606)	.	8,600	(0,4606)	10,465	13,944	(1,7780)	11,171	12,698	(1,7780)	14,361	16,705
50,001 - 80,000	(0,4606)	.	11,240	(0,4606)	14,338	20,115	(1,7780)	13,580	16,193	(2,10644)	15,688	18,567
80,001 - 100,000	(0,4606)	.	12,999	(1,7780)	9,933	13,186	(2,10644)	13,068	14,879	(3,13362)	16,307	18,726
100,001 - 500,000	(1,7780)	.	13,155	(1,7780)	19,272	36,637	(3,13362)	17,812	22,665	(4,15988)	23,657	32,200
500,001 - 1,000,000	(1,7780)	.	18,615	(2,10644)	14,896	27,430	(4,15988)	19,012	23,568	(6,18550)	21,888	28,079
1,000,001 - 1,500,000	(1,7780)	.	24,075	(2,10644)	17,035	35,904	(4,15988)	20,548	27,447	(6,18550)	23,588	32,933
1,500,001 - 2,000,000	(2,10644)	.	12,373	(2,10644)	19,174	44,377	(5,18550)	20,585	24,702	(7,23540)	26,641	33,835
2,000,001 - 2,500,000	(2,10644)	.	14,059	(2,10644)	21,313	52,851	(5,18550)	21,098	26,255	(7,23540)	27,425	36,440

Table 3. Values of $D = Np_L$ corresponding to f

f	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.9	1.0000	0.9562	0.9117	0.8659	0.8184	0.7686	0.7530	0.6567	0.5886	0.5000
0.8	1.4307	1.3865	1.3428	1.2995	1.2565	1.2137	1.1711	1.1286	1.0860	1.0432
0.7	1.9125	1.8601	1.8088	1.7586	1.7093	1.6610	1.6135	1.5667	1.5207	1.4754
0.6	2.5129	2.4454	2.3797	2.3159	2.2538	2.1933	2.1344	2.0769	2.0208	1.9660
0.5	3.3219	3.2278	3.1372	3.0497	2.9652	2.8836	2.8047	2.7283	2.6543	2.5825
0.4	4.5076	4.3640	4.2270	4.0963	3.9712	3.8515	3.7368	3.6268	3.5212	3.4196
0.3	6.4557	6.5254	5.9705	5.7496	5.5415	5.3451	5.1594	4.9836	4.8168	4.6583
0.2	10.3189	9.7682	9.2674	8.8099	8.3902	8.0039	7.6471	7.3165	7.0093	6.7231
0.1	21.8543	19.7589	18.0124	16.5342	15.2668	14.1681	13.2064	12.3576	11.6028	10.9272
0.0	*	29.1053	113.9742	75.5957	56.4055	44.8906	37.2133	31.7289	27.6150	24.4149

Table 4. Sampling plans for limiting quality of 0.05% based on Lot Sensitive Compliance Plans (LSP)

Lot Sizes	Single Sampling Plan based on LSP		Values of submitted quality accepted with design and probabilities (quality as percent nonconforming)				
	n	c	0.95	0.90	0.50	0.10	0.05
5,000 to 10,000	3,700	0	0.0012	0.0027	0.0190	0.062	0.081
>10,000 to 40,000	4,400	0	0.0010	0.0023	0.0160	0.052	0.068
>40,000 to 2,500,000	4,605	0	0.0010	0.0022	0.0150	0.050	0.065

Average Outgoing Quality (AOQ) curves

AOQ represents the average fraction defective including both accepted lots and those rejected and screened. This value becomes meaningful when applied over a series of lots and when rectifying inspection is being carried out as part of the sampling procedure. The AOQ will depend on the average quality of submitted lots. Nearly all submitted lots will be accepted if their quality levels are good. If the quality levels of submitted lots are very poor, many lots will be rejected and screened, and the AOQ will be below the average submitted fraction defective level.

The formula for AOQ was developed by considering that in lots of size N, (a) there are n items in the sample which after inspection contain no defectives because all discovered defectives are replaced; (b) there are N-n items which also contain no defectives if the lot is rejected; and (c) there are N-n items which contain $p(N\text{-}n)$ defectives if the lot is accepted. Thus, lots in the outgoing stage of inspection have an expected number of defective units equal to P_α $(N\text{-}n)$. This is then expressed as an average fraction defective, AOQ, given by the formula: AOQ = $P_\alpha\, p(N - n) / N$. Since lot size becomes large relative to sample size, AOQ is sufficiently approximated by AOQ = $P_\alpha p$. The plot of the AOQs against the corresponding incoming lot fraction defective level gives rise to the AOQ curve. This curve illustrates the behavior of AOQ with varying incoming lot quality.

