# Grammar, Numerals, and Number Words: A Wittgensteinian Reflection on the Grammar of Numbers

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## ABSTRACT

What account of meaning is sufficient for understanding expressions? Using numbers and number-words as familiar forms of expression, I shall try to make an apology of Wittgenstein's treatment of meaning. My apology is of two parts. The first part attempts to address two questions: Firstly, how do we account for our knowledge of mathematical objects such as those of numbers, numerals, and number-words? Secondly, how do we account for the meaningfulness of their application outside the domain of mathematics?

As a reply to the first, I shall say no more than affirm, following Wittgenstein, the simple fact that our knowledge of mathematical objects such as those of numbers, numerals, and number-words is accounted for by how we learn a given language. As a reply to the second, I shall argue that said meaningfulness may be explained solely by an appeal to Wittgenstein's account of language-games and consequently of grammar. The second part puts forward, albeit in a general way, the idea that the correctness of a move in language is made possible not only by rules of grammar, as Wittgenstein puts it, but also by rules of grammatical salience which are also embedded in language itself.

*Keywords:* Grammar, grammatical salience, meaning, meaning as use, numbers, numerals, number-words, Wittgenstein

Let me begin with the common sensical fact that some of us, if not all, under relatively normal conditions, are familiar with numbers, numerals, and number-words. We know that the symbol for our *concept* of number is called numeral. Similarly, we hold that a number-word designates a number. One apparent reason here is our ability to use the symbols 0, 1, 2, 3 to 9. These symbols, we are taught, are fundamental to our knowledge of both cardinal and ordinal series of numbers. We are trained as well that in an ordinal series, 2 comes after 1 or that before I reach 7, I should have counted from 1 to 6 in order. Hence, we are able to say that 1 comes *first* and 2 follows *second*, and so on.

In the same way, we are trained, under relatively normal conditions, that in a cardinal series, the symbols 0 to 9 can function as numerical configurations for numerical quantities. Thus, we are able to say *5 books, 3 Coke bottles, 7 dwarfs*, and so on, without any doubt whatsoever as to their veracity. We judge them as correct applications of numbers, number-symbols, and number-words. This is how we are taught to use them. We may go as far as to say that we acquire mastery of the series of natural numbers by repetitive *copying* perhaps, or *pointing*, or *memorizing* their names and symbols (Wittgenstein, 2009, pp. 62-64, [§§ 143-150]).[1]

It is not surprising to say, thus, that how we are taught to use numbers in our language somehow suggests that their meanings are fixed and determinate. This is to say that using them in our language means that their specific contents, in a fundamental way, are essentially ascertainable to such an extent that they warrant the understanding of their uses the sort of meaning it ascribes to the speaker. Otherwise put, we ascribe to numbers, along with their use, a sense or meaning of which, when used in our language, yields a correct understanding, of which the following hold true:

  (1) "One" stands for (or means) one.
  (2) "1" stands for (or corresponds to) the number-word one.
  (3) "Two" stands for (or means) two.
  (4) "2" stands for (or corresponds to) the number-word two.

These examples are, evidently, fundamental truisms. The role they play in our language, however, is sufficient to warrant some basic rules or guiding principles for using them. Thus we say that we have understood an utterance of the sorts "Jim has 'two' motorbikes" if and only if we have understood as well how "two" is supposed to be used in our language according to a rule, with the supposition though that we also know what "Jim" and "motorbikes" stand for or mean. Similarly, we say that we understand the meaning of "three" in the sentence "there are 'three' empty Coke bottles in front of me" if and only if, in fact, there are "three" empty Coke bottles in front of me. The use of number-words and consequently our knowledge of them, thus, are constituted through a system of determined meanings on the basis of how we apply what we learned about them through our language.

Apparently, this way of understanding numbers permits us to embrace a number of possibilities concerning their applications. We say, for example, that they are adequate in giving descriptions, reports, or even perhaps performances as the case may be. Consider, for instance, the following sentences:

(1)  The earth has *one* moon.
(2)  Matter has *five* phases.
(3)  Do it *thrice*.
(4)  Always think *twice*.
(5)  Jim is his *first* born.
(6)  Six is 6.

Surely, the uses of numbers and consequently of number words here, whether adjectival, ordinal or otherwise, are sufficient to provide or perhaps impress ways of using them for giving descriptions of facts or state of affairs for that matter. This of course presupposes the idea that the system of numbers we know, as well as how we use them, are intelligible or are capable of being understood under relatively normal conditions. The point here is that the meaning of numbers, numerals, or number-words—how we understand the content it specifies—is countenanced by our commitment to the knowledge we have of them as having the sort of meanings they have as we learn and acquire them.

But suppose we come across the following set of remarks:

(a)  Manong, 'yung "isa" po, "isa."
(b)  Manong, 'yung "isa" po, "dalawa."
(c)  Manong, 'yung "isa" po, "tatlo."

How shall we make sense of them?[2] These remarks, certainly, contain number-words. In fact, they contain *three* number-words: "isa," "dalawa," and "tatlo." Remark (a) for instance seems to suggest that "isa" here is to be understood as specifying the content of which the meaning is to be taken as "isa." Remark (b), however, seems to suggest that "isa" is to be understood as specifying the content of which the meaning is to be taken as "dalawa." Finally, we have remark (c) which seems to suggest that "isa" is to be understood as specifying the content of which the meaning is to be taken as "tatlo." This way of putting it, however, seems inadequate to grasp whatever meaning each remark contains. Prima facie, as we pointed out earlier, there is nothing in our knowledge of numbers that permits us to commit ourselves to the use of "isa" as specifying a content whose meaning is to be taken as "dalawa." In the same manner, there is nothing in our knowledge of numbers that permits us to commit ourselves to the use of "isa" as specifying a content whose meaning is to be taken as "tatlo." Interestingly, there is also nothing in our language that allows us to use "isa," "dalawa," and "tatlo," under relatively normal conditions, in this way. If there is nothing in our knowledge of numbers and language that permits us to use them in such a manner, how then shall we go on? Shall we then

rule out these remarks as utterly senseless or absurd? Or shall we take them instead as new instances of learning how to use numbers and consequently of number-words in some special ways or other?[3]

This set of remarks, (let us call it the "isa-challenge") it seems to me, poses a difficulty to our ordinary understanding of numbers, numerals, and number-words. Although prima facie *trivial*, the difficulty is to show how it is possible, within our ordinary understanding, if not knowledge, of numbers and consequently of language, how these remarks may hold true, or better yet may be meaningful, in the same sort of way that they do under ordinary circumstances. One interesting issue here may be drawn from our obliviousness to correlate forms of expressions with other expressions *as though* they mean the same. We tend to overlook the possibility that these forms of expressions may have different meanings. Similarly, we fail to see that they can have different functions, if not roles, in our language. The crucial point, however, concerns the relation between what an expression means and how it functions in our language. Since our forms of expressions have both meaning and function (roles) potentials in our language, it follows that our failure to appreciate how said relation holds results inevitably in misunderstanding. Given that we normally use numbers and their signs in our daily activities, misunderstanding them is not an unlikely possibility.

Part of the difficulty here, I surmise, arises from the idea that the remarks themselves do not fall under what we consider *ordinary* or *normal cases* on one hand. While we are familiar, for instance, with the number-words used, what they really mean, and how they come to mean what they mean remain essentially ambiguous. The mature Wittgenstein (2009), for instance, remarks that "it is only in *normal cases* that *the use of a word is clearly laid out in advance for us*; we know, are in no doubt, what we have to say in this or that case" (p. 61e, [§142] italics added). Given the said remarks, it does not *prima facie* appear that we have a normal case. We may argue, *a la* Wittgenstein then, that our normal, if not *ordinary*, understanding of numbers, numerals, and number-words thereby loses its point.[4] On the other hand, part of the difficulty also arises because we "do[es] not have *an overview* (übersicht) of the use of words" as Wittgenstein puts it (2009, p. 54e, [§122]; 1975, p. 51). In other words, our understanding of the grammar of the words used is "deficient in surveyability" (Wittgenstein, 2009, p. 52e, [§122]). There is deficiency in surveyability if we fail to understand the connection of words in a given sentence. There is failure of understanding as well in the absence of *seeing* how each word is connected to other words. *Seeing* the connections or the lack of it, thus guarantees, *in toto*, our understanding of the language being used *as though* we "get[s] tools in the toolbox of language ready for future use" (Wittgenstein, 1974, p. 49).

The problem before us therefore is twofold and may be expressed as follows: Firstly, how do we account for our knowledge of mathematical objects such as those of numbers, numerals, and number-words? Secondly, how do we account for the meaningfulness of their application outside the domain of mathematics? As a reply to the first, I shall say no more than affirm, *a la* Wittgenstein, the simple fact that our knowledge of mathematical objects such as those of numbers, numerals, and number-words is accounted for by how we learn a given language. I take, however, as a fundamental Wittgensteinian proviso the idea that it is only outside mathematics that our knowledge of mathematical objects such as those of numbers has meaning (Wittgenstein, 1956, p. 257, [V, §2]). Concerning the second question, I shall argue that said meaningfulness may be explained by an appeal to Wittgenstein's account of language-games and consequently of grammar. In the end, I shall put forward, albeit in a general way, the idea that the correctness of a move in language is made possible not only by rules of grammar, as Wittgenstein puts it, but also by rules of grammatical salience which are also embedded in language itself.

This paper, thus, is structured as follows: The section "Numbers, Numerals, and Number-Words" touches on the question 'How do we account for our knowledge of mathematical objects such as those of numbers?' In this section, I shall try to explore first, however briefly, how *Logicism* of Gottlob Frege (1960b) accounts for numbers. Then, I turn to how the early Wittgenstein looks at numbers, without precluding, in effect, what he says about numbers, or what I take to be what he says about them, in his later philosophy of mathematics. The section "The Meaning of a Word is its Use in the Language" lays some groundwork considerations for responding to the question "How do we account for the meaningfulness of their application outside the domain of mathematics?" One way of framing the problem here is by asking: What account of meaning-intelligibility may be satisfactory for understanding a given use of language? Here, I explore Wittgenstein's treatment of language-games and form of life. I shall try to make an apology of Wittgenstein's idea that meaning may be explained by an appeal to language-games and consequently to grammar. The section "The Grammar of Numbers and Number-Words" provides a Wittgensteinian reading of the "isa-challenge" as it takes into account the insights reached in the preceding section. The last section, the concluding part, shall attempt to develop, in a general way, some remarks concerning grammatical salience. It shall, however briefly, put forward the idea that there are correct grammatical moves in language because there are rules of grammatical salience.

## NUMBERS, NUMERALS, AND NUMBER-WORDS

How do we account for our knowledge of mathematical objects such as those of numbers, numerals, and number-words?

Let me respond to this problem by asking initially the question "How do we learn numbers?" The way this question helps, to borrow from Wittgenstein, is analogous to the way the question "How do we use numbers?" helps us understand the role they play in our language.[5]

Asking initially the questions: "How do we learn numbers?" and "How do we use numbers?" is much more *at home* in our language than the way the questions "Are numbers real?" and "Do numbers exist?" strike us.

This grammatical distinction of our questions permits us to look at the problem before us from two different routes. We may, for our purposes, call the first route as primarily concerned with meaning, if not meaning-intelligibility, whatever that entails. It calls for an account, in some general way or other, of our understanding of numbers. The second route, however, is not principally concerned with meaning, but with ontology, let alone metaphysics, of numbers, whatever that entails. It commits us into assuming, if not believing, that mathematical objects such as those of numbers, numerals, and number-words can be known to be true, let alone real, in the same sort of way that we regard empirical objects as true and real. In other words, the distinction allows us to look at the question, either as requiring an account of the meaning-intelligibility of numbers as part of given language or an account of the truth-possibility of numbers as part of our world. In either way, said account must be such that it warrants the understanding a clear view of our knowledge of numbers, numerals, and number-words.

Allow me then to situate this concern within the larger tradition of inquiry in contemporary philosophy of mathematics. Although said tradition is generally divided into realism, intuitionism, and formalism, I shall limit myself to a variant of realism known as logicism. I shall pass over intuitionism for the simple reason that it lacks a general thesis to begin with.[6] Similarly, since formalism is least defined as a philosophy of mathematics, I shall say nothing about it.[7] Let me state at the outset, however briefly, some fundamental assumptions pertinent to our inquiry. For our purposes, I take the following as desiderata:

(1)  That some of us, if not all, are familiar with or are in possession of some knowledge about numbers.