Average Total Inspection (ATI)

The ATI is directly related to inspection cost and is a function of submitted lot quality. It refers to the average number of pieces inspected per lot which includes those required for making the decision and those additional pieces required for 100% inspection of rejected lots. In single sampling, a sample of size n is always first inspected before reaching a decision on the disposition of the lot. Whenever a lot is rejected, the remaining N - n pieces will be inspected. This additional inspection is dependent on the incoming fraction defective and occurs in 1 - P_α proportion of lots submitted. ATI was calculated using the formula:

$$\text{ATI} = n + (N - n)\, Q_c \qquad \textbf{(10)}$$

where Qc = producer's average risk given by the Poisson distribution, that is, $Q_c = 1 - \textit{Poisson}\ (c,\ pm_c)$.

Efficiency of the plan, e(c, n)

Efficiency of the plan is expressed as the ratio.

$$\text{e(c, n)} = \text{ATI}\ (\underline{c}, \underline{n}) / \text{ATI (c, n)} \qquad \textbf{(11)}$$

where ($\underline{c}$, $\underline{n}$) is the notation used for the optimum plan, that is the plan that yields the minimum ATI among all plans satisfying $P(p_2) = 0.01$; and (c, n) is used to denote the plan that is in actual use.

Table 5. Tabulated values of OC and AOQ for LTPD = 0.05%, N = 1500,000

Sampling Plan	$P_{0.95}$	$P_{0.90}$	$P_{0.50}$	$P_{0.10}$	$P_{0.05}$
Based on ISO 2859-2 Sampling Plan is given by (3, 12500)					
p	0.010800	0.01386	0.02923	0.05292	0.06300
AOQ	0.010260	0.01247	0.01460	0.00529	0.00315
Based on LTPD Sampling Plan is given by (2, 10664)					
p	0.007500	0.01020	0.02490	0.04940	0.05990
AOQ	0.007125	0.00918	0.01245	0.00494	0.00299
Based on LSP Sampling Plan is given by (0, 4606)					
p	0.00110	0.00220	0.0152	0.0510	0.06500
AOQ	0.00104	0.00198	0.0076	0.0051	0.00325

Table 6. Comparison of ATI and efficiency of the sampling plans for LTPD = 0.05%, N = 1,500,000

Sampling Plan	ATI	Efficiency of Optimum Plan Against Other Plans			Percentage of Inspection Cost Savings by Using Optimum Plan (1 - Efficiency)*100		
		ISO 2859	LTPD	LSP	ISO 2859	LTPD	LSP
ISO 2859*	13,665	-	0.80	0.07	-	20.0	93.0
LTPD	17,035	0.80	-	0.09	20.0	-	91.0
LSP	197,625	0.07	0.09	-	93.0	91.0	-

Then 1 - e(c, n) gives the fraction of the actual inspection cost for lots of normal quality which could be saved by changing to the optimum plan.

An example

Evaluation of the performance of the three extended sampling plans generated from this study is shown below using a specific example. A production lot of size, N = 1,500,000 and LTPD = 0.05% was considered. The resulting sampling plans and operating characteristic values are shown in Table 5. The OC and AOQ curves are also presented in Figs. 1 and 2, respectively.

The most discriminating plan was given by the LSP-based plan that is attributed to the zero acceptance numbers. The least discriminating plan resulted from the ISO 2859-2 plans, which yielded the highest fraction defective level at the $p_{0.95}$ point. The LTPD-based plan gave similar discriminatory behavior as the LSP based plan. The AOQ curve was observed to rise at the beginning, pass the maximum point and then descend at the end. The maximum point of the curve corresponds to the AOQL value. AOQL was highest for the ISO 2859-2 plan, followed by the Dodge-Romig LTPD plan. Among the three plans, the LSP-based plan consistently yielded the lowest AOQL value.

Comparison of the extended sampling plans based on efficiency of inspection as given by ATI is presented in Table 6. Generally, the LSP-based plan demonstrated a large increase in ATI values at lot size equal to 1,500,000. The ATI of the ISO 2859-2 plan showed the