    (2) That some of us, if not all, are familiar with some basic rules of arithmetic, such as those of addition, subtraction, multiplication, and perhaps division.

    (3) That some of us, if not all, know how to count because of (1) and (2).

These assumptions are crucial to our inquiry. To see where our problem rests, let us go back to our basic question: "How do we account for our knowledge of mathematical objects such as those of numbers, numerals, and number words?" Otherwise put, let us find out how philosophers of mathematics deal with the question: "What is a number?"

One prominent response here may be taken from what logicism says about the objects of its inquiry or, better, what I take to be what it says about numbers.[8] I take, however, the logicism propounded by Frege (1960b), although Bertrand Russell and Rudolf Carnap are also representatives of logicism. I shall, however, be brief.

Frege's logicism may be motivated, in part, as an apology for arithmetic, and in another part, as a reaction to the Kantian idea that mathematical truths are synthetic a priori (Cook, 2009, p. 15; Hintikka, 2009, p. 273; Weiner, 2010, p. 33). Kant lays down this claim upon the fact that mathematical cognition proceeds from the "constructions" of concepts (Kant, 1998, p. 630 [A713/B741]).[9] Frege (1960b), in contrast, considers the question of mathematical truths as dependent upon "[...] a judgement about the ultimate ground upon which rests the justification for holding it to be true" (p. 3). Whether or not mathematical truths are synthetic a priori or analytic is therefore grounded on the sort of proof that it demands. Frege (1960b), for instance, writes:

> The problem becomes, in fact, *that of finding the proof of the proposition, and of following it up right back to the primitive truths. If, in carrying out this process, we come only on general logical laws and on definitions, then the truth is an analytic one*, [...] *If, however, it is impossible to give the proof without making use of truths which are not of a general logical nature, but belong to the sphere of some general science, then the proposition is a synthetic one.* For a truth to be a posteriori, it must be impossible to construct a proof of it without including an appeal to facts, i.e., to truths which cannot be proved and are not general [...] But if, on the contrary, its proof can be derived exclusively from general laws, which themselves neither need nor admit of proof, then the truth is a priori. (Frege, 1960a, p. 4, italics added)

The demand for proof or justification, in Frege's view, sets the tenor for understanding mathematical truths as essentially analytic.[10] To show that this is the case, Frege

has to contend with the question whether or not mathematical truths (arithmetical propositions) may be derived from the general laws of logic or whether or not mathematical truths can admit of proof or justification without appealing to facts.[11] If they can be so derived, then, they are essentially analytic and if not, then, they are essentially synthetic (Shapiro, 2000, p. 109). This attempt at derivation, if not reduction, of mathematics to logic, constitutes the main strategy of his logicism.[12]

The demand for *analyticity* of arithmetical truths, similarly, on the basis of the laws of logic, sets as well the tone for Frege's treatment of the nature of numbers. Stewart Shapiro (2000), for example, explains: "since he also held that arithmetic and real analysis are analytic, he believed that every truth about the natural numbers and every truth about the real numbers is knowable" (p. 109). How then are numbers known?

Frege's strategy, of course, is essentially to derive a definition of number on the basis of logic, i.e., from simple logical laws capable of proving the truths of arithmetic. The definition demanded here, Joan Weiner (2010) explains, must be such that it "preserve(s) whatever conceptual content is inherent in our pre-systematic views about arithmetic" (p. 37).[13] Frege (1960a), however, issues the reminder that

> [a] definition of a concept (of a possible predicate) must be complete; it must unambiguously determine, as regards any object, whether or not it falls under the concept (whether or not the predicate is truly assertible of it). [...] the concept must have a sharp boundary (Frege, 1960a, p. 159 [§56]).

How does Frege define number then? Frege's initial definition of number is predicative. He says that "the content of a statement of number is an assertion about a concept" (Frege, 1960b, p. 59, [§46]).[14] What this means is that when the number *one* is used, as in the statement, "the earth has *one* moon," something is predicated of the concept *moon of earth* — that *something* falls under it. Or again, if the number *zero* is used, as in "Venus has *zero* moons," something is predicated of the concept *moon of Venus* — that *nothing* falls under it. On this reading, the *concept of n number* is taken as a *second-order* concept which may be predicated to *first-order concepts*, where first-order concepts are concepts that hold of *objects* (Cook, 2009, p. 19; Maddy, 1990, p. 83).[15] What this amounts to thus may be expressed as follows: "The number which belongs to the concept F is the same as that which belongs to the concept G" (Frege, 1960b, p. 73, [§62]).

Frege (1960b), however, admits that this sort of relation is not a definition of number. It does not, in Frege's view, lead to an understanding of the concept of

number *per se*. It only fixes the sense of the phrase 'the number *n* belongs to' instead of the number *n* itself (Frege, 1960b, p. 68, [§56]). To define the *concept of number*, Frege (1960b) says, we need a definition of which the *number* appears as "a self-subsistent object that can be recognized as the same again" (p. 68, [§56]).[16]

Frege's attempted solution rests on finding a definition of *equinumerosity*. How do we know whether collections are *equinumerous*? We say that they do if they are of the same members. In other words, if there is a one-to-one correspondence between the members for each collection. For example, a glass for each bottle of Coke, if there are exactly two of those objects. Or a book for each student, if there are exactly two of those objects. To say that there is one-to-one correspondence between them is to say that the glasses are *equinumerous* with the bottles of Coke, or the books and students are equal in members.[17] Shapiro (2000) notes thus: "Frege showed how to define *equinumerosity* using only the resources of (so-called 'higher-order') logic, without presupposing natural numbers, or the notion of number generally" (p. 110).[18] On the basis of this formulation, Frege (1960b) arrives at the following definition: "The number which belongs to the concept F is the extension of the concept *equal* to the concept F" (pp. 79-80 [§68]).[19]

The early Wittgenstein's account of numbers, in contrast to Frege, is entirely different.[20] On one hand, Wittgenstein denies that numbers are self-subsistent objects. On the other hand, he rejects the idea that numbers are extensions of concepts that say something about the world. Instead, Wittgenstein considers number as a formal concept.[21] As a formal concept, a number can only be shown in the symbol for the object itself. In other words, it can only be presented in logical symbolism by a variable (Wittgenstein, 1955, p. 85, [4.1272]) and its value by the object that falls under it (Wittgenstein, 1955, p. 85, [4.127]).[22] Thus, any proposition or statement containing numbers is always expressed through the numerical sign that signifies it.[23] The numerical sign or the symbolism itself contains the actual correlation between the symbol and what the symbol signifies (Wittgenstein, 1975, p. 124, [§100]). For example, the proposition: "three students raised the flag" already contains the *concept* of number by using the word "three"—the concept "number" is itself already given.

Numbers, therefore, are always expressed in symbols or variables that signify them rather than in classes or in functions in the manner of Frege and Russell *as if* numbers *were* proper names.[24] What they mean, however, is always given by the rules of their use (Glock, 1996, p. 267). The question thus as to whether or not there are two, three or four objects or there are infinitely many objects is therefore *un-sayable*. But they can be made known through number-schemata—"names with

different meanings" (Wittgenstein, 1955, p. 141, [5.535]).[25] It is thus senseless to say that "1 is a number" as much as it is also senseless to say that "*a* is an object" (if *a* were the name of an object)—for what are they but what the *signs* or *names* signify?

Number as a formal concept, however, is closely associated with Wittgenstein's notion of formal series. To say that "it does" means that the properties belonging to formal concepts are internally related—they belong to symbolisms. A central concept here though is his treatment of operation. Max Black (1971), for example, explains that "Wittgenstein's emphasis upon operations seems best viewed as a *way of rendering prominent the associated rules* for the construction of complex symbols" (p. 258, italics supplied). For example, what is emphasized in the notations $(2 + 2 = 4)$, $(1, 4, 9, 16, 25, ...)$, $\{[p \supset q] \bullet p\} \supset q$ and even in the definition of numbers given in *Tractatus*, $(x = \Omega^0 x; \Omega'\Omega^v x = \Omega^{v+1} x)$ are the *associated rules* of operations that govern them, thereby indicating that the one and the other are *internally related* (Wittgenstein, 1955, p. 115, [5.2-5.21]).

In Wittgenstein's (1955) view, "the operation is the expression of a relation between the structures of its results and its base" (p. 115, [5.22]). Thus, the sum 4 is said to be internally related to 2 and 2 through the "+" sign. Similarly, that "36" is the next number following the series of numbers $(1, 4, 9, 16, 25...)$ expresses how the law of a series relates one number with another number (Wittgenstein, 1955, p. 83, [4.1252]).[26] The same holds true for 'q'. Precisely because it results in an operation, it "gives prominence to the rule for expressing one symbol as a function of another – or what comes to the same, the 'internal relations' of things *correlated by means of the function*" (Black, 1971, p. 260, italics added). The same may be said concerning the definition of number.

The idea here is that certain expressions may be constructed from other expressions on the basis of the associated rules of any given operation. In *Philosophical Remarks* for instance, Wittgenstein (1975) notes that since arithmetic is the grammar of numbers, "kinds of numbers can only be distinguished by the arithmetical rules relating to them" (p. 130, [§108]). The assumption here is that "the result of an operation can be the base of that very operation" (Wittgenstein, 1955, p. 115, [5.251]) provided that such related rules are given prominence. In *Notebooks* again, Wittgenstein (1961) remarks that "the concept of the operation is quite generally that according to which signs can be constructed according to a rule" (p. 90e, [22.11.16]). But for the signs themselves to be constructed according to a rule, there has to be structural similarity (Wittgenstein, 1961, p. 90e, [23.11.16]. Here, structural similarity is understood as similarities in *form*, because "*operation* and

*form-series* are equivalent" (Wittgenstein, 1961, p. 81e, [17.8.16]; 1955, p. 115, [5.23 – 5.232]).[27]

Numbers as formal concepts therefore make "sense" only within the context of propositions. Outside a given proposition, numbers mean nothing. Or better, it is only within the rules of logical symbolisms that numbers and their variable names have meaning.[28]

The mature Wittgenstein (2009), on the contrary, abandons this view and settles for the idea that numbers form a family (p. 36e, [§67]). One pertinent reason here is Wittgenstein's (1956) belief that our knowledge of numbers such as those of cardinal, ordinal, rational, or irrational numbers, is a "product of technique, of rules" and therefore their meaning "lies in our doing [the technique]" (p. 232, [IV, §15]).  In other words, our knowledge of numbers has meaning *only* in so far as they are part of such technique. Wittgenstein (1956) says thus: "it is only in mathematics that mathematical signs have meaning" (p. 274, [V, §16]).

It is not surprising, therefore, to say that our knowledge of numbers is largely a product of these conceptions. Whether or not we look at numbers as *extensions* of concepts as in Frege's view, or as a *formal concept* having a general form as in Wittgenstein's, our knowledge of numbers is learned from this frame. While we may choose to deny that this is the case, our first encounter with numbers suggests otherwise. We are taught that numbers are essentially stable and uniform such that when used, whether in language or in arithmetic, they warrant the understanding the sort of intelligibility they ascribe—the form of their expressions carved from rules of their use.  Interestingly, this form of expression provides us with the logic for viewing numbers and understanding them, to such extent that it also fixes and determines the meaning and the application of numbers in our language. Thus, when we use numbers, numerals, or number-words, we are able to grasp the meaning that their contents specify.  Within this frame,  significantly,  there is no question as to how numbers ought to be understood.

We may,  however,  object to this view and ask whether this logic of viewing numbers holds for all cases. In other words, whether it is still possible to grasp what numbers mean in the same sort of way when applied to, for example, special circumstances or not-so normal cases. A case in point is the example provided at the beginning of this paper—the "isa-challenge."  To wit:

    (a)  Manong, 'yung "isa" po, "isa."
    (b)  Manong, 'yung "isa" po, "dalawa."
    (c)  Manong, 'yung "isa" po, "tatlo."

The difficulty here is aggravated by the recurrent uses of the number-word "isa," not to mention the seemingly ambiguous applications of the number-words "dalawa" and "tatlo." It is tempting to view this example in the same sort of way that they are understood within the logic of numbers we know. One apparent reason here perhaps is the idea that the "form of expression" they exert on us, by virtue of their common forms, is most commonly taken as indicative of shared meaning (Hacker & Baker, 1985, p. 20). The so-called common forms that these numbers convey are thus taken at face value, as if they warrant the meaning of the words themselves.

However, careful reflection shows that this is not necessarily the case. Although we may grasp what "isa," "dalawa," and "tatlo" mean here, the question whether or not they mean the same or different remains. We may go as far as to affirm that what is here called "isa," "dalawa," and "tatlo" are similar to how we know and use them, yet this is something different. But that is as far as we can go.[29] We reach the end of the road, so it seems.

Similarly, we may say, as much as we like, that these are elliptical formulations of a much fuller construction of conversations; that we are not really talking about number-words, but something else. That may be true. We may, however, ask: What is it in the conversations that allows for the variations in formulations? Is it not possible that what we think we understand about an expression arises from our misleading associations of forms of expressions with the different regions of our language? (Wittgenstein, 2009, p. 47e, [§90]) Here, the notions "sameness" and "difference" are insightful, while the notions "look" and "see" are enlightening. Incidentally, the questions "How do we get out from here?" or "How shall we go on?" become pressing. Thus, we are left with the question: What do the remarks mean?

We may, on the other hand, flesh our way out by looking at the actual workings of the number-words here. While there is a clear prejudice for not doing so—and Wittgenstein (2009) understands that removing it is difficult (p. 116e, [§340])—it will certainly dispense the nimbus surrounding our language. By "looking and seeing" at the actual workings of language, we may, however, "let the fly out of the fly-bottle" (Wittgenstein, 2009, p. 110e, [§309]) since that is where our language is at home. But how can we possibly see the actual workings of our language? Wittgenstein's (2009) curative cue that "the meaning of a word is its use in the language" is here a form of discerning counsel (p. 25e, [§43]).

## THE MEANING OF A WORD IS ITS USE IN LANGUAGE

Our attempt to account for our knowledge of numbers, numerals, and number-words in the preceding section is hardly satisfactory. Although we are given some basic suggestions as to what numbers are, we seem to have ignored the possibility of how numbers may not be when used for instance outside their given domain. We may ask: How do numbers, numerals, and number-words mean when used in *this* or *that?* Shall we take them as different? Or shall we look for the meaning? We may reply in the following manner: Why not? But that is not a guarantee that we will find the answer we wish to obtain. Perhaps, we crave for meaning too much that we tend to overlook that the "use of language is an act" (Lee, 2001, p. 50). Is it not possible that the meaning we wish to obtain rests upon the act and not upon the language that describes it? How then shall we make sense of the remarks alluded to elsewhere? Perhaps one way of framing the question is this: What account of meaning-intelligibility is sufficient to understand a language? The succeeding discussion shall suggest an account of meaning-intelligibility following Wittgenstein's therapeutic reminder that "the meaning of a word is its use in the language" (Wittgenstein, 2009, p. 25e [§43]).

Let me, however, situate this matter within the larger perspective of the problem of meaning in the contemporary philosophy of language. To begin with, talk about meaning does not always involve talk about context. However, any talk of meaning is surely a talk about language. To talk about the problem of meaning is therefore to talk, in a general way, about language itself. How then shall we talk about language?

Referentialist philosophers such as Bertrand Russell, Rudolf Carnap, J. S. Mill, G. Frege, and the early Wittgenstein among others, for example, hold the general thesis that language is used to talk about things. The meaning of our expression is explained by what it refers to or stands for in the world. In this sense, the words we use function like names that stand for something. How we understand meaning is therefore explained by how words are associated with things in the process. We say, for example, that we understand what "dog," "horse," and "table" mean primarily because these words are associated with dog, horse, and table. Similarly, we say that we understand what a sentence means because we know the referents of its component words. On this view, meaning is identified with reference.

Ideational theories of language in the tradition of John Locke and Herbert Paul Grice, among others, in contrast, hold the view that the meaning of our expression is

mental and thus may be explained by the ideas that give rise to them or by the effects that they elicit from the hearers. Grice (2001), for example, remarks: "*what a speaker means* is to be explained in terms of the effect which he intends to produce in an actual or possible hearer; and *what a sentence in a language means* is to be explained in terms of directives with respect to the employment of that sentence, in a primitive (basic) way, with a view to inducing in a hearer a certain kind of effect" (pp. 68-69). In other words, the meanings of our expressions may be understood by considering (a) the utterer's meaning, (b) the sentence-meaning, and (c) the word-meaning, or by considering speaker-meaning and timeless meaning and from there deriving an understanding of meaning itself in terms of its effects.[30] In this view, meaning is reduced to the speakers' intentions, beliefs, thoughts, and psychological state.

Still, there are those who believe that language is used to do things. They argue, for example, that it is not simply *used* as a way to describe, name, or refer to something in the world. J. L. Austin (1962), for example, defends the idea that the use of language is essentially *performative,* whether explicit or implicit. To say that a use of language is performative is to say that it is meant to bring about the performance of an action instead of being just the simple act of saying something.[31] Similarly, John Searle (1969) holds that the use of language is an act. It is not the symbols, words, or sentences that matter in language, but rather "the production or issuance of the symbol or word or sentence in the performance of the speech act" (p. 16). In this view, meaning is a function of use and may be discerned on the basis of what speakers or users of language do with words and sentences.

The mature Wittgenstein, on the contrary, is critical of any theory of meaning or of language. One apparent reason here is his commitment to the view that the primary task of philosophy is no more than to describe the actual workings of our language— that it must not interfere with it (Wittgenstein, 2009, p. 55e, [§124]). Problems of meaning and understanding arise primarily because we tend "to sublimate the logic of our language" as though we have a clear view of how the different regions of our language function (Wittgenstein, 2009, p. 48e, [§94]—which results inevitably in the idling of language itself (Wittgenstein, 2009, p. 56e, [§132]). A second reason is his commitment to the view that the aim of philosophy is clarity— that it must "marshall recollections for a particular purpose" (Wittgenstein, 2009, p. 55e, §127]) so that we can have a clear overview of language. In this sense, the goal of philosophy is essentially therapeutic (Hacker, 1996, p. 111), and that can only be done by "assembling reminders" or "methods" by way of looking and seeing, or by way of taking a wider look at the different regions of our language (Wittgenstein, 2009, p. 36e, [§66, §340]; 1956, p. 127, [§6]).

How then shall we look at language? A good place to begin, it seems to me, is to take a wider look at the role of context in our language. The context, as it were, provides the background conditions within which words acquire meaning or sense. It lays down the boundary to ascertain the meanings of words. For example, the use of words "foul" and "fault" acquire sense and meaning only within certain contexts. One does not use 'fault" to describe an error in baseball or basketball; one uses "foul" instead. Similarly, one does not use "foul" to refer to any violation of the rules in either table tennis or tennis; "fault" is the more appropriate word. To a greater extent, it also lays down as well certain arbitrary rules of language that direct how words themselves are supposed to be used. Wittgenstein's examples like "slab," "pillar," "beam," and "block" are illustrative of this point. Understanding what "slab," "pillar," "beam," and "block" mean is given by the context within which they were uttered. The context thus lays the boundaries of the use of language.

The later Wittgenstein talks of these boundaries in essentially the same way. In *Philosophical Investigations*, Wittgenstein (2009) remarks: "whether a sign is a word or a sentence depends on the situation in which it is uttered or written" (p. 28e, [§49]). How a sign is used, *inter alia,* is therefore determined by the context or by the linguistic circumstances that surround the sign itself. Outside a given context, Wittgenstein (1958) argues, the sign is, *in ipso,* "utterly dead and a trivial thing" (p. 4).[32] The whole operation of the sign is, *ceteris paribus,* dependent on its context. Thus he notes: "the sign (the sentence) gets its significance from the system of signs, from the language to which it belongs. Roughly, understanding a sentence means understanding a language" (Wittgenstein, 1958, p. 5).

A corollary notion to the aforementioned is the idea that a word has meaning only within the context of a proposition. Elsewhere, Wittgenstein (1975) remarks that "if we say 'a word has meaning in the context of a proposition,' then that means that it's only in a proposition that it functions as a word, and this is no more something that can be said than that an armchair only serves its purpose when it is in space" (p. 58, [§12]). Equally, Wittgenstein (2005) asks: "[C]an one understand something other than a proposition? Or, conversely: Doesn't it only become a proposition when one understands it? So: Can one understand something other than *as a proposition*?" (p. 2e).

The underlying notion here is the idea that the sense of a word or the meaning of a sentence is always accompanied by its context of significant use.[33] Whereas, we may, in an essential way, know what the component words mean or what the sentence means, we cannot ignore the circumstances within which they are actually used. The context of significant use, as it were, provides the ramparts for appreciating

the sense or the thought that words or sentences express. On one hand, words and sentences can have different contexts of use and therefore can have different senses or meanings. We may, for example, grasp what they mean in one context, but this is no warrant that they mean the same in another context. In other words, they (words and sentences and the like) are always interwoven with context and environment. The context of use contributes to how words are used as well as to how sentences are understood. Alfred Sidgwick (1895), for example, remarks that "words and propositions, like everything else, exist not in an average or a fixed environment but in numbers of special and changing environments which affect their character" (p. 284). On the other hand, the context also provides the possibilities of different uses such that it allows for the variations of meaning. Interestingly, it makes explicit that meaning is not confined solely to words and sentences in isolation. Rather, meaning is discerned through an appreciation of the particular circumstance or linguistic context in which words and sentences occur. The context of use, in this sense, not only brings about the possibility of understanding what words and sentences mean but also provides the basis for distinctions, sameness as well as differences in uses, thereby delineating what potential meanings may be elicited from them.[34]

Indirectly, this reechoes what Gottlob Frege (1960b) remarks in *The Foundations* concerning context.[35] Frege (1960b) issues the following reminders:

(1) Always to separate sharply the psychological from the logical, the subjective from the objective;

(2) Never to ask for the meaning of a word in isolation, but only in the context of a proposition;

(3) Never to lose sight of the distinction between concept and object. (p. xxii)

For Frege, the meaning of a word is determined by the part it plays in a given proposition.[36] How a word is used in the proposition shows how it is to be understood. There is no atomistic meaning of a word. The context carries the elements through which we determine the meaning of a word. Otherwise, if the meaning of a word were to be sought outside its context, then, there lies the danger of seeking the meaning elsewhere, and thus one is forced to determine the meaning of a word in terms of "mental pictures" or "individual acts of minds," which for Frege (1960b) is a clear violation of the separation between the logical and the psychological determinations of meanings (p. xxii).

This is not to say, nevertheless, that the meaning of a word is determinable *solely* through context. This is not necessarily the case.[37] To confine meaning *only* to context is to fixate, if not deflate, the likelihood to compose new sentences or new propositions. Rupert Read (2000), for example, remarks: "Frege's context principle is irreconcilable with compositionality; it makes our understanding of new sentences completely mysterious" (p. 77). Peter Michael Hacker (1996) in the same way argues that compositionality in the manner of Frege, (a) "mistakenly assumes that the distinction between sense and nonsense are drawn once and for all by reference to circumstance-invariant features of type sentences, rather than being, in many *different* ways, circumstance-dependent," and (b) "disregards the very different *uses* to which sentences of the same form may be put" (p. 105, italics Hacker's).

An exception perhaps to this context principle is Wittgenstein's maxim that "the meaning of a word is its use in the language."[38] Consider the following passages:

> For a large class of cases of the employment of the word "meaning" – though not for all – this word can be explained in this way: the meaning of a word is its use in the language.  And the *meaning* of a name is sometimes explained by pointing to its *bearer*. (Wittgenstein, 2009, p. 25e, [§43], italics Wittgenstein's)

> …a meaning of a word is a kind of employment of it. For it is what we learn when the word is incorporated into our language (Wittgenstein, 1969, p. 66 [§61]).

> Understanding a word may mean: *knowing* how it is used; *being able to* apply it (Wittgenstein, 1974, p. 47, italics Wittgenstein's).

> But if we had to name anything which is the life of the sign, we should have to say that it was its *use* (Wittgenstein, 1958, p. 4).