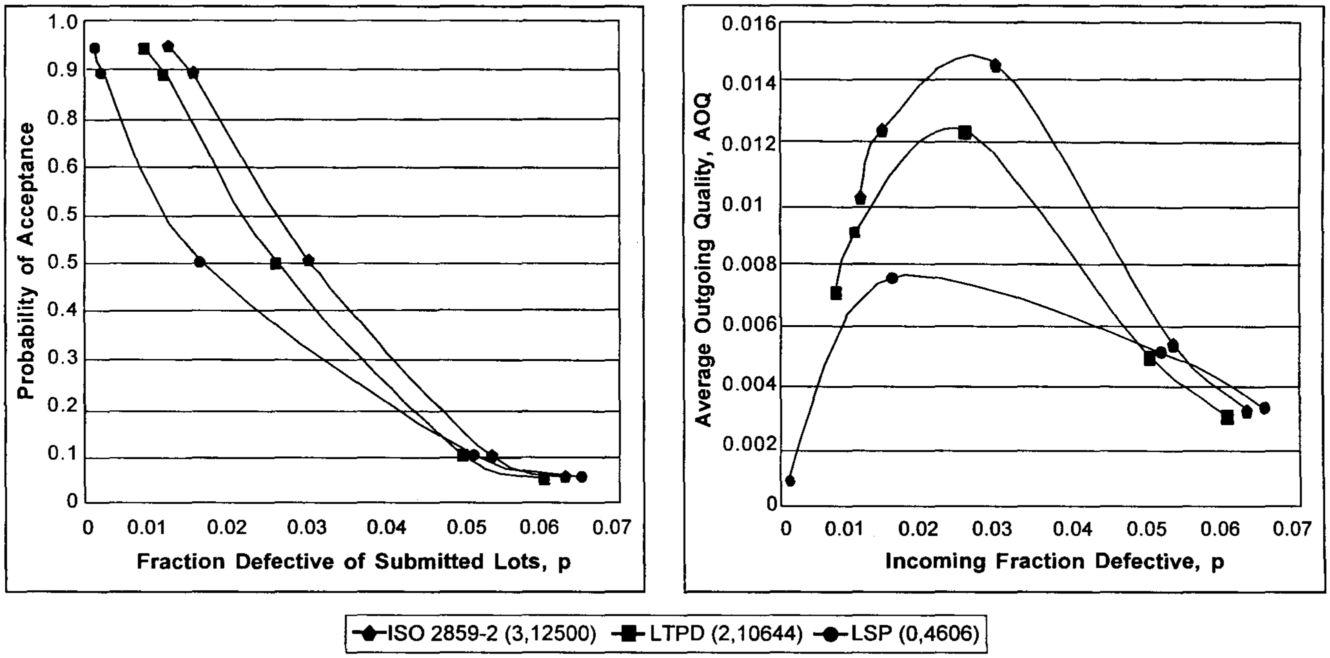

Fig. 1. OC Curves for LQ=0.05%, N=1,500,000

Fig. 2. AOQ Curves for LQ=0.05%, N=1,500,000

smallest magnitude of increase with increasing lot size. It was identified as the optimum plan, that is the plan with the lowest ATI value for the given LQ and lot size. The summary presented in Table 6 shows the calculated efficiency of using the optimum plan over the other types of plans. The corresponding value of 1 - e(c, n) for the identified optimum plan was also calculated. These value represents the percentage of actual inspection costs that could be saved by using the optimum plan over the other types of plans. The ISO 2859-2 was most efficient for N = 1,500,000, yielding the lowest ATI value of 13,665. It was observed that the greatest percentage of inspection cost savings would be gained by not using the LSP-based plan. At this lot size, the ATI values calculated from the LSP-based plan were the highest, indicating that in terms of inspection cost, this plan becomes uneconomical for large lot sizes.

CONCLUSION

Three existing LTPD/LQ indexed sampling schemes were extended to cover the limiting quality levels appropriate for high precision processes. The results yielded a series of tables providing single acceptance sampling plans for LTPD/LQ—values ranging from 0.002% to 0.50% or 20 ppm to 5000 ppm. The most discriminating among the three plans evaluated was given by the LSP-based plan which specified zero acceptance numbers. The ISO 2859-2 plan was the least discriminating, resulting in the highest observed AQL values. Best performance was shown by the Dodge-Romig LTPD plans which demonstrated the same level of discriminatory power as the LSP-based plan, but was not limited to the use of zero acceptance numbers. Efficiency was also demonstrated by the LTPD plans resulting from the condition that they should provide the required LTPD protection with minimum ATI. This study resulted in the identification of appropriate single sampling plans for use by manufacturing processes operating at very low levels of fraction defective.

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DEFINITION OF TERMS AND NOTATIONS

Terms

Acceptable Quality Level (AQL) is the maximum percent defective or the maximum number of defects per hundred units that, for purposes of sampling inspection, can be considered satisfactory as a process average.

Average Outgoing Quality Limit (AOQL) is the maximum ordinate of the AOQ curve that represents the worst possible average quality that would result form a recifying inspection program.

Lot Tolerance Percent Defective (LTPD or p_2) or Limiting Quality (LQ) is the poorest level of quality that the consumer is willing to accept in an individual lot.

Average Total Inspection (ATI) is the total amount of inspection required by the sampling program.

Notations

n	sample size
c	acceptance number
P_a	probability of acceptance
p_1	process average
p_2	Lot tolerance percent defective or limiting quality
p	p_i/p_2