*Prima facie*, the slogan "the meaning of a word is its use in the language" may seem to provide the ramparts through which problems about meaning may be easily responded to. One obvious reason here is the connection between language and context. Since a given context provides the background condition for the use of language, it seems but logical to suppose that the basis for meaning rests upon the context. Is it not for this reason that some remarks are out of context? In the same way, given that the background condition presents the range of possible contexts for the use of language, it follows that it also presents an accounting of potential meanings for specific linguistic utterances.[39] Consider, for instance, what potential meanings are suggested by the remark "ang haba ng araw, a" in the following contexts:

(a)  Having a vacation,

(b)  Cleaning one's room,

(c)  Disagreement with someone, and

(d)  Being alone.

It may be noted of course that there is nothing wrong in fact with drawing a parallel between the meaning of a word and its use.[40] Sometimes, one really grasps the meaning of a word *at a stroke* on the basis of how the words are used (Wittgenstein, 2009, p. 59, [§139]).[41] The problem, however, arises, as Hacker (1996) puts it, when one "identifies the meaning of an expression with its use" (p. 125).[42] At one point, there is a fundamental difference between the "meaning of a word" and the "use of a word." While it is possible that meaning and use are closely connected, it does not necessarily follow that they mutually imply each other. Within the grammar of language, there is a constant opposition between "words" and "meanings."[43] On one hand, the quest for meaning is, to some extent, identified with: (a) what the expression refers to, (b) the ideas that expressions evoke, and (c) responses and dispositions that expressions produce through their utterances.[44] On the other hand, the quest for meaning is faced with the difficulty of (a) determining "what is and what is not to count as revealing the use of a term" and (b) identifying "how meaning is to be analyzed in terms of use" (Alston 1963, pp. 107-108). At another point, although it is a truism that every difference in meaning is a difference in use, it is quite impossible *to determine in advance* whether or not the difference in use is indeed a difference in meaning (Wittgenstein, 2009, pp. 156-157, [§§555-557], italics added). Hans-Johann Glock (1996) thus suggests that the term "use" has to be unpacked in order to see "the aspects of use that are relevant to meaning" (p. 378).

As it were, the term "use" may have different meanings.[45] Firstly, one may refer to what Gilbert Ryle (1953) calls "stock use". Ryle frequently talks about the "ordinary" or "stock use" of words as opposed to the non-stock use of words. Here, the words "ordinary" or "stock", Ryle (1953) asserts, "can serve merely to refer to a use without describing it" (p. 169). The words "book," "pencil," "paper," "fish-knife," and so on, thus have stock uses. In other words, their meanings are almost shared by everyone because their stock uses are open to view. Secondly, one may also consider what J. L. Evans (1953) calls syntactical or grammatical use. Words are meaningful only when they are combined with other words to form phrases or sentences.

Although words themselves have meanings, they do not occur in isolation. Most often, words are combined with other words to form phrases or sentences to be meaningful. Here, it may be helpful to consider the distinction between (a) words and (b) sentences. While learning the meaning of words may seem initially plausible,

learning the meaning of a sentence, as one may initially hope, is not.[46] Learning what a sentence means demands a more elaborate process than learning the meaning of words, since the latter does not include the possibility of learning the meaning of a sentence. Finally, one may look upon the "semantical use" of words. This suggests that an expression of some kind has a meaning or at least seems to indicate, to denote, and even to connote, that an expression means something. The assumption here is that while words are syntactically ordered, they do not have meanings unless they are interpreted on the basis of some semantical rules. Here, the semantical rules provide the possibility of correlating the words with other expressions whose meanings are already known (Abelson, 1957, p. 53). Understanding what an expression means then, for example, is explained by reference to (a) the meanings of words contained in the expression and (b) how the expression is constructed on the basis of words that form it—its semantic structure. In other words, the meaning of an expression rests entirely upon, or is determined by, the meaning of its parts and how the parts are syntactically combined.[47] The relevant question, however, is whether this notion of semantical use is sufficient to decipher the meaning of a given expression.

Be that as it may, Wittgenstein (2009) admits that words have different meanings depending on their use (p. 157e, [§558]). The word "is" in "Princess *is* sick," for example, is different from the "is" in "two plus two *is* four." The "is" in the former functions as a *copula* connecting the subject and its compliment whereas the "is" in the latter signifies or indicates, so it seems, equality. How a word is used, thus, serves as a determining condition for understanding the meaning of a word. The "use," so to speak, teaches the meaning of a word. In *Philosophy of Psychology*, Wittgenstein (2009) writes:

> Just don't think you knew in advance what 'state of seeing' means here! Let the use *teach* you the meaning (p. 223e, [§250], Wittgenstein's emphasis).

> Let the use of words teach you their meaning. (Similarly, one can often say in mathematics: let the *proof* teach you *what* was being proved.) (p. 231e, [§303]).

Interestingly, while it is plausible that the use of a word may possibly teach its meaning, it remains crucial to ask what is it in "use" that allows the likelihood of understanding the meaning of a word or the meaning of a given sentence. In other words, how is a shared understanding of meaning on the basis of use possible at all? Or what account of meaning-intelligibility is sufficient to understand a language?

One apparent response here is provided by Wittgenstein's account of language-games and form of life. Wittgenstein (2009) views language-game as any system

of linguistic transactions, like the primitive language of giving or obeying *orders* (p. 14e, [§23]), or any form of language with which one learns the uses of words (Wittgenstein, 1958, p. 17), or as the *totality* of language (Wittgenstein, 1958, p. 108) by which human beings communicate with one another in concrete human activities. Entrenched within this language-game are certain kinds of beliefs, values, attitudes, patterns of behavior, fundamental ideas as well as forms of practices, which may be viewed as pertinent bases for common human understanding. Precisely because these things are entrenched within the language-game that one plays, one may say that said language-game constitutes as well a shared form of life. Thus, the language-game or the totality of language, as it is played, is understood as already *given*. It is part of the background condition as an account of one's form of life. In Wittgenstein's (2009) words, "what has to be accepted, the given, is – one might say – *forms of life*" (p. 238e, [§345], Wittgenstein's italics).[48] The given, Hans Johan Glock (1996) claims, may also be associated with culture or social formation (p. 125). Similarly, in *The Blue and Brown Books,* Wittgenstein (1958) suggests that learning a language is like learning a culture (p. 134). The phenomenon of language, in this case, is not to be taken simply as consisting of meaningful signs or symbols that one learns. Instead, it *ought* to be understood, as Marie McGinn (1997) suggests, as something that is already embedded, if not inherent, in "the lives of those who speak it" (p. 44).

A key feature of this concept, nevertheless, is the idea that the use of language is always an act of participation or sharing in a specific human form of life; that independent of any human form of life, any use of language, is utterly senseless and dead. The use of language acquires meaning or sense, as Barry Stroud (1996) asserts, only because of the "distinctive role of the expression in all those human activities in which it is or might be employed" (p. 301). Utterances or words such as *makibaka, magsolian na tayo ng kandila, bahala na ang nasa taas, konting tiis, pasensya ka na anak, gawat ang buhay*, including unique gestures of approval or disapproval, respect and reverence, and so on, are better understood when communicated out of common human experiences and shared principles and values. Interestingly, they are also better understood when the speakers themselves are exposed to, or familiar with, the same contingent conditions—whether social, political, or economic conditions. Each utterance or word draws its meaning from the actual usage that it has in a given human form of life.[49] The human form of life or the human activity, *in ipso*, one may suppose, gives *life* to the very signs and symbols, words and sentences, remarks and utterances, of the language that one uses— the language of everyday (Wittgenstein, 2009, p. 54, [§120])—for where else can one learn the use of language except from everyday language? (Wittgenstein, 2009, p. 57, [§134])[50] Likewise, the so-called signs and symbols of mathematics—including its various operations, its conceptions of numbers and numerals, as well as its peculiar language

of symbolisms — ought to be understood within this context. Wittgenstein (1956) argues that "it is essential to mathematics that its signs *should also be used in mufti. It is their use outside mathematics*, in other words the *meaning* of the signs that makes the sign-game mathematics" (p. 257, [V, §2], italics added).

The use of language, thus, is learned and developed within the confines of a particular form of life. When Wittgenstein says that one learns a language as one learns to speak it, one also learns, implicitly, the uses of words and how they come to have meanings. When one, similarly, explains how words are used, such as "color-words," "number-words" or how the terms "unethical," "dogs," "demonstrations," "rallies," and so on are used — in the language-game one plays — then, it becomes clearer how words come to have meaning or what makes it possible for language to acquire meaning.[51] Consequently, one also learns what the dictum "*speaking* of a language is part of an activity, or of a form of life" means (Wittgenstein 2009, p. 15, [§23]). To the extent that such is the case, the speaking of language unveils the range of possibilities through which said form of life is communicated and made understood. The various uses of words or the combinations thereof achieve their purposes and functions within the context of a given social environment — where social processes, personal ideals and intentions, political events, and other human activities or perhaps even collective desires for both destruction and war are shaped, molded, and articulated as components of one's form of life — the unquestionable given that holds these pieces in place. True enough, "words have meanings only in the stream of life" (Wittgenstein cited in Garver, 1996, p. 151).

Similarly, a deeper understanding of words or any linguistic act, as it were, is achieved through an act of participation or sharing in the form of life within which said linguistic act is performed. Participation in a given human form of life may circumscribe, if not perhaps define, the various uses of words and how sentences function differently at different times. This assumes nonetheless that one is already exposed to, or acquainted with, several features of language which said form of life carries. Being exposed to or familiar with the features of language provides the signposts or compasses by means of which one sees the whole of how language is supposed to work.[52] At one point, it emphasizes the fact that the said form of life is the basis of understanding the use of language. At another point, it also discloses the possibility of achieving a more profound understanding of words or any linguistic act, for that matter, through the said participation itself. One sees, after a closer examination, that the said form of life brings together certain relevant features or categories for giving descriptions, whether of values or of attitudes, or perhaps experiences, which may be shared by anyone, including agreements as to what constitutes truth or falsity; there is agreement, for instance, "not in opinions, but

rather in form of life" (Wittgenstein, 2009, p. 94e, [§241]). One learns the value, if not the meaning, of an act, a state of affair, or an attitude perhaps, through these relevant features, and from there one shapes a possible understanding of language which one can share with another.  Still, one also becomes familiar with the different regions of language and its corresponding concepts, as well as how said regions or concepts may be appropriately applied, for instance, in giving descriptions, articulating an order, or perhaps narrating an event. To some extent, the act of providing a description of an act may be viewed as well as a vehicle within which one discovers how words themselves function or how they are supposed to be understood. Consequently, one realizes that there exists, implicitly, a proportional relationship between one's embedding in any given form of life and one's use of language. One discovers that one's understanding of language is as deep as one's participation in the said form of life—as one gets deeply involved, so one's grasp of language goes deeper.

One may thus imagine, if not at all consider, how the remark "manong, yung isa po, isa" or how the word "isa" in the following human activities may be made understood:

    (1) Riding a jeepney, and paying one's fare;
    (2) Buying a guyabano at a fruit stand;
    (3) Identifying an item;
    (4) Describing an order;
    (5) Liquidating a receipt;
    (6) Giving information;
    (7) Issuing a prescription; and
    (8) Reiterating a reminder.

To the unfamiliar mind, or to someone who is not exposed to any of these language-games, the remark may seem certainly troubling. Of course, it remains possible that one understands what the remark is supposed to mean, but that does not warrant that one knows which way to go or that one finds one's way about. In other words, it does not warrant that one plays the same language-game. Here, the notions "sameness" and "difference" are certainly instructive.

Knowing which way to go or finding one's way about demands adherence, if not obedience, to certain patterns or standards of correctness to which one's understanding of expressions may conform. Since part of the essence of language is that it arises or grows out of the foundations of the human form of life, it follows that it is also part of such essence, logically speaking, that there be ways in which expressions or uses of words may be properly evaluated.[53] Here, it is crucial to

consider that these so-called patterns or standards of correctness emerge out of the shared human form of life; as words are used within it, so the use itself establishes regularity in the way words are employed and understood in a given language-game. The use of words, implicitly, fixes the meaning of words, including the range of what may be considered appropriate responses.[54]

Understanding what is said thus and for what purpose something is said depends on what sort of responses are given. The remark "*Anak, gulatin mo nga ang tulya*," for instance, when properly understood, yields a response of sorts that is adequately related to the language-game being played.[55] Zosimo Lee (2001), for example, notes that there is *understanding* when one can continue the language-game or when one understands *what is being done* with the expression. Understanding an expression thus requires, among other things, "some way in which those expressions are used, some regularities or general practices to which an individual speaker's performance can conform or fail to conform" (Stroud, 1996, p. 303).

The possibility, therefore, of having a shared understanding of meaning is grounded on the firm foundation of the human form of life. While it is possible that words may have meanings in isolation from the human form of life, they are senseless and dead apart from the concrete reality of various human activities that language-users share—it is a language-game that cannot be played. Attempting to grasp the meaning of words or language itself independent of its social context is like saying that "one plays patience by oneself" (Wittgenstein, 2009, p. 96e, [§248]). The learning of language may never be private. It springs and grows within a given form of life, as Wittgenstein asserts.

One may suppose, then, that the problem in understanding the possibility of shared meaning or understanding how language itself is possible arises because of one's penchant to understand language in abstraction or to view it in isolation from its "actual employment." This penchant for abstraction blurs or disregards the essential features relevant to understand the actual workings of language—to isolate language from its home or from where it ordinarily lives is, in effect, to turn it idle.[56] Marie McGinn (1997) aptly explains:

> The tendency to isolate language, or abstract it from the context in which it ordinarily lives, is connected with our adopting a theoretical attitude towards it, and with our urge to explain *how* these mere signs (mere marks) can acquire their extraordinary power to mean or represent something. Wittgenstein's aim is to show us that in this act of abstraction we turn our backs on everything that is essential to the actual functioning of language; it

> is our act of abstracting language from its employment within our ordinary lives that turns it into something dead, whose ability to represent now cries out for explanation. (McGinn, 1997, p. 44)

The problem, I surmise, arises because one attempts to explain what language is *as if* it possesses an essence that one may later on discover—"*the essence is hidden from us*" (Wittgenstein, 2009, p. 48e, [§92]) or the idea that there is a hidden logical syntax of language and the world (Hacker 2012, p. 3).[57] Wittgenstein (2009) argues that the attempt to explain language or the craving to look for the essence of language independent of its social context is brought about by the illusion that "what is peculiar, profound and essential to one's investigation resides in its trying to grasp the incomparable essence of language" (p. 49e, [§97]). It fails, on the contrary, to take into account that there is nothing essential in language except (a) how language itself is employed within the grammar of everyday human activity as a natural component of a form of life and (b) how words themselves are combined to describe the sort of activity being communicated. Wittgenstein (2009) thus claims that "the more closely we examine actual language, the greater becomes the conflict between it and our requirement" (p. 51e, [§107]).

How language is employed, then, is what is essential. Its actual usage is its essence. The actual usage, it seems, constitutes, as far as Wittgenstein is concerned, the primordial use of language—"if the words 'language', 'experience', 'world' have a use, it must be as humble a one as that of the words 'table', 'lamp', 'door'" (Wittgenstein, 2009, p. 49e, [§97]). Its distinctive role in the human form of life is the "proto-phenomenon"—"*the language-game that is being played*" (Wittgenstein, 2009, p. 175e, [§654], italics Wittgenstein's) where everything that one needs to understand "lies open to view" (p. 55e, [§126]). In other words, everything that one needs to know about language is already given by its use or its grammar. Garver and Lee (1994) thus note, "the method of describing the forms of language, or the uses of language, therefore, is grammar" (p. 153). Understanding language, then, or how a shared understanding of meaning is possible, is essentially grounded on understanding the grammar of language, since, as Wittgenstein (2009) argues, "essence is expressed by grammar" (p. 123e, [§371]).

The "isa-challenge," therefore, may be made understood by paying attention to its grammar. Understanding its grammar here may eventually show the idea that the use of number-words in civil life does not always follow a fixed algorithm of meaning in the same sort of way that they do in the realm of arithmetic. Like any sign or symbol, the use of number-words, too, is dependent upon the human form of life which provides the basis for distinguishing the meanings of utterances.

Wittgenstein (2009) thus says, "[E]very sign *by itself* is dead. *What* gives it life? – In use it *lives*. Is it there that it has a living breath within it? – Or is the *use* its breath?" (p. 135e, [§432]).

## THE GRAMMAR OF NUMBERS AND NUMBER-WORDS

We have, in the preceding section, laid some groundwork considerations sufficient to account for the determination of meaning. We made an attempt to ground meaning intelligibility by defending the idea that meaning is discernible on the basis of use and that use is hinted by grammar. In this section, we shall try to explore the idea of grammar further and how grammar contributes to the understanding of expressions.

Consider again, the following remarks:[58]

    (a)  Manong, 'yung "isa" po, "isa";
    (b)  Manong, 'yung "isa" po, "dalawa"; and
    (c)  Manong, 'yung "isa" po, "tatlo".

Initially, it may be supposed that the remarks themselves are simple, akin to Wittgenstein's example of a primitive language of giving orders and buying apples. Secondly, although they are considered as such, they do not fall within what Wittgenstein considers a normal case—they do not present a clear overview of how the words themselves are used. Finally, the remarks themselves are complete because they are uttered within their own context and are actually spoken to communicate a thought. The remarks, thus, are examples of the use of language in *actual use* and not simply a concatenation of words understood in abstraction. The problem, however, is how to understand what they mean since (a) the use of number-words does not cohere with how one normally uses numbers and (b) the use of number-words does not cohere with how one ordinarily understands numbers.

It may be noted nevertheless that the remarks themselves are language-game specific. They are remarks uttered for a specific purpose within a particular social context. The meanings, therefore, are already embedded in the actual use—except that one has to look and see at the actual use of language itself.

Wittgenstein's concern with the actual usage of language is hinted at the opening of *Philosophical Investigations*. Wittgenstein (2009) asks, "But what is the meaning of the word 'five'? – No such thing was in question here, only how the word 'five' is used" (p. 6e, [§1]). The same hint is also provided in another important work. In *Philosophical Grammar*, he remarks that when talking about numbers or any statement

that involves numbers, one is not "looking for a definition of the concept of number," instead, one is concerned primarily with "an exposition of the grammar of the word *number* and of *numerals*" (Wittgenstein, 1974, p. 321). One may then ask, what is grammar?

Wittgenstein (2005) views grammar as a (normative) description of language (p. 146e; 2009, p. 146e, [§496]). As a normative description, it states the rules for use of language. The rules determine the conditions of the use of words and constitute its sense and meaning – it is grammatical rules that determine meaning (Wittgenstein, 1974, p. 184, [§133]).[59] Meaning, thus, is how a word is assigned in language (Wittgenstein, 2009, p. 18e, [§29]) or the *place* of a word in grammar (Wittgenstein, 1974, p. 59, [§23]). It is therefore laid down in grammar (Wittgenstein, 2005, p. 51e).

Given that Wittgenstein considers actual linguistic usage a language-game, grammar essentially defines the role of rules that govern these language-games, including how they are to be played. Grammar, thus, Wittgenstein (1974) maintains, "has somewhat the same relation to the language as the description of a game, the rules of a game, have to the game" (p. 60, [§23]).[60] For example, just as the rules of chess constitute the game of chess and the kind of moves allowed to play it, grammar similarly constitutes the actual usage of language and the kind of moves it allows to use it. In the same way, just as the game of chess assigns a specific move to each piece thereby defining what moves are possible, grammar similarly sets the parameters and limits of words, thereby defining what moves in language are allowed.[61] In addition, since Wittgenstein (2005) considers grammar a "ledger of language" (p. 48e), it determines, on one hand, how language itself is to be explicated and, on the other hand, how the particular grammar of a word is established in advance (p. 51e).

Interestingly, although grammar is understood as such, its rules are nevertheless arbitrary (Wittgenstein, 2005, p. 186; 2009, p. 146e, [§497]) and therefore not accountable to any reality (1974, p. 184, [§133]) nor justifiable by any reference to it (2005, p. 148e).[62] Grammar, however, given sufficient surveyability, may characterize "the way we represent things, how we look at matters" (2009, p. 54e, [§122]). For instance, one ought not to say that there are numbers "isa," "dalawa," "tatlo," or "apat" because there are numbers "isa," "dalawa," "tatlo," or "apat," as if that *were* similar to the statement that one ought to say that "princess is sick" just because "princess is sick."[63] The number-grammar or the use of words is not reflected in the nature of numbers or the words one uses (Wittgenstein, 1967, p. 65e, [§357]), akin to the idea that essence resides in the nature of things. On the contrary, it is the rules for the use of number-words or words, as it were, that determine in

advance what may be properly understood as numbers or what may be properly labeled as sick.[64] Likewise, it is the rules for the use of words that determine *in toto*, the nature of numbers (Wittgenstein, 2009, p. 123e, [§§371-373]). It is possible, nonetheless, to have a system of numbers which is different from what one has learned except that one does not just go that way (Hacker, 2009, p. 343).[65]

Turning then to the remarks earlier given, one may suppose that the "isa-challenge" involves basic linguistic shifts in both concepts and meanings. Baker and Hacker (2009) remark that generally, "[any] transposition of a concept from one system or language-game into another involves a shift in meaning" (p. 18). This, one may presume, is an evident truism. While number-words are ordinarily applied within mathematics, the fact that they are also employed in civil life means that, to say the least, their meanings and applications similarly involve shifts in both meaning and use. The shift in meaning, thus, is determined by the use or by how the concepts are applied in civil life. The use, in other words, specifies the grammar of the word.

Remark (a) "Manong, 'yung 'isa' po, isa," is a case in point. It extends the use of the number-word "isa" outside the domain of mathematics. The use of the number-word "isa," thus, is extra-mathematical. One may suppose, apparently at least, that the use of the number-word "isa" here is not a mathematical expression. Neither is it an expression having a mathematical function. The number-word "isa" is simply used as part of ordinary things or in civil life. In *Tractatus*, Wittgenstein (1955) writes:

> Mathematical propositions express no thoughts. In life it is never a mathematical proposition which we need, but we use mathematical propositions only in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics. (In philosophy the question "Why do we really use that word, that proposition?" constantly leads to valuable results.). (p. 169, [6.21– 6.211])

To say that it is simply used as part of ordinary things or in civil life is to say that, *inter alia*, as Wittgenstein (1961) says, "[one uses] numbers to apply to ordinary things, etc., which in fact says no more than that numbers occur in our quite ordinary sentences" (p. 67e, [20.6.15]. The point, thus, is a general one. The use of the number-word "isa" in civil life forms part of the background conditions of language. The fact that numbers are applied in civil life indicates one's knowledge for numerical concepts. This point is significant for two reasons. At one point, it shows one's ability to play these concepts along with other concepts in the background conditions. At another point, it shows the stability of one's knowledge for numerical concepts.[66] Here, the concepts of play and stability are grammatically important.

They form the basis for understanding the shared forms of representation, common articulations, and communal practices that constitute a human activity.

The use of the number-word "isa," for example, may be explained on this basis. When one says "manong, 'yung 'isa' po, 'isa'," it is plausible to suppose that what one actually says is (a) a form of representation, (b) an articulation, and (c) a practice, that constitute a shared human activity. Of course, it is legitimate to ask questions of the sorts (a) to what is it a form of representation of, (b) what does it articulate, and (c) in what sense does it constitute a practice? To these questions, one may easily respond in the following manner. As a form of representation, it is simply the way one represents a thing or an act, or a state of affairs. For example, when someone utters the word "isa" in a given context F, the utterance *in ipso* becomes an element in one's language—a means of representation. By uttering the word, one has given that word a role in one's language-game. To say that it has a role in one's language-game is to say no more than what the role plays in the language.[67] What does it articulate then? Simply put, it is a precise articulation of what it represents. Given *this* context, the use of the number-word "isa" functions as an elliptical articulation, thereby suggesting that the very act of paying the fare or the fare itself may be so articulated in diverse ways. As far as the second question is concerned, one certainly may say "'yung isa po, isa" instead of the more elaborate sentence "'yung isang pasahe po, para sa isang pasahero."  The use of the number-word "isa," therefore, is a shortened form of the latter.[68] Contrast it for instance with the following expressions expressing the same meaning:

(a)  Manong, heto po ang bayad ko.
(b)  Boss, bayad po.
(c)  Manong, pasahe po.
(d)  Manong, 'yung isang pasahe po, para sa isang pasahero lang.

Surely, when one says, "'yung isa po, isa," given *this* context, what one really means is any of the above sentences. It is a cause of wonder of course in what sense remark (a) means the same thing as these sentences. Why should not it be different? If by the sentence "manong, 'yung isa po, isa" means any of the above, why does it have to be articulated in a different way? Part of the reason is grounded on shared practices. Since the context is very familiar, in this case, one is exposed to many forms of representation and articulation. The remark in (a) is no exception. It is part of the shared practices in the context of paying one's fare. That is simply how the fare is said or articulated—it is something that is already given or something that is already laid out in advance.

An interesting feature, on the other hand, of this form of representation is the use of the number-word "isa." One may surmise that its role goes more than mere representation of the fare in the precise sense possible. One may further surmise that the use itself is a rule for using the number-word. It is a grammatical move and as such, it lays down the rule for the use of the word "isa"—as a form of representation in *this* context. The use of the word "isa" here, however, has two levels of meaning. The "isa" in the first may be understood as a grammatical marker, while the "isa" in the second may be understood as a grammatical contrast. As a grammatical marker, the word "isa" establishes the convention of using the word "isa"—hence, as a representation. It refers to the fare itself as its referential marker. Instead of saying "bayad po," one uses "isa" instead. The word "isa" thus makes the fare itself evident and the act of paying the fare more intelligible and precise. It thus marks the rule for representing the fare in the context of the number-word "isa." In other words, the number-word "isa" is designated as having *this* meaning and functions as *a* form of representation in *this* language-game. Consider for instance what may be the meaning of the word "isa" in the following human activities or the remark itself for that matter:

(1) Buying a guyabano at a fruitstand;
(2) Identifying an item;
(3) Describing an order; and
(4) Liquidating a receipt.

This is to say that the word "isa" is already imbued with a certain meaning that the above activities do not have. One may say, in this regard, that the word "isa" in activities (1, 2, 3, and 4) is not used in the same sort of way as the word "isa" in remark (a) of the "isa-challenge" such that the word "isa" determines in advance all other expressions of fare in this given context—that it means this rather than that. Consider, for example, the following thought-formulation: Let N refer to the number word "isa" to mean the shortened form of "bayad po." Let F refer to the context for uttering "bayad po." Let G refer to a non-empty domain containing members Ni, Nii, Niii and Niv as forms of expression.

N is the language of F if and only if:

There exists in F a convention that if N (F) = {Ni ... Niv}, then, any utterance in G in context F, amounts to N only if any member in G is reducible to or is identical with N under context F.[69]

As a grammatical contrast, however, the word "isa" contrasts the scope of what is marked. It establishes the convention of how to describe the numbers that the fare

covers. Its use, as it were, lies open to view. One immediately understands what the second "isa" means and the role that it exactly plays within the given context. Of course, there are evident truisms in this language-game. There are norms that have to be followed, except that in this scenario, these norms are rather described in terms of number-words. One such norm is the obligation to pay one's fare. Another is to do it with honesty. Thus, one may say, instead, *manong, 'yung isa po, isa*, which may be understood as a variant of "manong, bayad po." In this way, remark (a) functions in the same sort of way as Wittgenstein's "slab" example. While it is possible to think of the remark in (a) as a sentence, the convention forbids that it be taken as an assertion or statement of the number-word "isa." As Barry Stroud (1996) remarks, "[it has] a life or meaning in a way that no other sounds or marks do in that [context]" (p. 302). To some extent, that is exactly the point that is being described. Words have meanings only within the context of human activities and their having "this" role in this specific language-game. Interestingly, when one says *"manong, 'yung 'isa' po, isa,"* one relies heavily on one's mastery to understand this language. Since Wittgenstein (1958) argues that sentences have "life" only as part of the system of language (p. 5), it necessarily follows that sentences are to be understood within that language or the language that gives the sentences "life." It constitutes, as it were, a move in the language. In so doing, however, one has to be a master of a technique of understanding language (Wittgenstein, 2009, p. 87e, [§199]). This technique, one may surmise, is learned and developed through one's embedding in language; in other words, embedding in the "customs," "usages," and "institutions" of language (Wittgenstein 2009, p. 87e, [§199]). Barry Stroud (1996), for instance, defines this technique as "the technique of acting and responding linguistically in appropriate ways, of being a human language-speaker, and so being capable of the sorts of activities and reactions that language makes possible" (p. 302). Thus, the remark *"manong, 'yung 'isa' po, isa"* may be taken as a custom of sorts or a certain way of using language—that within the context of *this* human activity, it is already an institution, of which the use of number-words lies open to view and may not be mistaken to mean another. This is to say that I need not justify the use of number-words in this way, [since], "I have reached a bedrock, and my spade is turned ... this is simply what I do" (Wittgenstein, 2009, p. 91e, [§217]).

One may suppose, thus, that this is what is meant by Wittgenstein's (2009) remark that language is founded on convention (p. 120e, [§355]). The remark *"manong, 'yung 'isa' po, 'isa'* is part of such convention. How one pays one's fare, or how one does it, regardless of how one says or articulates it, forms part of the convention that one learns. To some extent, this constitutes the bedrock of language. One learns this convention by being exposed to it. One understands what it means only by doing it. One imbibes its practice by participating in its activity. It is simply what

one learns in the process. The way it is done and how it is developed, including the agreement for its possibility as a shared practice, is part of one's form of life or part of one's human activity—"what has to be accepted, the given, is – one might say – *forms of life*" (Wittgenstein, 2009, p. 238e, [§345]).

One may ask, however, does one know whether one follows the convention properly? Similarly, one may also ask whether what one says or does is in accord with a rule.[70] The answer is, no! The decision involved in following a convention, or in this case, in paying one's fare through remark (a) does not rest on one's choice. The fact that it is already a custom or a way of life means that it is already a rule with a definite meaning. In other words, "[one] no longer has any choice because the rule is already stamped with a particular meaning," it runs through the whole of space— "when I follow [a] rule, I do not choose. I follow the rule blindly" (Wittgenstein, 2009, p. 92e, [§219]). Thus, in following a convention, it is not rules that matter, but form of life.

Remark (b), however, takes a different direction. The grammatical marker "isa" here allows for a shift in meaning. Instead of referring to the grammatical contrast "isa" in (a), it points to "dalawa" *as though* "isa" may be so referred. What is interesting, however, is the move it allows in the language such that it transposes the meaning of "isa" into "dalawa." Given the said context, this move nonetheless is not a trivial thing. The scope of the rule may very well be extended and stretched provided that it does not violate the convention established by the grammatical marker for the number-word "isa" laid down earlier. Thus, the use of the number-word "isa" in remark (b) follows the same convention alluded to earlier, except that it now includes a wider coverage. As a grammatical contrast, the use of the number-word "dalawa" establishes the convention that whenever the marker exceeds "isa," a fitting description is "dalawa." The use of "dalawa" thus also forms part of *this* human activity. It is part of the context as one of its regulative rules – which may be diverse. To some extent, it is precisely this feature that allows for the possibility of transpositions in meaning. Since the rules are established to regulate the context, they may be formulated in diverse ways. This of course is not the same as what constitutes the context—this belongs to constitutive rules. Unlike regulative rules, constitutive rules are not formulated in the same way that regulative rules are. Imagine what may happen, for instance, if the rule for the use of the number-word "isa" in *this* context is violated. Since the number-word "isa" is laid down as a convention to refer to paying one's fare, one ought not to disobey it. Disobeying the rule or breaking it is tantamount to not engaging in the activity itself.[71] Thus, between the grammatical marker and the contrast, the former takes pre-eminence— it is necessarily prior to the latter. It may be said nonetheless that for any form of

expression involving the use of the number-word "dalawa" in *this* context, in so far as its marker remains the same, it may be included as a possible meaning of the grammatical contrast "isa" under *this* context. Consider, for instance, the extended version of the thought-formulation given earlier. To wit:

> N is the language of F if and only if:
>
> There exists in F a convention that if N = $\{Ù^0{}^\prime N = N, Ù^{+1+0}{}^\prime N = Ù^\prime N, Ù^{+1+1+0}{}^\prime N = ÙÙ^\prime N\}$ where N (F) = $\{Ni \ldots Niv\}$, then, any utterance in G in context F amounts to N only if N is reducible to or is identical with N within the range of N = $\{Ù^0{}^\prime N = N, Ù^{+1+0}{}^\prime N = Ù^\prime N, Ù^{+1+1+0}{}^\prime N = ÙÙ^\prime N\}$.[72]

The conditions attached to the number-word "dalawa" are also the same conditions attached to the number-word "tatlo" in remark (c). Suffice it to say that similar to remarks (a) and (b), remark (c) is also a rule-laying remark. It also establishes the convention for paying the fare involving three passengers. This is to say that whenever (c) is uttered in *this* context, it simply means that the fare covers more than "isa."

The variations thus in the use of number-words are generally grammar-dependent. Here, it is helpful to be guided by the idea that numbers form a family (Wittgenstein, 2009, p. 36e, [§67]). While the *use* demonstrates the possibilities of using number-words, grammar nevertheless establishes its correct application. It makes the *use* of number-words conform to the rule in a given context or language-game. In so far as grammar lays down the use of words in advance, it also makes clear how one ought to go on.

## CONCLUDING REFLECTION

In the foregoing, I have tried to provide some basis necessary for the determination of meaning. Since similar expressions usually have different meanings, it is necessary to lay down the basis for distinguishing meanings. Using numbers and number-words as examples, I tried to defend the idea that the meaning of expressions may be discerned on the basis of grammar. By focusing on grammar, it becomes evident that meaning is essentially context-dependent—everything lies open to view in grammar. More precisely, it is the *use* that determines the meaning of words. Grammar is hinted by *use*. However, the notion of *use* here ought to be understood in the context of a shared human activity—the background of meaning. This, I surmise, is consistent with Wittgenstein's claim that words have meanings only in the stream of life. Be that as it may, I find interest nonetheless in the question of whether Wittgenstein's account of language contains, aside from grammatical rules, rules of

grammatical salience or whether Wittgenstein's grammatical anatomy of language is not simply laid out in grammar but is also governed by grammatically salient features of language that are embedded in the stream of life. In this final section, I argue, however briefly, that this is possible and that there may be rules of grammatical salience.

The idea that there may be rules of grammatical salience is very *un*-Wittgenstein. One reason bears connection to Wittgenstein's paradox on rules. The paradox states that "no course of action could be determined by a rule" because (a) any course of action can be brought into accord with it and (b) any course of action can also be brought into conflict with it (Wittgenstein, 2009, p. 87e, [§201]). A second reason concerns the labyrinth of language. Wittgenstein remarks that "Language is a labyrinth of paths. You approach from *one* side and know your way about; you approach the same place from another side and no longer know your way about" (Wittgenstein, 2009, p. 88e, [§203]). Still, a third reason arises from the idea that when one follows a rule, one does not choose but rather follows the rule *blindly* (Wittgenstein, 2009, p. 92e, [§219]). On the basis of these reasons, it is thus plausible to suppose that, indeed, there are no rules of grammatical salience. In the larger context of Wittgenstein's account of language, such idea simply does not fit and is, therefore, non-existent. It is my suggestion, nevertheless, that there may be rules of grammatical salience. I begin, however, by asking initially the question "When is a use of language grammatically salient?" The way this question helps is analogous to the way the question "What do you mean?" leads to an understanding of the problem "How do you know your way about?" and, implicitly, "How do you know which way to go?"

Now, one answer to the question "When is a use of language grammatically salient?" is suggested by the very idea that (a) a language-user knows exactly one's way about and (b) a language-user knows exactly which way to go. On the contrary, knowing one's way about and knowing which way to go involve an understanding of what grammatical moves are allowed in a given language-game. Correct grammatical moves are suggested by shared practices—they are embedded in customs, institutions. They constitute a shared form of life. One's shared form of life is the locus of agreement, of judgment, and of consensus. Grammatical moves are thus consensus and agreement on form of life—they are already given. The thesis, then, that there may be rules of grammatical salience is articulated as follows: There are rules of grammatical salience because there are correct grammatical moves in language. Or perhaps the converse may prove better: There are correct grammatical moves in language because there are rules of grammatical salience.

The correctness of a move in language, though it need not be plausible, is often suggested by the agreement that it is such a correct move—that at least two or more language-users share the idea that the move in fact is correct or the move may be said to conform to what is being practiced. As is often the case, there is correct judgment in a move when there is as well an understanding of what is being done with language. When a language-user, for example, understands what is being done with language, one knows as well what appropriate responses may be. Thus, when one says, "manong, 'yung isa po, isa lang," the person to whom the utterance is addressed acknowledges that in fact *this* is what one means—that one gives a fare for only one passenger. More often than not, persons who are exposed to this sort of language-game know exactly the ways to go. The ways to go, one may say, are already laid down in practice— that is, in form of life. Compare it for instance with the following scenarios:

(a) Manong, magkano hanggang UP?
(b) Dadaan po ba ito sa Welcome Rotonda?
(c) Miss, Visayas Avenue?
(d) Ale, alam n'yo po ba kung saan ang CHED?

Even with these remarks, there are appropriate responses. They, too, are also laid down in advance. Whether what is being uttered is suggestive of the idea that one does not know which way to go, an appropriate response may clear the way and thus dispense the nimbus that surrounds one's seeming ignorance on what is being done with language. But how does one know whether the responses are appropriate? Shared human behavior, common customs, and collective practices that constitute the human form of life are the systems of reference by which one knows what responses are appropriate. Implicitly, these are also the systems by which one judges whether a move in language is incorrect.

The point I am suggesting, therefore, is a general one. There are rules of grammatical salience because there are correct grammatical moves in language.

## ACKNOWLEDGMENT

## ENDNOTES

[1] Wittgenstein (2009) remarks that "when a child learns this language, he has to learn the series of number-words a, b, c ... by heart. And he has to learn their use" (p. 9e, [§9]). The same point is also expressed in *The Blue and Brown Books*. Wittgenstein (1958) says that they have to be learned by heart since "there was no feature comparable to this (introduction of numerals) in the learning of language"(p.79, [§2]).

[2] Here, perhaps Wittgenstein's advice is enlightening. Wittgenstein (2001) says:

> You must not try to avoid a philosophical problem by appealing to common sense answer: present it as it arises with most power. You must allow yourself to be dragged into the mire, and get out of it (pp. 108-109).

[3] Instead of paraphrasing what Wittgenstein says in passage 140 of the *Philosophical Investigations* concerning other possibilities, processes, and so on other than the thought we originally have, I raise this question.

[4] Here, I am paraphrasing what Wittgenstein himself says in the middle part of passage 140 in the *Philosophical Investigations*.

[5] The manner of framing the question here reechoes Wittgenstein's style of writing evident in *The Blue and Brown Books*.

[6] Intuitionism in the philosophy of mathematics is varied. Roy Cook (2005) notes that "intuitionism comes in two or three major forms" such as those of L. E. J. Brouwer's, Arend Heyting's, and Michael Dummett's (p. 387).

[7] Peter Simmons (2009) writes:

> [Formalism] is much harder to pin down exactly what formalism is, and what formalists stand for. As a result, it is harder to say what clearly belongs to formalist doctrine and what does not. It is also harder to say what count as considerations for and against it, with one very clear exception (p. 291).

[8] The standard view of logicism may be expressed as follows: "Logicism is the thesis that mathematics is reducible to logic, hence nothing but a part of logic" (Carnap, 1983, p. 41). To say that mathematics is reducible to logic is to say that (a) "mathematical concepts are derivable from the fundamental concepts of logic" and (b) "all valid mathematical sentences in any conceivable domain of any size are derivable from the fundamental statements of logic"(Carnap, 1959, pp. 140-141).

[9] Jaakko Hintikka (2009) notes:

> It was the use of "constructions" in the form of *ekthesis* and auxiliary constructions that made mathematical truths synthetic for Kant. (The force of the term in his philosophy of mathematics is thus reminiscent of the meaning of "synthetic" in synthetic geometry.) In contrast, logical truths were for Kant based on the law of contradiction and hence analytic. (p. 275)

[10] Frege (1960b) asserts, however, later in *The Foundations*, that arithmetical truths are analytic:

> From all the preceding it thus emerged as a very probable conclusion that the truths of arithmetic are analytic and a priori; and we achieved an improvement on the view of Kant (pp. 118-119, [§109]).

[11] In *The Basic Laws of Arithmetic*, Frege (1964) reiterates the fundamental idea that arithmetic is a branch of logic and that "the simple laws of numbers may be derived from logical means alone" without appealing to either experience or intuition (p. 29 [§0]).

[12] Jaakko Hintikka (2009) however, explains:

> [...] when Frege first conceived the program of logicism, the development of modern logic had not yielded a system of logic which he could use as a target of a reduction of mathematics to logic. Hence he had to create such a logic himself (p. 276).

[13] In *The Basic Laws of Arithmetic*, Frege (1964) provides some principles of definition (p. 90 [§33]).

[14] Michael Dummett (1991) phrases this, however, as: "The content of an ascription of number consists in predicating something of a concept (p. 88)" instead of the usual "the content of a statement of number is an assertion about a concept."

[15] Concepts for Frege (1960a) are species of function expressions, whose values are truth-values. They are predicate referents of a function of an argument. I thank the anonymous referee for pointing this out.

[16] This allows Frege (1960b) to introduce numbers as proper names. He says:

> When we have thus acquired a means of arriving at a determinate number and of recognizing it again as the same, we can assign it a number word as its proper name (p.73, [§62]).

[17] Here, I share Stewart Shapiro's (2000) reading of Frege's treatment on equinumerosity (p. 109).

[18] Jaakko Hintikka (2009) similarly notes: "Frege's insight was that the notion of the equinumerosity (equicardinality) of two sets can be characterized purely logically" (p. 276).

[19] William Demopoulus and Peter Clark (2005) explain that Frege introduced this "cardinality operator" "in order to pass from the analysis of numerical properties to the numbers" (p. 134).

[21] G. E. Anscombe (1996) illustrates a more detailed explanation of formal concepts (pp. 123-124).

[22] For the early Wittgenstein, names only symbolize the "combinatorial possibilities of objects." They are expressed through symbolisms as dummy names for objects. I thank the anonymous referee for clarifying this point.

23 Wittgenstein (1975) writes: "[A]nd if we say numbers are structures we mean that they must always be of a kind with what we use to represent them. I mean: numbers are what I represent in my language by number schemata" (p. 129, [§107]).

24 Frege claims that for any proposition 'ɸ( )', the blank may be filled with any proper name— in this case, the empty place can be filled with a numeral. G. E. Anscombe (1996) explains, thus, that for Frege, "*proper names* include ordinary proper names, clauses in sentences, definite descriptions, and *numerals*" (p. 124). Severin Schroeder (2006), in the same way, explains that in Wittgenstein's view "the only correct use of a formal concept is such that it can be expressed by a bound variable" (p. 90).

25 G. E. Moore (1955) recasts Wittgenstein's lectures on infinity and cardinal numbers. He explains that although the latter does not point out that there is a mistake in treating infinity as logical product or that it can be possibly enumerated, he nonetheless demonstrates that there is a difference in meaning when one says "'there are an infinite number of shades of grey between black and white', we 'mean something entirely different' from what we mean by *e.g.*, 'I see three colours in this room', because, whereas the latter proposition can be verified by counting, the former cannot. He said that 'There are an infinite number' does not give an answer to the question How many are there? Whereas 'There are three' does give an answer to this question" (p. 4).

26 If one were to ask "How do you know that '36' is the next number?", a typical Wittgensteinian response might be "Because, that is the way I calculate" (Wittgenstein, 1974, p. 243).

27 This point is also expressed in *Philosophical Remarks*. Wittgenstein (1975) writes: "I want to say that numbers can only be defined from propositional forms, independently of the question which propositions are true or false" (p. 125, [§102]).

28 Here, I paraphrase Wittgenstein's (1956) remarks in *Remarks on the Foundations of Mathematics* (p. 274, [V, §16]).

29 This is how far we can go because we cannot guess how a word functions. "[We] have to *look* at its application and learn from that" (Wittgenstein, 2009, p. 116e [§340).

30 Paul Grice (1989), however, also introduces some models of conversational implicature pertinent to his account of meaning. See his "Some models for Implicature" in *Studies in the Way of Words*.

31 Austin (1962), however, distinguishes three types of performatives: perlocutionary, illocutionary and locutionary acts. See Lecture VIII of his *How to Do Things with Words*.

32 Barry Stroud (1996) explains that Wittgenstein's concern here is the distinction between meaningful utterances "from those which are dead or mean nothing" (p. 300). He remarks that "those sounds or marks are meaningful which have a distinctive role or use in a system of signs like a human language" (p. 300).

33 Consider for example the role of the *context of use* in what Wittgenstein (1969) says in the following passages:

> "I know that that's a tree." Why does it strike me as if I did not understand the sentence? though it is after all an extremely simple sentence of the most

ordinary kind? It is as if I could not focus my mind on any meaning. Simply because I don't look for the focus where the meaning is. As soon as I think of an everyday use of the sentence instead of a philosophical one, its meaning becomes clear and ordinary. (p. 348e, [§347])

Just as the words "I am here" have a meaning only in certain contexts, and not when I say them to someone who is sitting in front of me and sees me dearly, and not because they are superfluous, but because their meaning is not *determined* by the situation, yet stands in need of such determination. (p. 348, [§348] italics in the original)

[34] Zosimo Lee (2001), for example, succinctly writes:

A key notion is the concept of *difference* including the distinction between what are the same and what are different. Semantic contrasts are one way through which we learn what distinctions are possible. Some word-pairs are polar opposites; some indicate nuances and shades of meaning. Different language-games — activities using language — are made possible because of the distinctions embedded in language. The same expression can mean different things because the expression is being *used* in different ways. (p. 51, italics in the original)

[35] Peter Milne (1986) explains that Frege's adoption of the context principle is necessitated by the demand to determine whether a given sign in a sentence stands for its content or whether it stands for itself. Frege eventually rejects this in favor of his more superior theory of sense and reference.

[36] James Conant (1998) notes that Wittgenstein extended Frege's context principle "not only to words but to sentences as well and their role within the context of circumstance of significant use" (p. 233).

[37] Quoting Michael Dummett, Oswald Hanfling (1980) issues a caveat: "The utterance of a sentence does not require a particular context to give it point, but is governed by a general convention – at least in certain types of situation – that in uttering them we are understood as saying that their reference is truth" (Hanfling, p. 197).

[38] Katherine J. Morris (1994), however, offers an alternative reading for the context principle. Against Baker and Hacker for instance, she claims that Wittgenstein's recurrent uses of questions and qualifications (e.g., "here we might say," "here is one possibility") ought to be taken as *modal expressions.* She says for example:

Wittgenstein's qualifiers and questions are playing an important modal role here: that they are not a mere matter of style, but are part of the content of what he calls the description of the use of our words (p. 301).

[39] Take for example Wittgenstein's reminder concerning variety of uses in his *Lectures on the Foundations of Mathematics.* Wittgenstein (1976) says:

An expression has any amount of uses. How, if I tell you a word, can you have the use in your mind in an instant? You don't. You may have in your mind a

certain picture or pictures, and a piece of the application, a representative piece. The rest can come if you like (p. 20).

[40] The idea that the meaning of a word is determined by its use is also shared for example by Peter Strawson and J. L. Evans, among others. Consider for example the following quotes:

> To give the meaning of an expression . . . is to give general directions for its use to refer to or mention particular objects or persons; to give the meaning of a sentence is to give general directions for its use in making true or false assertions (Strawson 1950, p. 327, Strawson's italics).

> ... the meaning of a word is simply the rules which govern its use, and to ask for its meaning is to ask for the rules. Any sound or mark can acquire meaning provided that rules are given, whether explicitly in definitions or implicitly by usage, determining its correct employment (Evans 1953, p. 9).

[41] Wittgenstein (2009) though often speaks of the meaning of an expression as (i) being determined by its use, (ii) an explanation of the meaning of a word (p. 158e, [§560]), and (iii) a correlate of the understanding – "But we understand the meaning of a word when we hear or say it; we grasp the meaning at a stroke, and what we grasp in this way is surely something different from the 'use' which is extended in time" (p. 593, [§138]). See also P. M. S. Hacker's (1996) summary (p. 125f).

[42] Severin Schroeder (2006) explores and discusses several objections to the slogan "meaning is use" (pp. 168-180).

[43] Wittgenstein (2009) makes reference to this opposition in §2 of *Philosophical Investigations*. Marie McGinn (1997), on the other hand, explains this opposition in detail (p. 47f).

[44] Here, I follow William P. Alston's (1963b) reading. I think this is what Wittgenstein is working against when he introduced the variety of uses that words have. The same problems were also discussed by Gilbert Harman (1968).

[45] This reading is an influence of Raziel Abelson (1957). Also, see Newton Garver's (1965) discussion on use and mention.

[46] In another important work, J. L. Evans (1961) offers a more elaborate discussion on the role of use in ascertaining the meaning of words or of sentences. Evans writes:

> If, then, we transpose the concept of meaning to the sphere of sentences, what are we to say of meaning in relation to words? If "having meaning" in the sense of "being meaningful" has to do exclusively with sentences, is there any sense of the term "meaning" in which it can be applied to words?

> It is precisely at this point that one feels inclined to introduce the notion of "use," and to say that what was formerly intended in talking about the meaning of words can be translated in a much less misleading way, and in a logically much more appropriate way, into the language of "use". (pp. 256-257)

[47] Jerrold Katz and Jerry Fodor (1963) offer an elaborate characterization of the semantic theory. Similarly, John Searle (1980) also has an interesting discussion on some of the problems associated with the traditional semantic theory started by Frege.

[48] Hacker and Baker (1985), however, argue that this remark, if taken out of context, may lead to "a variety of conjectural interpretations" (p. 239). See Hacker's discussions (pp. 239f). E. F. Thompkins (1990), on the other hand, argues that the term *form of life* is non-Wittgensteinian. Thompkins (1990) remarks that *Lebensform* is Wittgenstein's word and not *form of life*. He argues that "it is legitimate to discuss what he means by *form of life* only if the rules of the language-game played with it correlate adequately with those of the game that he plays with *Lebensform*" (p. 181). To some extent, his basis seems to stem from Wittgenstein's teaching on the "impreciseness of meaning" as he himself puts it and Quine's (1960) thesis on indeterminacy of translation (pp. 26-30, [§7]). One might argue, however, that it is possible to interpret a completely unknown language (if interpretation is a form of translation as well) on the basis of "shared human behavior." (Wittgenstein, 2009, p. 88e, [§206])

[49] In §655-656 of the *Philosophical Investigations*, for instance, Wittgenstein (2009, p. 175e) issues a caveat that what is important is the taking account of the language-game being played and not its explanation. One has to look on the language-game as the *primary thing*. If one knows exactly what language-game is being played, then, one knows as well which way to go or one finds out one's way about. And to do this requires that one pay attention to the *actual usage* of the language. One must "look and see" how the language itself is spoken.

[50] Wittgenstein (2009) asks the question: "But how is this sentence applied – that is, in our everyday language? For I got it from *there*, and nowhere else" (p. 57e). Similarly, in *The Big Typescript*, Wittgenstein (2005) explicitly claims, "For when I speak about language – word, sentence, etc. – I have to speak in everyday language. – But is there any other?" (p. 58e).

[51] The idea here is borrowed from Rush Rhees (1959-1960).

[52] Zosimo Lee (2001), for example, remarks that "perhaps one has to be exposed to or acquainted with a lot or even the whole of language (as well as to have paid attention to the features of language) to be able to see the parts and notice how the parts hang together" (p. 51). Being exposed to or acquainted with language also means being able to have an overview of how words are used and how such uses are taught—thereby circumscribing what may be possibly grasped.

[53] Hacker and Baker (1985) quote Wittgenstein:

> I want to say: it is a feature of our language that it springs up // it grows // out of the foundations of forms of life, regular actions // that it springs up from the soil of firm forms of life, regular forms of actions (p. 242).

[54] It is tempting to consider that these standards of correctness are similar to what Gershon Weiler (1967) calls "*fait accompli* of established rules of use" (p. 427). Weiler, however, does not share the idea that the meaning of words may be possibly understood in terms of how they are used. He writes:

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In order to learn the use of these expressions correctly, they must already have a correct use. And if, further, we hold that their use is their meaning, then it follows that they must already have their specific meaning. Consequently, their meaning cannot be derived from or explained by referring to our learning the language. If the use of language is rule-following behaviour then the justification of the rules involved cannot be the learning of the rule-following behaviour itself. (Weiler, 1967, p. 426)

Alice Crary (2000) on the other hand argues that it is not right to view Wittgenstein as advocating the claim that use fixes or determines the meaning of a word (p. 142, n3). She points out that Wittgenstein never did make such a claim.

[55] In certain areas in Nueva Ecija, the remark "Anak, gulatin mo nga ang tulya" amounts to an order. To say "gulatin ang tulya" is to say that one must boil the tulya. I learned what this remark means through Nana Juling.

[56] One perhaps may ask, *a la* Wittgenstein, "[I]s the word ever actually used in this way in the language in which it is at home?" (Wittgenstein, 2009, p. 53e, [§116]) If not, one needs to bring the word back to its everyday use. Otherwise, such attempt will only lead to the idling of language (Wittgenstein, 2009, p. 56e, [§132]).

[57] The idea that there is a hidden logical syntax of language is central, for example, in the early Wittgenstein. This hidden isomorphism is brought about by understanding the forms and possibilities of atomic facts that constitute the fixed order of the world (Wittgenstein, 1955, p. 35, [2.021–2.023]). Understanding the forms and possibilities of atomic facts is part of the requirement of logic. Again, he says, "The limits of my language mean the limits of my world. Logic fills the world: the limits of the world are also its limits" (p. 149, [5.6–5.61]). On this view, one is thus held captive of the crystalline purity of logic thereby indicating that the world of atomic facts cannot be separated from the perspective of language. Alexander Maslow (1961), for instance, explains that "an investigation of the formal aspect of language is at the same time also an investigation of the formal aspects of the world" (p. 2). Consequently, this yields the idea that "to give the essence of propositions means to give the essence of all description, therefore the essence of the world" (Wittgenstein, 1955, p. 127, [5.4711]). Similarly, in *Philosophical Investigations*, Wittgenstein (2009) recasts this view. He notes for instance that "thought, language, now appear to us as the unique correlate, picture, of the world" (p. 49e, [§96]). The world has an *a priori* order fixed by logic— the essence of thinking (p. 49e, §97). The language of logic is the ideal that *must* occur in reality (p. 50e, [§101]). The picture and the *a priori* order of the world— are requirements of logic (p. 51, [§107]). Wittgenstein (2009) nonetheless repudiates this view. For one, he takes notice that "the language-game in which they are to be applied is missing?" (p. 49e, [§96]). For another, he takes notice as well that this view is "unwalkable"— it is slippery since it has no friction. To be able to walk requires friction so "back to the rough ground" (p. 51e, [§107]).

[58] We may of course consider other forms of expressions quite similar to this one. We may, instead of using number-words use the following forms of expressions:

| | |
|---|---|
| Seven is a dog. | April is summer. |
| Tuesday is female. | May is hot. |
| Three is five. | June is fine. |

These expressions, like the ones we have, are ambiguous. While we are familiar with what the component words mean, we are left with the question of whether what we know about them suffices to ascertain their meaning. It seems to me that we need to look and see whether they are in fact the sort of expressions we think they are. Given sufficient contextual knowledge, it is correct to suppose that they are not used in the same way that they usually are. For example, Seven, Tuesday and Three do not function as number-words here. The same may be said of April, May, and June—that they are not month-words. The problem, however, arises in the absence of contextual knowledge. In the philosophy of language, expressions like these may be likened to the challenge of metaphor. Again, while we may use day-word, month-word, and number-word to say and mean something and take them as obvious examples of using them in different ways, it is still possible that people will take it to mean something else. I thank the anonymous referee for raising this issue.

[59] Wittgenstein (2005) explains:

> Grammatical rules, as they currently exist, are rules for the use of words. Even if we transgress them we can still use words meaningfully. Then what do they exist for? To make language-use as a whole uniform? (Say for aesthetic reasons?) To make possible the use of language as a social institution? And thus – like a set of traffic rules – to prevent a collision? (But what concern is it of ours if that happens?) The collision that mustn't come about must be the collision that cannot come about! That is to say, without grammar it isn't a bad language, but no language. (p. 147e)

In the same way, Wittgenstein (2009) says that rules stand like signposts (p. 44e, [§85]). They establish order (p. 45e, [§87]). One learns to react to a given sign because it is the result of one's training thereby suggesting the emergence of custom for the use of language (p. 86e, [§198]).

[60] P. M. S. Hacker (2012) has written an extensive discussion on this.

[61] In *The Big Typescripts*, Wittgenstein equates the rules of language or grammar with a set of traffic rules thereby suggesting that they are of practical importance to us. The rules do not tell us that we always have to use this or that word in such and such a way. Instead, the rules are there simply to describe the workings of our language for practical purposes. Wittgenstein (2005) writes:

> … one does have to admit that the grammar of a language as a generally recognized institution is a set of traffic rules. For it isn't essential to language as such that we always use the word "table" this way; rather, this is just a practical arrangement, as it were (p. 147e).

[62] Wittgenstein (1967) explains in *Zettel* that there is a certain temptation to justify the rules of grammar by verification or by pointing to whatever verifies it. He remarks:

> One is tempted to justify rules of grammar by sentences like "But there really are four primary colours". And the saying that the rules of grammar are arbitrary is directed against the possibility of this justification, which is

constructed on the model of justifying a sentence by pointing to what verifies it (p. 61e, [§331]).

In a later passage, he suggests how we come to have color-grammar: He says:

> We have a colour system as we have a number system. Do the systems reside in our nature or in the nature of things? How are we to put it? – *Not* in the nature of numbers or colours (p. 65e, [§357]).

[63] This reading takes its form from Hacker (2009) but with modifications.

[64] To some extent, this suggests to what extent grammar rests on convention (Wittgenstein, 1974, p. 190, [§138]). Part of the convention on numbers is how one knows "isa," "dalawa," and so on, i.e., how one learns it by acquiring language. Thus, when one says "lima," it simply shows the way "lima" is understood—the number-word "lima" simply means this "number" or its symbol "5" or ✋.

[65] Although it is possible to invent new language or introduce new forms of descriptions, its requirement is too farfetched. Wittgenstein (2009), for example, requires that it be done in exactly the same way that one has learned a language, i.e., it must also be embedded in a form of life— "to imagine a language is to imagine a form of life" (p. 11e, [§19]). To pursue the analogy, one may say, "to invent, propose or introduce a new language is likewise to invent, propose or introduce a form of life.

[66] This reading is a modification made from Baker and Hacker (1985, p. 236).

[67] Here, I paraphrase some passages from *Philosophical Investigations* (Wittgenstein, 2009, p. 29e, [§50]).

[68] Wittgenstein (2009, p. 11e, [§19]) of course has an interesting remark on this shortening of sentences. He raises the question "But why shouldn't I conversely have called the sentence ['yung isang pasahe po, para sa isang pasahero] a lengthening of the sentence ['yung isa po, isa lang]?"

[69] This suggests that whenever one says N, "'yung isa po, isa lang," as a shortened form of "bayad po," in a given context F where there exists as well other forms of expressions having the same meaning, any utterance within the context thereby suggests one and only one meaning, which may be reduced to N. In other words, whatever form of representation is uttered in F, it means "bayad po." This thought formulation is patterned after Brian Loar's (1976).

[70] The question of whether one follows a rule or whether one's act is in accord with a rule is a difficult problem. It is tempting to suppose that one actually follows a rule whenever one says or does something, which is more often the case. But, as Wittgenstein (2009) suggests, no one follows a rule privately (p. 87e, [§202]). To follow a rule is a matter of practice. It requires an agreement (a) that it accords with a rule, (b) that it constitutes a practice, and (c) that this act or remark of sort is its correct application. Similarly, something is said to constitute a practice or that something accords with a rule only if one is directed by a compass or signpost regularly— there is regularity in the use of the

said signpost whatever it may be (Wittgenstein, 2009, p. 86e, [§198]). This regularity, so to speak, is what relates a given rule and a given act. Hence, the agreement consists in form of life.

[71] Zosimo Lee (2001) briefly explains in an appended note the distinction between constitutive and regulative rules.

[72] This is suggestive of the idea that N may accommodate sentences with more than one meaning provided that the reduction of N to N = $\{\grave{U}^0N = N, \grave{U}^{0+1}N = N, \grave{U}^{0+1+1}N = N\}$ holds true in F. If Ni, Nii, Niii and Niv mean N, then, under context F, $N^{+0+1+1}$ also means N without necessarily being identical with N itself.

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